2014 International Conference on Unmanned Aircraft Systems (ICUAS)
May 27-30, 2014. Orlando, FL, USA

Neural Dynamic Surface Hypersonic Flight Control Using Minimal-Learning-Parameter Technique

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Abstract— **This paper presents dynamic surface control for longitudinal dynamics of a generic hypersonic flight vehicle in presence of unknown dynamics. For the attitude subsystem, the minimal-learning-parameter technique is combined with dynamic surface design. The uniform ultimate boundedness stability is guaranteed via Small-gain Theorem. The singularity problem is removed and the simpler adaptive algorithm is easy to implement since the online updating computation burden is greatly reduced. Simulation result shows the feasibility of the proposed method.**

I. Introduction

Hypersonic flight vehicles (HFVs) provide a promising and cost-effective technology in need of commercial as well as military applications for space access and prompt global reach capabilities. However, stemming from the highly coupled and nonlinear nature of its dynamic behavior, the design of flight control systems for hypersonic vehicles poses many challenges. Different from traditional flight vehicles, due to the high mach numbers and high altitude, the flight control is extremely sensitive to changes in atmospheric conditions as well as physical and aerodynamic parameters.

Adaptive control is widely studied for the dynamics since with large envelope the control system has to adapt to the environment. High gain observer based neural control [1] are implemented on hypersonic flight dynamics. In [2], robust flight control schemes are proposed for the hypersonic vehicle with uncertainties and external disturbances using disturbance observer [3], [4] and neural networks (NNs). In [5], the longitudinal dynamics of the vehicle are studied using time-scale decomposition to reduce the complexity of T-S modeling. Similar design is studied in [6] by considering possible sensor/actuator failures.

Back-stepping design [7], [8], [9] is an explicit tool for systematic nonlinear design and it is widely applied on control of nonlinear systems [10], [11]. There exist many results on hypersonic flight control such as [12], [13]. The hypersonic flight dynamics are written in the linearly parameterized form [14] and then robust adaptive dynamic inversion with back-stepping arguments is conducted in case of actuator constraint. Improved DSC design [15] is studied by taking the constraints on system states with adaptive gain design. Since the dynamics cannot be known exactly, intelligent control with universal approximation [16], [17], [10], [18] is widely studied. In [19], [20], [21], the neural back-stepping discrete design is studied on the longitudinal dynamics of HFV.

In this paper, we study the NN based adaptive DSC control for the longitudinal motion of an airbreathing hypersonic vehicle [22]. To avoid the singularity and the computation burden, we focus on direct dynamic surface control with minimal learning parameters. To reduce the computation burden, we took the similar idea from [23], [24] where the “minimal learning parameter” technique is proposed in view of the estimation of the norm information of NN weights [25], [26].

This paper is organized as follows. Section II describes the longitudinal dynamics of a generic hypersonic flight vehicle. Sections III and IV present the adaptive controller design and the stability analysis. The simulation result is included in Section V. Section VI presents several comments and final remarks.

II. Hypersonic Air Vehicle Model

The control-oriented model of the longitudinal dynamics of a generic HFV considered in this study is given by [22]. This model is comprised of five state variables $X_A=[V,h,\alpha,\gamma,q]^T$ and two control inputs $U=[\delta_e,\Phi]^T$.

$$\begin{aligned}
\dot{V} &= \frac{T\cos\alpha-D}{m}-g\sin(\theta_p-\alpha) && (1)\\
\dot{h} &= V\sin\gamma && (2)\\
\dot{\gamma} &= \frac{L+T\sin\alpha}{mV}-\frac{g\cos\gamma}{V} && (3)\\
\dot{\alpha} &= q-\dot{\gamma} && (4)\\
\dot{q} &= \frac{M}{I_{yy}} && (5)
\end{aligned}$$

For the altitude subsystem, the tracking error of the altitude is defined as $\tilde{h}=h-h_r$ and the flight path command is chosen as

$$\gamma_d=\arcsin\left[\frac{-k_h(h-h_r)-k_I\int(h-h_r)\,dt+\dot{h}_r}{V}\right] \quad (6)$$

if $k_h>0$ and $k_I>0$ are chosen and the flight-path angle (FPA) is controlled to follow γ_d, the altitude tracking error is regulated to zero exponentially [1].

*This work was supported by National Science Foundation of China (Grants No: 61304098, 61134004, 61273153), Natural Science Basic Research Plan in Shaanxi Province (Grant No: 2014JQ8326) and NWPU Basic Research Funding (Grant No: JC20120236).

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Define $X=[x_1,x_2,x_3]^{\mathrm{T}}$, $x_1=\gamma$, $x_2=\theta_p$, $x_3=q$ where $\theta_p=\alpha+\gamma$. The attitude subsystem can be derived as

$$\begin{aligned}\dot{x}_1 &= f_1(x_1)+g_2(x_1)x_2\\ \dot{x}_2 &= x_3\\ \dot{x}_3 &= f_3(x_1,x_2,x_3)+g_3(x_1)\delta_e\\ y &= x_1\end{aligned}\tag{7}$$

where

$$\begin{aligned}f_1 &= \frac{P_1\varphi_0\sin x_d}{V}+\frac{SV\left(C_L^0-x_1\right)}{2m}-\frac{g\cos x_1}{V}\\ g_1 &= \frac{SV}{2m}\\ f_3 &= P_2\varphi_0+P_3\varphi_1V^2\\ g_3 &= P_4V^2\end{aligned}$$

with

$$\begin{aligned}P_1 &= \frac{1}{m}\left[C_T^{x_d^3},C_T^{x_d^2},C_T^{x_d},C_T^0\right]\\ P_2 &= \frac{z_T}{I_{yy}}\left[C_T^{x_d^3},C_T^{x_d^2},C_T^{x_d},C_T^0\right]\\ P_3 &= \frac{\rho S\bar{c}}{2I_{yy}}\left[C_{M,x_d}^{x_d^2},C_{M,x_d}^{x_d},C_{M,x_d}^0\right]\\ P_4 &= \frac{\rho S\bar{c}c_e}{2I_{yy}}\end{aligned}$$

