2014 International Conference on Unmanned Aircraft Systems (ICUAS)
May 27-30, 2014. Orlando, FL, USA

Simulation and robust trajectory-tracking for a Quadrotor UAV

Sergio Salazar*, Iván González-Hernández, Ricardo Lopez and Rogelio Lozano

***Abstract*— In this article, a robust control algorithm using sliding modes is proposed for the efficient regulation on the trajectory tracking tasks, in the nonlinear, multivariable, quadrotor system model, that ensures the asymptotic convergence to a desired trajectory (reference signal - $r(t)$) in presence of all possible uncertainties and external disturbances. A smooth piecewise continuous function trajectory is proposed where the corresponding derivatives are bounded. Furthermore, we assume that the disturbances on the vehicle are bounded and the signal $r(t)$ are available on line. The proposed algorithm employs a sliding surface based on the errors generated from the current state of the path in order to reach the desired reference $r(t)$. The stability analysis of the closed-loop control system is proven via the use of Lyapunov theory. Finally, a numerical simulation of tracking a smooth trajectory is performed to demonstrate the validity and effectiveness of the proposed robust algorithm in presence of disturbances onto the vehicle.**

I. INTRODUCTION

Technological developments in the realm of trajectory-tracking have led to innovative concepts in the mission management of current and next generation air, land and sea vehicles. Particularly, the highly nonlinear and coupled dynamics in this type of vehicles are adequate for studying either linear and nonlinear control techniques on the trajectory tracking tasks. For example, in a recent article, by Zongyu and Lin [4], an application of hyperbolic tangent function is made to the problem of trajectory tracking for Quadrotors where the main objective is to ensure the asymptotic convergence to any desired trajectory in presence of parametric and external uncertainties employing a continuous adaptive control law with an online approximator. Afterwards, in [9] the authors present a trajectory tracking controller utilizing nested saturation control algorithm to track satisfactorily the desired trajectory in real-time application.

In recent years, several Quadrotor related papers have presented their experimental flight results [1], [8]. The nonlinear trajectory tracking control strategies, proposed by Kendoul [11], [12], [13] were performed by a real-time autonomous flight and showed great autonomous capability of taking off, hovering and landing. The control law designs in [10], [12] adopted nested saturation technique [14] to account for the actuator saturation limit explicitly and gave the analytical expressions of parameter selection. These nonlinear controller syntheses, the design parameter selections and the stability proofs were quite intricate. So, we can mention that the state of the art in the topic of trajectory tracking is wide with good contributions that improve the performance of the Quadrotors to accomplish a given mission a desired optimal trajectory.

In this article, as part of our recent research to assess the potential and robustness of the control algorithms in the trajectory tracking for Unmanned Aerial Vehicle (UAV) applications, we investigated the potential of a nonlinear design tool called sliding mode methodology for trajectory tracking of small size Quadrotor. The control law is very well known for its robustness against disturbances and invariance during the sliding regime. A state feedback control algorithm were developed for tracks a reference signal. Thus, we propose a sliding mode control scheme for the trajectory tracking tasks in the mini-Quadrotor UAV system model. The advantage of this methodology is that it is possible to reduce and eliminate disturbances by selecting the appropriate gain in the sliding mode controller and the performance of the overall system were tested in a numerical simulation.

This paper is organized as follows: Section II presents the dynamical model of the mini-Quadrotor UAV via a Lagrange approach. A smooth piecewise continuous function trajectory is proposed in section III. The robust control algorithm based on sliding mode methodology for the trajectory-tracking is described in Section IV. The effectiveness of the sliding mode methodology proposed to the trajectory-tracking is evaluated through a series of simulations in Section V. Finally, Section VI gives a brief conclusion of the proposed robust trajectory tracking

I. Gonzalez and R. Lopez are with UMI-LAFMIA 3175 CNRS at Automatic Control Section CINVESTAV-IPN México D.F., México. `{igonzalez,jlopez}@ctrl.cinvestav.mx`
S. Salazar and R. Lozano are with UMI-LAFMIA 3175 CNRS at CINVESTAV-IPN México D.F., México. `{ssalazar,rlozano}@ctrl.cinvestav.mx`

This work was supported by CONACyT, UMI-LAFMIA 3175 CNRS

$*$ Corresponding author

978-1-4799-2376-2/14/$31.00 ©2014 IEEE

control design algorithm.