For the velocity subsystem, we have

$$\begin{aligned}\dot{V} &= f_V(V,x_1,x_2)+g_V(x_1,x_2)\Phi\\ y_V &= V\end{aligned}\tag{8}$$

with $f_V=\beta_b\varphi_0\cos x_d-P_0\varphi_1V^2-g\sin x_1$, $g_V=\beta_a\varphi_0\cos x_d$, $P_0=\frac{\rho S}{2m}\left[C_D^{x_d^2},C_D^{x_d},C_D^0\right]$, $\varphi_0=\left[x_d^3,x_d^2,x_d^1,1\right]^T$, $\varphi_1=\left[x_d^2,x_d^1,1\right]^T$, $\beta_a=\frac{1}{m}[\beta_1,\beta_3,\beta_5,\beta_7]$, $\beta_b=\frac{1}{m}[\beta_2,\beta_4,\beta_6,\beta_8]$, $x_d=\alpha$.

Assumption 1: There exist constants $b_{\min}$ and $b_{\max}$ such that $b_{\max}\geq|g_i|\geq b_{\min}>0$, $i=1,3,V$.

The goal pursued in this study is to design a dynamic controller δ_e and Φ to steer system altitude and velocity from a given set of initial values to desired trim conditions with the tracking reference h_r and V_r.

Lemma 1: [27] For any given real continuous function $f(x)$ with $f(0)=0$, if the continuous function separation technique and the RBF NN approximation technique are used, then $f(x)$ can be denoted as

$$f(x)=\bar{\theta}(x)Ax\tag{9}$$

where $\bar{\theta}(X)=[1,\theta_1(X),\theta_2(X),\cdots,\theta_l(X)]^T$, $A^T=\left[\varepsilon,\omega^T\right]$, $\varepsilon^T=[\varepsilon_1,\varepsilon_2,\cdots,\varepsilon_n]$ is a vector of the approximation errors, and

$$\omega=\begin{bmatrix}\omega_{11}^* & \omega_{12}^* & \cdots & \omega_{1n}^*\\ \omega_{21}^* & \omega_{22}^* & \cdots & \omega_{2n}^*\\ \vdots & \vdots & \cdots & \vdots\\ \omega_{l1}^* & \omega_{l2}^* & \cdots & \omega_{ln}^*\end{bmatrix}\tag{10}$$

is a weight matrix.

III. Controller Design for Attitude Subsystem

Attitude subsystem (2)-(5) is represented by strict-feedback formulation in system (7) that is suitable for the DSC-based design procedure.

Step 1 The FPA surface error is defined as $z_1=x_1-x_{1d}$. Taking the derivative of z_1 and using (7), we have

$$\begin{aligned}\dot{z}_1 &= \dot{x}_1-\dot{x}_{1d}\\ &= f_1(x_1)+g_1(x_1)x_2-\dot{x}_{1d}\end{aligned}\tag{11}$$

According to Lemma 1, we have

$$\begin{aligned}f_1(x_1) &= \bar{\theta}_1(x_1)A_1x_1\\ &= \bar{\theta}_1(x_1)A_1z_1+\bar{\theta}_1(x_1)A_1x_{1d}\end{aligned}\tag{12}$$

Let $C_{\theta 1}=\|A_1\|$, the normalized term $A_1^m=\frac{A_1}{\|A_1\|}=\frac{A_1}{C_{\theta 1}}$ and $\omega_1=A_1^m z_1$, $\nu_1=\bar{\theta}_1(x_1)A_1x_{1d}$.

Define $z_2=x_2-\alpha_2$. So we have

$$\begin{aligned}\dot{z}_1 &= \dot{x}_1-\dot{x}_{1d}\\ &= g_1(x_1)x_2+\bar{\theta}_1(x_1)C_{\theta 1}\omega_1+\nu_1-\dot{x}_{1d}\end{aligned}\tag{13}$$

It is known that

$$\left\|\bar{\theta}_1(x_1)A_1x_{1d}\right\|\leq b_{\min}\lambda_{11}\varphi_1(x_1)\tag{14}$$

where $\lambda_{11}=b_{\min}^{-1}\|A_1x_{1d}\|$ and $\varphi_1(x_1)=1+\|\theta_1\|$.

Define α_{2c} as the virtual controller and let it pass the first-order filter α_2.

$$\tau_2\dot{\alpha}_2+\alpha_2=\alpha_{2c},\alpha_{2c}(0)=\alpha_2(0)\tag{15}$$

Define $y_2=\alpha_2-\alpha_{2c}$, then we have

$$\dot{y}_2=\dot{\alpha}_2-\dot{\alpha}_{2c}=-\frac{y_2}{\tau_2}-\dot{\alpha}_{2c}\tag{16}$$

Referred to [25], [28], $\dot{\alpha}_{2c}$ is continuous function and has a maximum value M_2.

Define $\lambda_{12}=b_{\min}^{-1}C_{\theta 1}^2$ and $\tilde{\lambda}_{11}=\lambda_{11}-\hat{\lambda}_{11}$, $\tilde{\lambda}_{12}=\lambda_{12}-\hat{\lambda}_{12}$ where $\hat{\lambda}_{11}$ and $\hat{\lambda}_{12}$ are the estimation accordingly.

Consider the following Lyapunov function

$$V_1=\frac{1}{2}\left(z_1^2+y_2^2+b_{\min}\tau_{11}^{-1}\tilde{\lambda}_{11}^2+b_{\min}\tau_{12}^{-1}\tilde{\lambda}_{12}^2\right)\tag{17}$$

where τ_{11} and τ_{12} are positive design constants.

Calculate the derivative of V_1

$$\begin{aligned}\dot{V}_1 &= z_1\dot{z}_1+y_2\dot{y}_2-b_{\min}\tau_{11}^{-1}\tilde{\lambda}_{11}\hat{\lambda}_{11}-b_{\min}\tau_{12}^{-1}\tilde{\lambda}_{12}\hat{\lambda}_{12}\\ &= z_1\left[g_1(x_1)x_2+\bar{\theta}_1(x_1)C_{\theta 1}\omega_1+\nu_1-\dot{x}_{1d}\right]\\ &+ y_2\left(-\frac{y_2}{\tau_2}-\dot{\alpha}_{2c}\right)-b_{\min}\tau_{11}^{-1}\tilde{\lambda}_{11}\dot{\hat{\lambda}}_{11}-b_{\min}\tau_{12}^{-1}\tilde{\lambda}_{12}\dot{\hat{\lambda}}_{12}\\ &= z_1\begin{bmatrix}g_1(x_1)z_2+g_1(x_1)\alpha_{2c}+g_1(x_1)y_2\\ +\bar{\theta}_1(x_1)C_{\theta 1}\omega_1+\nu_1-\dot{x}_{1d}\end{bmatrix}\\ &+ y_2\left(-\frac{y_2}{\tau_2}-\dot{\alpha}_{2c}\right)\\ &- b_{\min}\tau_{11}^{-1}\tilde{\lambda}_{11}\dot{\hat{\lambda}}_{11}-b_{\min}\tau_{12}^{-1}\tilde{\lambda}_{12}\dot{\hat{\lambda}}_{12}\end{aligned}\tag{18}$$

It is worth noticing that

$$\begin{aligned}
C_{\theta1}\bar{\theta}_1(x_1)\omega_1 z_1 &= C_{\theta1}\bar{\theta}_1(x_1)\omega_1 z_1 - \gamma_1^2\omega_1^T\omega_1 + \gamma_1^2\omega_1^T\omega_1 \\
&= -\gamma_1^2\left(\omega_1 - \frac{C_{\theta1}\bar{\theta}_1(x_1)z_1}{2\gamma_1^2}\right)^2 \\
&+ \frac{C_{\theta1}^2\bar{\theta}_1^T(x_1)\bar{\theta}_1(x_1)z_1^2}{4\gamma_1^2} + \gamma_1^2\omega_1^T\omega_1 \\
&\le \frac{C_{\theta1}^2\bar{\theta}_1^T(x_1)\bar{\theta}_1(x_1)z_1^2}{4\gamma_1^2} + \gamma_1^2\omega_1^T\omega_1 \\
&\le b_{\min}\frac{\hat{\lambda}_{12}}{4\gamma_1^2}\bar{\theta}_1^T(x_1)\bar{\theta}_1(x_1)z_1^2 \\
&+ b_{\min}\frac{\tilde{\lambda}_{12}}{4\gamma_1^2}\bar{\theta}_1^T(x_1)\bar{\theta}_1(x_1)z_1^2 + \gamma_1^2\omega_1^T\omega_1 \\
z_1 v_1 &\le b_{\min}\hat{\lambda}_{11}\varphi_1(x_1)\|z_1\| + b_{\min}\tilde{\lambda}_{11}\varphi_1(x_1)\|z_1\| \\
&\le g_1\hat{\lambda}_{11}\varphi_1(x_1)\|z_1\| + b_{\min}\tilde{\lambda}_{11}\varphi_1(x_1)\|z_1\| \\
g_1(x_1)z_1 z_2 &\le b_{\max}z_1^2 + \frac{b_{\max}}{4}z_2^2 \\
g_1(x_1)z_1 y_2 &\le b_{\max}z_1^2 + \frac{b_{\max}}{4}y_2^2 \\
z_1\dot{x}_{1d} &\le \frac{1}{4}z_1^2 + \dot{x}_{1d}^2 \le \frac{1}{4}z_1^2 + M_1{}^2 \\
y_2\dot{\alpha}_{2c} &\le \frac{1}{2}\left(\rho_2 y_2^2 + \rho_2^{-1}M_2^2\right)
\end{aligned}$$