II. QUADROTOR MATHEMATICAL MODEL

Quadrotor unmanned aircraft, consisting of four individual rotors of “X” arrangement, is an attractive rotary-wing vertical take-off and landing (VTOL) Unmanned Aerial Vehicle (UAV) for both military and civilian usages. In this type of vehicles, vertical motion is created by collectively increasing and decreasing the speed of all four rotors; pitch or roll motion is achieved by the differential speed of the front-rear set or the left-right set of rotors, coupled with lateral motion; yaw motion is realized by the different reaction torques between the (1,3) and (2,4) rotors. The main thrust is the sum of the thrusts of each motor, as shown in Figure 1.

Let $\mathcal{I} = \{i_I, j_I, k_I\}$ be the inertial frame, $\mathcal{B} = \{i_B, j_B, k_B\}$ denote a set of coordinates fixed to the rigid aircraft as is shown in Figure 1. Let $q = (x, y, z, \phi, \theta, \psi)^T \in \mathbb{R}^6 = (\xi, \eta)^T$ be the generalized coordinates vector which describe the position and orientation of the flying machine, so the model could be separated in two coordinate subsystems: translational and rotational. They are defined respectively by

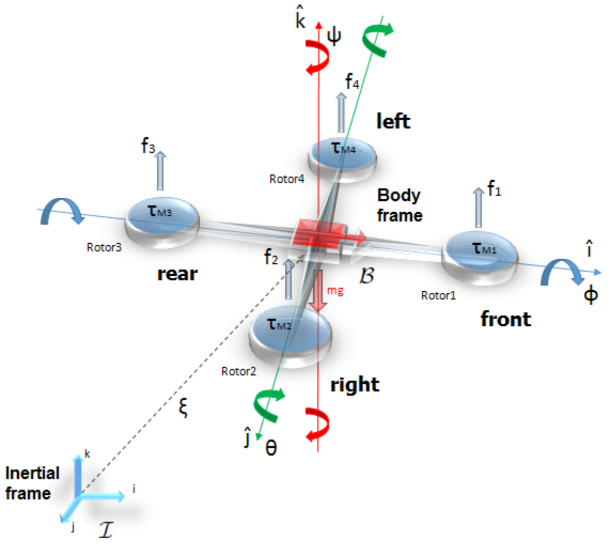

Fig. 1. The coordinate system of the Quadrotor aircraft platform.

- $\xi = (x, y, z)^T \in \mathbb{R}^3$: denotes the position of the vehicle’s mass center relative to the inertial frame $\mathcal{I}$.
- $\eta = (\phi, \theta, \psi)^T \in \mathbb{R}^3$: describe the attitude of the aerial vehicle, *i.e.* roll, pitch and yaw angles respectively.

As for the modeling of our Quadrotor, a few reasonable assumptions are made:

- **Assumption 1.** Quadrotor fuselage is a rigid body.
- **Assumption 2.** The structure is symmetrical.
- **Assumption 3.** The center of gravity (CoG) and the body fixed frame origin are assumed to coincide.
- **Assumption 4.** The propellers are rigid.

The dynamic model is obtained via Lagrange approach. We can decompose the equations into translational and rotational displacement. As first step, we obtain the Lagrangian of the aerial vehicle, which is given by $L(q, \dot{q}) = (T_{trasl} + T_{rot}) - U$ where,

$$\begin{aligned} T_{trasl} &= \frac{1}{2} m \dot{\xi}^T \dot{\xi} \\ T_{rot} &= \frac{1}{2} \dot{\eta}^T \mathbb{J} \dot{\eta} \\ U &= mgz \end{aligned}$$

.

The model of the full Quad-rotor aircraft dynamics is obtained from the Euler-Lagrange equations with external generalized force $F_{\mathcal{B}}$ and generalized moment τ_η where $F_{\mathcal{I}}$ is the translational force applied to the Quad-rotor aircraft due to the control input. Then

$$F_{\mathcal{B}} = (0, 0, u)^T \tag{1}$$

where u is the sum of mechanical thrust forces: $u = f_1 + f_2 + f_3 + f_4$ with $f_i = k_i \omega_i^2$ for i=1,2,3,4, $k_i > 0$ is a constant and ω_i is the angular speed of motor i, as shown in Figure 1. This force vector can be expressed in the inertial frame as

$$F_{\mathcal{I}} = R^{\mathcal{B} \to \mathcal{I}} F_{\mathcal{B}} \tag{2}$$

where $R^{\mathcal{B} \to \mathcal{I}}$ is the rotation matrix which is defined by three Euler angles $\eta = (\phi, \theta, \psi)^T \in \mathbb{R}^3$ and $R \in SO(3)$,

$$R^{\mathcal{B} \to \mathcal{I}} = \begin{pmatrix} c_\theta c_\psi & s_\psi c_\theta & -s_\theta \\ c_\psi s_\theta s_\phi - s_\psi c_\phi & s_\psi s_\theta s_\phi + c_\psi c_\phi & c_\theta s_\phi \\ c_\psi s_\theta s_\phi + s_\psi c_\phi & s_\psi s_\theta s_\phi - c_\psi c_\phi & c_\theta c_\phi \end{pmatrix} \tag{3}$$