Then we have

$$\begin{aligned}
\dot{V}_1 &= z_1\dot{z}_1 + y_2\dot{y}_2 - b_{\min}\tau_{11}^{-1}\tilde{\lambda}_{11}\hat{\lambda}_{11} - b_{\min}\tau_{12}^{-1}\tilde{\lambda}_{12}\hat{\lambda}_{12} \\
&\le g_1(x_1)z_1\alpha_{2c} + b_{\min}\frac{\hat{\lambda}_{12}}{4\gamma_1^2}\bar{\theta}_1^T(x_1)\bar{\theta}_1(x_1)z_1^2 \\
&+ b_{\min}\frac{\tilde{\lambda}_{12}}{4\gamma_1^2}\bar{\theta}_1^T(x_1)\bar{\theta}_1(x_1)z_1^2 + \gamma_1^2\omega_1^T\omega_1 \\
&+ g_1(x_1)\hat{\lambda}_{11}\varphi_1(x_1)\|z_1\| + b_{\min}\tilde{\lambda}_{11}\varphi_1(x_1)\|z_1\| \\
&+ b_{\max}z_1^2 + \frac{b_{\max}}{4}z_2^2 + b_{\max}z_1^2 + \frac{b_{\max}}{4}y_2^2 \\
&+ \frac{1}{4}z_1^2 + M_1{}^2 - \frac{y_2^2}{\tau_2} + \frac{1}{2}\left(\rho_2 y_2^2 + \rho_2^{-1}M_2^2\right) \\
&- b_{\min}\tau_{11}^{-1}\tilde{\lambda}_{11}\dot{\hat{\lambda}}_{11} - b_{\min}\tau_{12}^{-1}\tilde{\lambda}_{12}\dot{\hat{\lambda}}_{12} \qquad (19)
\end{aligned}$$

The virtual control is proposed as

$$\begin{aligned}
\alpha_{2c} &= -k_1 z_1 + \dot{y}_d - \frac{\hat{\lambda}_{12}}{4\gamma_1^2}\bar{\theta}_1^T(x_1)\bar{\theta}_1(x_1)z_1 \\
&- \hat{\lambda}_{11}\varphi_1(x_1)\tanh\left(\frac{\hat{\lambda}_{11}\varphi_1(x_1)z_1}{\delta_1}\right) \qquad (20)
\end{aligned}$$

where k_1, γ_1 and δ_1 are positive design parameters.

Substituting (20) into (19), then we have

$$\begin{aligned}
\dot{V}_1 &\le -k_1 b_{\min}z_1{}^2 + (b_{\min} - g_1(x_1))\frac{\hat{\lambda}_{12}}{4\gamma_1^2}\bar{\theta}_1^T(x_1)\bar{\theta}_1(x_1)z_1^2 \\
&+ \frac{1}{4}(b_{\max}+1)z_1^2 + (b_{\max}+1)M_1{}^2 \\
&- g_1(x_1)z_1\hat{\lambda}_{11}\varphi_1(x_1)\tanh\left(\frac{\hat{\lambda}_{11}\varphi_1(x_1)z_1}{\delta_1}\right) \\
&+ b_{\min}\frac{\tilde{\lambda}_{12}}{4\gamma_1^2}\bar{\theta}_1^T(x_1)\bar{\theta}_1(x_1)z_1^2 + \gamma_1^2\omega_1^T\omega_1 \\
&+ g_1(x_1)\hat{\lambda}_{11}\varphi_1(x_1)\|z_1\| + b_{\min}\tilde{\lambda}_{11}\varphi_1(x_1)\|z_1\| \\
&+ b_{\max}z_1^2 + \frac{b_{\max}}{4}z_2^2 + b_{\max}z_1^2 \\
&+ \frac{b_{\max}}{4}y_2^2 - \frac{y_2^2}{\tau_2} + \frac{1}{2}\left(\rho_2 y_2^2 + \rho_2^{-1}M_2^2\right) \\
&- b_{\min}\tau_{11}^{-1}\tilde{\lambda}_{11}\dot{\hat{\lambda}}_{11} - b_{\min}\tau_{12}^{-1}\tilde{\lambda}_{12}\dot{\hat{\lambda}}_{12} \qquad (21)
\end{aligned}$$

Define $zz_1 = \hat{\lambda}_{11}\varphi_1(x_1)\|z_1\| - \tanh\left(\frac{\hat{\lambda}_{11}\varphi_1(x_1)z_1}{\delta_1}\right)$.

Since we have the following inequality

$$g_1(x_1)zz_1 \le g_1(x_1)\delta_1 \le b_{\max}\delta_1$$

The adaptive law is proposed as

$$\begin{aligned}
\dot{\hat{\lambda}}_{11} &= \tau_{11}\left[\varphi_1(x_1)\|z_1\| - \delta_{11}\left(\hat{\lambda}_{11} - \lambda_{11}^0\right)\right] \qquad (22) \\
\dot{\hat{\lambda}}_{12} &= \tau_{12}\left[\frac{1}{4\gamma_1^2}\bar{\theta}_1^T(x_1)\bar{\theta}_1(x_1)z_1^2 - \delta_{12}\left(\hat{\lambda}_{12} - \lambda_{12}^0\right)\right] \qquad (23)
\end{aligned}$$

where δ_{11} and δ_{12} are positive design constants, λ_{11}^0 and λ_{12}^0 are initial values of $\hat{\lambda}_{11}$ and $\hat{\lambda}_{12}$ respectively.