Then the control torques generated by the four rotors are

$$\tau_\eta = \begin{pmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{pmatrix} = \begin{pmatrix} (f_3 - f_1)\, l \\ (f_2 - f_4)\, l \\ \{(f_1 + f_2) - (f_3 - f_1)\}\, d \end{pmatrix} \tag{4}$$

Let l denote the distance from the rotors to the center of mass and d is the drag coefficient produced by coordinated reactive torque involving the four rotors because of the geometry of the mini-Quadrotor UAV. Since the lagrangian contains no cross terms in the kinetic energy, combining $\dot{\xi}$ and $\dot{\eta}$ vectors in the Euler-Lagrange equation can be partitioned into the dynamics for the ξ coordinates and the η dynamics. So, we obtain

$$\begin{array}{l} F_{\mathcal{I}} = m\ddot{\xi} + mg \\ \tau_\eta = J\ddot{\eta} + \dot{J}\dot{\eta} - \frac{1}{2}\frac{\partial}{\partial \eta}\left(\dot{\eta}^T J \dot{\eta}\right) \end{array} \tag{5}$$

Defining the Coriolis terms and gyroscopic and centrifugal terms as

$$C(\eta, \dot{\eta})\,\dot{\eta} \;=\; \dot{J}\dot{\eta} - \frac{1}{2}\frac{\partial}{\partial \eta}\left(\dot{\eta}^T J \dot{\eta}\right) \tag{6}$$

The equations of motion of the Quadrotor can be expressed as

$$m\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = u\begin{pmatrix} -\sin\,\theta \\ \sin\,\phi\,\cos\,\theta \\ \cos\,\phi\,\cos\,\theta \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix} \tag{7}$$

$$\mathbb{J}\ddot{\eta} \;=\; \tau_\eta - C(\eta, \dot{\eta})\dot{\eta} \tag{8}$$

III. TRAJECTORY GENERATION

To start, we propose a smooth piecewise continuous function trajectory as reference signal $r(t)$ for the mini-Quadrotor UAV defined by the following polynomial

$$r(t) = at^2 - bt^3 \tag{9}$$

then, the reference velocity is given as

$$\dot{r}(t) = 2at - 3bt^2 \tag{10}$$

and

$$\ddot{r}(t) = 2a - 6bt \tag{11}$$

$$r^{(3)}(t) = -6b \tag{12}$$

We choose the trajectory initial conditions so that $a = 0.208$ and $b = 0.00231$.

IV. NONLINEAR CONTROL DESIGN FOR TRAJECTORY-TRACKING

In this section we present a control law for the trajectory-tracking of the mini-Quad-rotor UAV based on sliding-mode. The control objective is to design a controller such that the tracking error $\|x(t) - r(t)\|$ converges to the origin asymptotically in order to track a reference signal $r(t)$, where

- $r(t)$ and its derivatives are bounded for all $t \geq 0$.
- the signal $r(t)$ is available on line.

The collective input u in (7), is essentially used to make the altitude reach a desired value. The control input τ_ψ is used to set the yaw displacement to zero, τ_θ is used to control the pitch and the horizontal movement in the $x - axis$. Similarly τ_ϕ is used to control the roll and horizontal displacement in the $y - axis$. In order to simplify, let us propose a change of the input variables

$$\tau = J\tilde{\tau} + C(\eta, \dot{\eta})\,\dot{\eta} \tag{13}$$

where

$$\tilde{\tau} = \begin{pmatrix} \tilde{\tau}_\phi \\ \tilde{\tau}_\theta \\ \tilde{\tau}_\psi \end{pmatrix} \tag{14}$$

So, rewriting equation (7) we have

$$\begin{array}{c} \begin{pmatrix} m\ddot{x} \\ m\ddot{y} \\ m\ddot{z} \end{pmatrix} = \begin{pmatrix} -u\,\sin\,\theta \\ u\,\cos\,\theta\,\sin\,\phi \\ u\,\cos\,\theta\,\cos\,\phi - mg \end{pmatrix} \\ \begin{pmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{pmatrix} = \begin{pmatrix} \tilde{\tau}_\phi \\ \tilde{\tau}_\theta \\ \tilde{\tau}_\psi \end{pmatrix} \end{array} \tag{15}$$

where x and y are the coordinates in the horizontal plane, and z is the vertical position, ψ is the yaw angle around the $z - axis$, θ is the pitch angle around the $y - axis$, and ϕ is the roll angle around the $x - axis$. The control inputs u, $\tilde{\tau}_\phi$, $\tilde{\tau}_\theta$ and $\tilde{\tau}_\psi$ are the total thrust or collective input (directed out the bottom of the aircraft) and the angular moments, respectively.

A. Altitude and Yaw control

To altitude control, we consider the dynamic in the z-axis from (7),

$$\ddot{z} \;=\; \frac{1}{m}(u\cos\phi\cos\theta - mg) \tag{16}$$

using the following control input

$$u = \frac{mc_1 + mg}{\cos\phi\cos\theta} \tag{17}$$

where the variable c_1 is a PD controller, which is defined by

$$c_1 = -k_{D_z}\dot{z} - k_{P_z}(z - z_d) \tag{18}$$

with k_{D_z},$k_{P_z} > 0$ and z_d the altitude desired. From (17) we can see that control u is necessary bounded in: $-\frac{\pi}{2} < \phi < \frac{\pi}{2}$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ because when θ and ϕ are $\frac{\pi}{2}$ the altitude control u is indeterminated so there exists a singularity. Then, using (17) into (16) we get

$$\ddot{z} = c_1 \tag{19}$$

Now, the yaw angular position $\ddot{\psi}$ can be controlled by applying

$$\tilde{\tau}_{\psi} = -a_{\psi_1}\dot{\psi} - a_{\psi_2}\left(\psi - \psi_d\right) \tag{20}$$

where ψ_d is the desired yaw angular position. Afterwards, substituting (17) into (7) and provided that $\cos\phi\cos\theta \neq 0$, we obtain

$$\begin{array}{ll}
m\ddot{x} = -\frac{(c_1+mg)}{\cos\,\phi}\tan\,\theta & (a)\\
m\ddot{y} = (c_1+mg)\tan\,\phi & (b)\\
m\ddot{z} = c_1 & (c)\\
\ddot{\phi} = \tilde{\tau}_{\phi} & (d)\\
\ddot{\theta} = \tilde{\tau}_{\theta} & (e)\\
\ddot{\psi} = -a_{\psi_1}\dot{\psi} - a_{\psi_2}\left(\psi - \psi_d\right) & (f)
\end{array} \tag{21}$$

By selecting the correct values for a_{ψ_1}, a_{ψ_2} ensure a fast and stable response in the yaw axis.

B. Roll-angle and y-trajectory control

As we know, attitude control is the most important part of our Quad-rotor aircraft system, it stabilizes the orientation of the vehicle. If c_1 is small enough means that the vehicle has achieved the required altitude and hence this variable $c_1 \to 0$ for a time T, so the dynamics on $x - axis$ and $y - axis$ of (21) is reduced to

$$\ddot{x} = -g\frac{\tan\,\theta}{\cos\,\phi} \tag{22}$$

$$\ddot{y} = g\,\tan\,\phi \tag{23}$$

First of all, we consider the dynamics in *Roll axis* given by (21.d) and (23). We will implement a non-linear controller design based on Robust Nested Saturations. This type of control allows in the limit to guarantee arbitrary bounds for ϕ, $\dot{\phi}$, y and $\dot{y}$. To further simplify the analysis, we will assume that

$$\tan\,\phi \approx \phi \quad \exists \quad |\phi| < \frac{\pi}{6}$$

which is arbitrarily small and the gravitational constant

$$g = 1\;(normalized)$$

Then, rewriting (21.d) and (23) we obtain

$$\begin{array}{l}
\ddot{y} = \phi\\
\ddot{\phi} = \tilde{\tau}_{\phi}
\end{array} \tag{24}$$

and if we differentiate twice $\ddot{y} = \phi$ implies that

$$y^{(iv)} = \tilde{\tau}_{\phi} \tag{25}$$

which represents four integrators in cascade as shown in Figure 2.