The following conclusion can be derived

$$\begin{aligned}
\tilde{\lambda}_{11}\left(\hat{\lambda}_{11} - \lambda_{11}^0\right) &\le -\frac{1}{2}\tilde{\lambda}_{11}^2 + \frac{1}{2}\left(\lambda_{11}^* - \lambda_{11}^0\right)^2 \qquad (24) \\
\tilde{\lambda}_{12}\left(\hat{\lambda}_{12} - \lambda_{12}^0\right) &\le -\frac{1}{2}\tilde{\lambda}_{12}^2 + \frac{1}{2}\left(\lambda_{12}^* - \lambda_{12}^0\right)^2 \qquad (25)
\end{aligned}$$

Finally

$$\begin{aligned}
\dot{V}_1 &\le -\left[k_1 b_{\min} - \frac{1}{4}(b_{\max}+1) - 2b_{\max}\right]z_1^2 \\
&- \left[\frac{1}{\tau_2} - \frac{b_{\max}}{4} - \frac{1}{2}\rho_1\right]y_2^2 \\
&+ \gamma_1^2\omega_1^T\omega_1 + \frac{b_{\max}}{4}z_2^2 \\
&- \frac{1}{2}b_{\min}\delta_{11}\tilde{\lambda}_{11}^2 - \frac{1}{2}b_{\min}\delta_{12}\tilde{\lambda}_{12}^2 + P_1 \\
&\le -\alpha_1 V_1 + \gamma_1^2\omega_1^T\omega_1 + P_1 + \frac{b_{\max}}{4}z_2^2 \qquad (26)
\end{aligned}$$

where

$$\alpha_1 = \min\left\{\begin{array}{l} 2\left(k_1 b_{\min} - \frac{1}{4}(b_{\max}+1) - 2b_{\max}\right), \\ 2\left(\frac{1}{\tau_2} - \frac{b_{\max}}{4} - \frac{1}{2}\rho_1\right), \delta_{11}\tau_{11}, \delta_{12}\tau_{12} \end{array}\right\} \qquad (27)$$

and

$$\begin{aligned} P_1 &= \frac{b_{\min}}{2}\left(\lambda_{11}^* - \lambda_{11}^0\right)^2 + \frac{b_{\min}}{2}\left(\lambda_{12}^* - \lambda_{12}^0\right)^2 \\ &+ \frac{1}{2}\rho_1^{-1}M_2^2 + (b_{\max}+1)M_1^2 + b_{\max}\delta_1 \end{aligned} \tag{28}$$

Step 2 Taking the derivative of z_2 and using (7), we have

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_2 = x_3 - \dot{\alpha}_2 \tag{29}$$

Define $z_3 = x_3 - \alpha_3$ and $\alpha_{3c}$is the virtual controller and let it pass the first-order filter α_3.

$$\tau_3\dot{\alpha}_3 + \alpha_3 = \alpha_{3c}, \alpha_{3c}(0) = \alpha_3(0) \tag{30}$$

Define $y_3 = \alpha_3 - \alpha_{3c}$, then

$$\dot{y}_3 = \dot{\alpha}_3 - \dot{\alpha}_{3c} = -\frac{y_3}{\tau_3} - \dot{\alpha}_{3c} \tag{31}$$

The virtual control is designed as

$$\alpha_{3c} = -k_2z_2 + \dot{\alpha}_2 \tag{32}$$

The derivative of z_2 is calculated

$$\begin{aligned} \dot{z}_2 &= \dot{x}_2 - \dot{\alpha}_2 = z_3 + y_3 + \alpha_{3c} - \dot{\alpha}_2 \\ &= -k_2z_2 + z_3 + y_3 \end{aligned} \tag{33}$$

The Lyapunov function candidate is selected as

$$V_2 = V_1 + \frac{1}{2}\left(z_2^2 + y_3^2\right) \tag{34}$$

The derivative of V_2 is calculated as

$$\dot{V}_2 = \dot{V}_1 + z_2\dot{z}_2 + y_3\dot{y}_3 \tag{35}$$

It is worth noticing that

$$\begin{aligned} z_2\dot{z}_2 &= z_2(-k_2z_2 + z_3 + y_3) \\ &\le -k_2z_2^2 + \frac{1}{2}\rho_2z_2^2 + \frac{1}{2}\rho_2^{-1}z_3^2 + \frac{1}{2}\rho_3z_2^2 + \frac{1}{2}\rho_3^{-1}y_3^2 \\ y_3\dot{y}_3 &= y_3\left(-\frac{y_3}{\tau_3} - \dot{\alpha}_{3c}\right) \\ &\le -\frac{1}{\tau_3}y_3^2 + \frac{1}{2}\rho_4y_3^2 + \frac{1}{2}\rho_4^{-1}M_3^2 \end{aligned} \tag{36}$$

Using the conclusion (26), we have

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + z_2\dot{z}_2 + y_3\dot{y}_3 \\ &\le -\alpha_1V_1 + \gamma_1^2\omega_1^T\omega_1 + P_1 + \frac{b_{\max}}{4}z_2^2 \\ &- k_2z_2^2 + \frac{1}{2}\rho_2z_2^2 + \frac{1}{2}\rho_2^{-1}z_3^2 + \frac{1}{2}\rho_3z_2^2 + \frac{1}{2}\rho_3^{-1}y_3^2 \\ &- \frac{1}{\tau_3}y_3^2 + \frac{1}{2}\rho_4y_3^2 + \frac{1}{2}\rho_4^{-1}M_3^2 \\ &\le -\alpha_1V_1 + \gamma_1^2\omega_1^T\omega_1 - \left(k_2 - \frac{1}{2}\rho_2 - \frac{1}{2}\rho_3 - \frac{b_{\max}}{4}\right)z_2^2 \\ &- \left(\frac{1}{\tau_3} - \frac{1}{2}\rho_3^{-1} - \frac{1}{2}\rho_4\right)y_3^2 + \frac{1}{2}\rho_2^{-1}z_3^2 + P_2 \end{aligned} \tag{37}$$

where $P_2 = P_1 + \frac{1}{2}\rho_4^{-1}M_3^2$.

Step 3 Similarly, taking the derivative of z_3 and using (7), we have

$$\dot{z}_3 = \dot{x}_3 - \dot{\alpha}_3 = f_3(x_1,x_2,x_3) + g_3(x_1)\delta_e - \dot{\alpha}_3 \tag{38}$$

According to Lemma 1, we have

$$\begin{aligned} f_3(x_1,x_2,x_3) &= \bar{\theta}_3(X)A_3X \\ &= \bar{\theta}_3(X)A_3\begin{bmatrix} z_1 + y_d \\ z_2 + \alpha_2 \\ z_3 + \alpha_3 \end{bmatrix} \\ &= C_{\theta 3}\bar{\theta}_3(X)\omega_3 + \nu_3 \end{aligned} \tag{39}$$

where $C_{\theta 3} = \|A_3\|$, the normalized term $A_3^m = \frac{A_3}{\|A_3\|} = \frac{A_3}{C_{\theta 3}}$ and $\omega_3 = A_3^m\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$, $\nu_3 = \bar{\theta}_3(X)A_3(y_d + \alpha_2 + \alpha_3)$.

The Eq.(38) could be derived as

$$\dot{z}_3 = \dot{x}_3 - \dot{\alpha}_3 = C_{\theta 3}\bar{\theta}_3(X)\omega_3 + \nu_3 + g_3(x_1)\delta_e - \dot{\alpha}_3 \tag{40}$$

Similarly we have

$$\|\nu_3\| \le b_{\min}\lambda_{31}\varphi_3(X) \tag{41}$$

where $\lambda_{31} = b_{\min}^{-1}\|y_d + \alpha_2 + \alpha_3\|$ and $\varphi_3(X) = 1 + \|\theta_3\|$ and $\lambda_{32} = b_{\min}^{-1}C_{\theta 3}^2$.

The controller and adaptive law are proposed as

$$\begin{aligned} \delta_e &= k_3z_3 + \dot{\alpha}_3 - \frac{\hat{\lambda}_{32}}{4\gamma_3^2}\bar{\theta}_3^T(X)\bar{\theta}_1(X)z_3 \\ &- \hat{\lambda}_{31}\varphi_3(X)\tanh\left(\frac{\hat{\lambda}_{31}\varphi_3(X)z_3}{\delta_3}\right) \end{aligned} \tag{42}$$

$$\dot{\hat{\lambda}}_{31} = \tau_{31}\left[\varphi_3(X)\|z_3\| - \delta_{31}\left(\hat{\lambda}_{31} - \lambda_{31}^0\right)\right] \tag{43}$$

$$\dot{\hat{\lambda}}_{32} = \tau_{32}\left[\frac{1}{4\gamma_3^2}\bar{\theta}_3^T(X)\bar{\theta}_3(X)z_3^2 - \delta_{32}\left(\hat{\lambda}_{32} - \lambda_{32}^0\right)\right] \tag{44}$$

where k_3, δ_3, τ_{31}, τ_{32}, γ_3, δ_{31}, δ_{32} are positive design constants and λ_{31}^0 and λ_{32}^0 are initial values of $\hat{\lambda}_{31}$ and $\hat{\lambda}_{32}$, respectively.

Theorem 1: Consider the closed-loop system composed of system (7), virtual controllers (20), (32), controller (42) and adaptive updating laws (22), (23), (43), (44). All the signals in the closed-loop system are uniformly ultimately bounded (UUB).

Proof The following Lyapunov function is considered

$$V_3 = V_2 + \frac{1}{2}\left(z_3^2 + b_{\min}\tau_{31}^{-1}\tilde{\lambda}_{31}^2 + b_{\min}\tau_{32}^{-1}\tilde{\lambda}_{32}^2\right) \tag{45}$$

The derivative of V_3 is

$$\dot{V}_3 = \dot{V}_2 + z_3\dot{z}_3 - b_{\min}\tau_{31}^{-1}\tilde{\lambda}_{31}\dot{\hat{\lambda}}_{31} - b_{\min}\tau_{32}^{-1}\tilde{\lambda}_{32}\dot{\hat{\lambda}}_{32} \tag{46}$$

where $\tilde{\lambda}_{31} = \lambda_{31} - \hat{\lambda}_{31}$ and $\tilde{\lambda}_{32} = \lambda_{32} - \hat{\lambda}_{32}$.

Similar to the analysis in Step 1 and Step 2, the derivative of V_3 is

$$\dot{V}_3 \le -\alpha_3V_3 + \gamma_1^2\omega_1^T\omega_1 + \gamma_3^2\omega_3^T\omega_3 + P \tag{47}$$

where $P = P_2 + P_3$, $P_3 = b_{\max}\delta_3 + \frac{b_{\min}}{2}\left(\lambda_{31}^* - \lambda_{31}^0\right)^2 + \frac{b_{\min}}{2}\left(\lambda_{32}^* - \lambda_{32}^0\right)^2$.

$$\alpha_3 = \min\left\{\begin{array}{l} \alpha_1, 2\left(k_2 - \frac{1}{2}\rho_2 - \frac{1}{2}\rho_3 - \frac{b_{\max}}{4}\right), \\ 2\left(k_3 b_{\min} - \frac{b_{\max}+1}{2\tau_3}\rho_5 - \frac{1}{2}\rho_2^{-1}\right), \\ 2\left(\frac{1}{\tau_3} - \frac{1}{2}\rho_3^{-1} - \frac{1}{2}\rho_4 - \frac{b_{\max}+1}{2\tau_3}\rho_5^{-1}\right), \\ b_{\min}\delta_{31}, b_{\min}\delta_{32} \end{array}\right\}$$

By selecting appropriate parameters [25], $\alpha_3 > \alpha_0 > 0$ where α_0 is positive constant.