Now, we assume external perturbations in the control input of the system (25),

$$y^{(iv)} = \tilde{\tau}_{\phi} + \gamma \tag{26}$$

where γ is the external perturbation which exist in any realistic problem, affecting the trajectory-tracking with $\|\gamma\| < L$ for $L > 0$ and is arbitrarily small. Moreover, the assumption considers that the dynamic system works in a small linear region ($\tan\,\phi \approx \phi$) and the perturbations in the input of the system affect directly the stability. Therefore, we propose a robust control algorithm motivated in the sliding modes control (see [2], Chapter 14), to ensure the stability on the trajectory-tracking of our system (mini-Quad-rotor UAV) in the presence of external perturbations. From Figure 2, we can obtain the following differential equations set,

$$\begin{array}{l}
\dot{y}_1 = y_2 = \dot{y}\\
\dot{y}_2 = y_3 = \ddot{y} = \phi\\
\dot{y}_3 = y_4 = y^{(3)} = \dot{\phi}\\
\dot{y}_4 = y^{(4)} = \tilde{\tau}_{\phi} + \gamma = \ddot{\phi}
\end{array} \tag{27}$$

We define tracking errors as

$$\begin{array}{l}
e_1 = x_1 - r\left(t\right) = x - r\left(t\right)\\
e_2 = x_2 - \dot{r}\left(t\right) = \dot{x} - \dot{r}\left(t\right)\\
e_3 = x_3 - \ddot{r}\left(t\right) = \ddot{x} - \ddot{r}\left(t\right) = \theta - \ddot{r}\left(t\right)\\
e_4 = \dot{\theta}
\end{array} \tag{28}$$

Asymptotic tracking will be achieved if we design a state feedback control law to ensure that $e_i(t)$ for $i = 1, 2, 3, 4$ is bounded and converges to zero as t tends to infinity. For instance, boundedness of e_1 will ensure boundedness of x because $r(t)$ is bounded. We need also to ensure boundedness of tracking errors. For that, we restrict our analysis to the case where the system

$$\dot{\upsilon} = f_0\left(\upsilon, \zeta\right) \tag{29}$$

with $\upsilon = \left[e_1, e_2, e_3\right]^T$, $\dot{\upsilon} = \left[\dot{e}_1, \dot{e}_2, \dot{e}_3\right]^T$ and $\zeta = e_4$. The previous system (29) is bounded-input-bounded-state stable. This will be the case for any bounded input ζ and any initial state $\upsilon\left(0\right)$ if the system $\dot{\upsilon} = f_0\left(\upsilon, \zeta\right)$ is input-to-state stable. So, from this point on, we concentrate our attention on showing boundedness and convergence of $e(t)$. Thus, we start with the system

$$\begin{array}{l}
\dot{e}_1 = e_2\\
\dot{e}_2 = e_3\\
\dot{e}_3 = e_4
\end{array} \tag{30}$$

where e_4 is viewed as the "*control input*". We want to design e_4 to stabilize the origin. For this linear system (in the controllable canonical form), we can achieve this task by the linear control