Then we have

$$\dot{V}_3 \leq -\alpha_0 V_3 + \gamma^2\|\omega\|^2 + P \tag{48}$$

where $\gamma = \sqrt{\gamma_1^2 + \gamma_3^2}$, $\omega = [\omega_1 \omega_3]'$.

From (13), (29), (40), we begin with the $\sum\tilde{z}\omega$ subsystem as

$$\sum\tilde{z}\omega : \left\{\begin{array}{l} \dot{z}_1 = g_1 x_2 + C_{\theta 1}\bar{\theta}_1(x_1)\omega_1 + \nu_1 - \dot{x}_{1d} \\ \dot{z}_2 = \dot{x}_2 - \dot{\alpha}_2 = x_3 - \dot{\alpha}_2 \\ \dot{z}_3 = g_3\delta_e + C_{\theta 3}\bar{\theta}_3(X)\omega_3 + \nu_3 - \dot{\alpha}_3 \\ \tilde{z} = H(z) = z \end{array}\right. \tag{49}$$

According to Definition 2 in [25], the conditions of ISpS for the system $\sum\tilde{z}\omega$ could be satisfied with the functions $\kappa_3(s) = s^2$ and $\kappa_4(s) = \gamma^2 s^2$ of class K and we can get a gain function $\gamma_z(s)$. On the other hand, the $\sum\omega\tilde{z}$ subsystem could be expressed as

$$\sum\omega\tilde{z} : \left\{\begin{array}{l} \omega_1 = A_1^m z_1 \\ \\ \omega_3 = A_3^m \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \end{array}\right. \tag{50}$$

leading to

$$\|\omega\| \leq \gamma'\|z\| \tag{51}$$

where $\gamma' \leq 1$. Then the condition of small gain theorem could be satisfied by choosing $\gamma < 1$ so that the closed-loop system is UUB. This completes the proof.

IV. Controller Design for Velocity Subsystem

From (8), define the velocity tracking error $z_v = V - V_d$.

Similar to Step 3 in attitude controller design, we design the following controller

$$\begin{aligned} \Phi &= -k_v z_v + \dot{V}_d - \frac{\hat{\lambda}_{v2}}{4\gamma_v^2}\bar{\theta}_v^T(X_v)\bar{\theta}_1(X_v)z_v \\ &- \hat{\lambda}_{v1}\varphi_v(X_v)\tanh\left(\frac{\hat{\lambda}_{v1}\varphi_v(X_v)z_v}{\delta_v}\right) \end{aligned} \tag{52}$$

where $\varphi_v = 1 + \|\theta_v\|$, $X_v = [V, x_1, x_2, \dot{V}_d]^T$.

$$\dot{\hat{\lambda}}_{v1} = \tau_{v1}\left[\varphi_v(X)\|z_v\| - \delta_{v1}\left(\hat{\lambda}_{v1} - \lambda_{v1}^0\right)\right] \tag{53}$$

$$\dot{\hat{\lambda}}_{v2} = \tau_{v2}\left[\frac{1}{4\gamma_v^2}\bar{\theta}_v^T(X_v)\bar{\theta}_v(X_v)z_v^2 - \delta_{v2}\left(\hat{\lambda}_{v2} - \lambda_{v2}^0\right)\right] \tag{54}$$

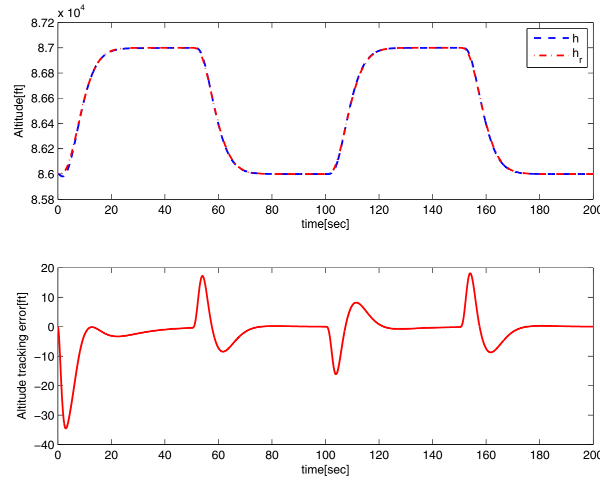

Fig. 1. Altitude Tracking

where k_v, δ_v, τ_{v1}, τ_{v2}, γ_v, δ_{v1}, δ_{v2} are positive design constants and λ_{v1}^0 and λ_{v2}^0 are initial values of $\hat{\lambda}_{v1}$ and $\hat{\lambda}_{v2}$, respectively.

V. Simulation

In this section, we verify the effectiveness and performance of the proposed controller. The initial values of the states are set as $v_0 = 7850$ft/s, $h_0 = 86000$ft, $\alpha_0 = 0.00645$rad, $\gamma_0 = 0$, $q_0 = 0$. The velocity tracks the step command with 300ft/s for every 100 seconds. Meanwhile, the altitude follows the square command with period 100s and magnitude 1000ft. The reference commands of h_r and V_r are generated by the following filter

$$\begin{aligned} \frac{h_r}{h_c} &= \frac{0.16}{(s^2 + 0.76s + 0.16)} \\ \frac{V_r}{V_c} &= \frac{0.16^2}{(s^2 + 0.76s + 0.16)^2} \end{aligned}$$

The control gains for the dynamic surface controller are selected as $k_v = 5$, $k_h = 1.2$, $k_I = 0.1$, $k_1 = 20$, $k_2 = 5$, $k_3 = 20$. Parameters for adaptive laws are selected as $\tau_{ij} = 0.1$, $\delta_{ij} = 5$, the initial values of λ_{ij} are set to be zero and $\gamma_i = 0.6$, $i = 1, 3, v$, $j = 1, 2$. The filter parameters are selected as $\tau_2 = \tau_3 = 0.1$. The number of NN nodes are set as $N_1 = 5$, $N_3 = 5^3$, $N_v = 5^4$ with their centers x_1, X and X_v being evenly spaced in $[-0.1; 0.1]$, $[-0.1; 0.1] \times [-0.3; 0.3] \times [-0.1; 0.1]$ and $[7850; 8450] \times [-0.1; 0.1] \times [-0.3; 0.3] \times [-5; 5]$.

The system performance is depicted in Figs.1-2 and the control inputs are illustrated in Figs.3-4. It shows that with the proposed controller the altitude and velocity can follow the reference very well. The elevator deflection demonstrated in Fig.3 exhibits larger input in the first period than the second period since the system is not responding at trimmed state. The simulation results shows that with "Minimal Learning Parameter" the controller could achieve good tracking performance for HFV in presence of unknown system dynamics.

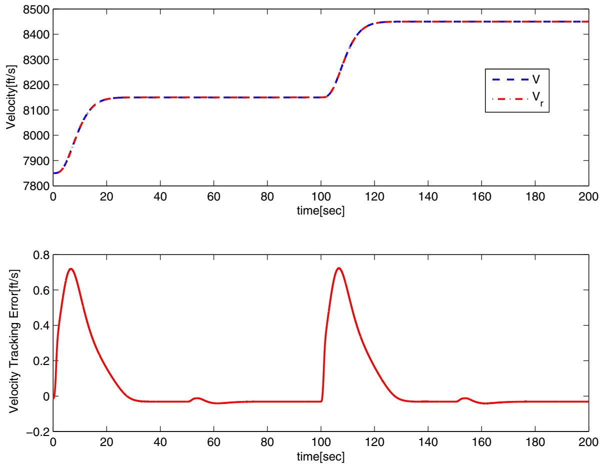

Fig. 2. Velocity Tracking

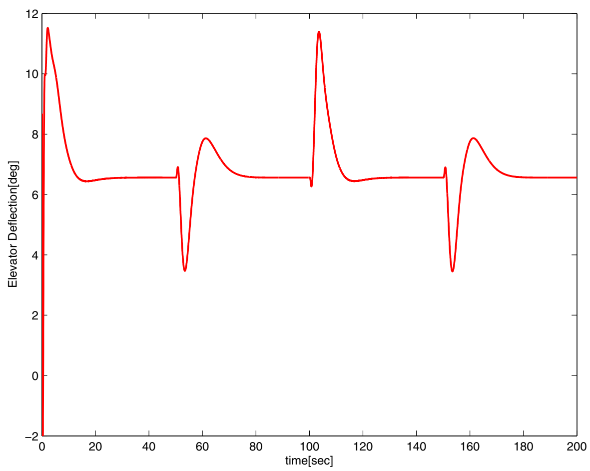

Fig. 3. Elevator Deflection

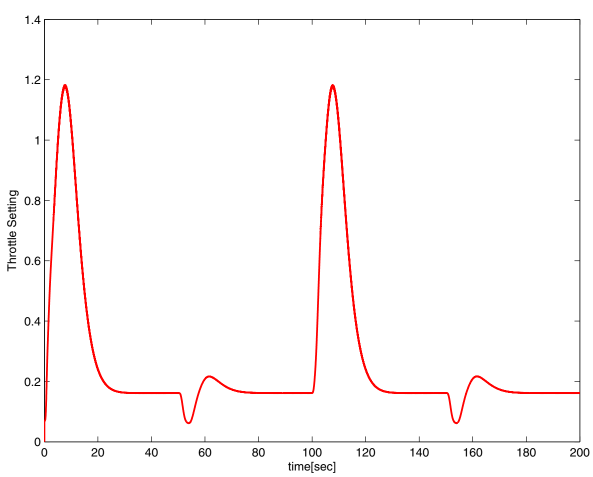

Fig. 4. Fuel Equivalence Ratio

VI. Conclusions

This paper presents the neural dynamic surface control of hypersonic flight dynamics. The "explosion of complexity" of back-stepping design is eliminated. The highlight is that the attitude controller could greatly reduce the online updating computation burden and it avoids the singularity problem. Thus the algorithm is easy to implement. For the velocity subsystem, similar direct neural controller is designed. Simulation with control oriented model of hypersonic dynamics is presented to show the effectiveness of the proposed algorithm.

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