$$e_4 = -\left(k_1 e_1 + k_2 e_2 + k_3 e_3\right) \tag{31}$$

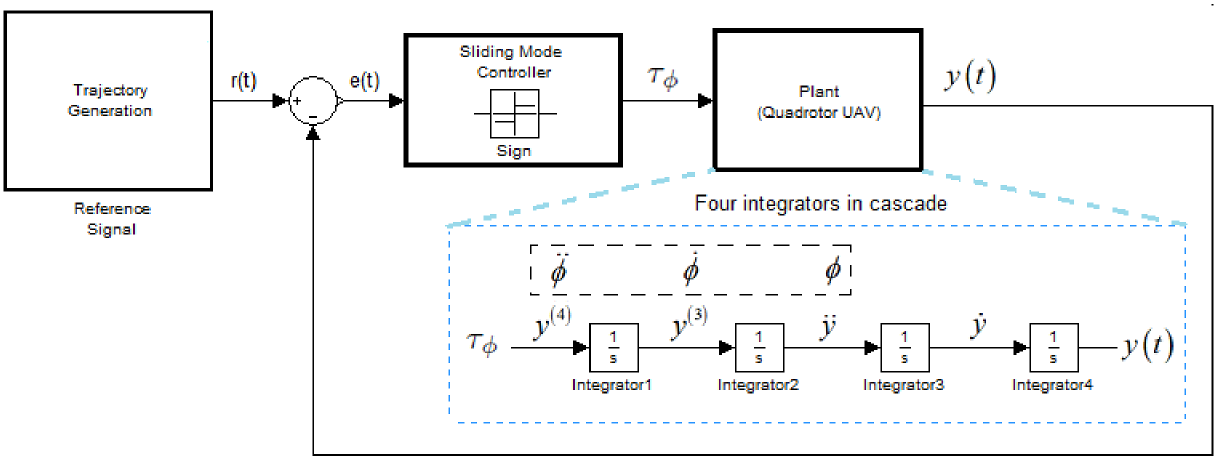

Fig. 2. Block diagram of four integrators in cascade for the trajectory tracking control onto the y-axis.

where the design coefficients k_1 to k_3 are chosen such that the reconstruction error dynamics dominant characteristic polynomial is Hurwitz, i.e.,

$$s^3 + k_3 s^2 + k_2 s + k_1 = 0$$

Then, to design the sliding mode control, we start by designing the "*sliding manifold*" along which the sliding motion is to take place. For this trajectory-tracking problem, the sliding manifold denoted by s is defined by the tracking errors as follow

$$s = (k_1 e_1 + k_2 e_2 + k_3 e_3) + e_4 \tag{32}$$

and

$$\begin{aligned} \dot{s} &= (k_1 \dot{e}_1 + k_2 \dot{e}_2 + k_3 \dot{e}_3) + \dot{e}_4 \\ \dot{s} &= (k_1 \dot{e}_1 + k_2 \dot{e}_2 + k_3 \dot{e}_3) + \ddot{\theta} \\ \dot{s} &= (k_1 e_2 + k_2 e_3 + k_3 e_4) + (\tilde{\tau}_\phi + \gamma) \end{aligned} \tag{33}$$

Therefore, if the control law enforces the trajectories in the phase space such that $s = 0$ in (32), then the tracking errors converge asymptotically to origin due to

$$s = (k_1 e_1 + k_2 e_2 + k_3 e_3) + e_4 = 0 \tag{34}$$

In order to provide global asymptotic stability about the origin, we propose the following Lyapunov candidate function given as

$$V = \frac{1}{2} s^2 \tag{35}$$

The time derivative of the Lyapunov function candidate in (35) can be computed as follows

$$\dot{V} = s\dot{s} \tag{36}$$

differentiating with respect to time, we have that

$$\dot{V} = s\left[(k_1 e_2 + k_2 e_3 + k_3 e_4) + (\tilde{\tau}_\phi + \gamma)\right] \tag{37}$$

We can proceed by designing the input control ($\tilde{\tau}_\phi$) with a pure switching component ν,

$$\tilde{\tau}_\phi = -(k_1 e_2 + k_2 e_3 + k_3 e_4) + \nu \tag{38}$$

and substituting it into (37) we obtain

$$\dot{V} = s(\gamma(t) + \nu) = s\gamma(t) + s\nu \leq |s| L + s\nu \tag{39}$$

then

$$\dot{V} \leq |s| L + s\nu \tag{40}$$

selecting

$$\nu = -\rho \, sign(s) \tag{41}$$

where $\rho > 0$ and $sign(s) = \frac{s}{|s|}$ substituting into (40) we obtain

$$\dot{V} \leq |s| L + s\left(-\rho \frac{s}{|s|}\right) = |s| L - |s| \rho \tag{42}$$

therefore

$$\dot{V} \leq -|s| (\rho - L) \tag{43}$$

This means that, the origin is stable and the trajectory errors $e_i(t)$ for $i = 1, 2, 3, 4$ are bounded. At last, using (41) into (38), we have

$$\tilde{\tau}_\phi = -k_1 e_2 - k_2 e_3 - k_3 e_4 - \rho \, sign(s) \tag{44}$$

This control law (44) will drive the mini-Quadrotor UAV to the desired reference signal $r(t)$ in a finite time, if and only if

$$\rho > L \tag{45}$$

where L is the upperbound of the external perturbation. This condition ensures a stable performance of the trajectory-tracking for our mini-Quadrotor UAV onto $y - axis$.

C. Pitch-angle and x-trajectory control

We will use for x-trajectory control the same procedure used in the previous section for y-trajectory control. We now consider the dynamics in Pitch axis given by (21.e) and (22). As before, we will use a control strategy such that after a finite time θ is small enough such that $\tan\ \theta \approx \theta$. Therefore, we have

$$\begin{aligned} \ddot{x} &= -\theta \\ \ddot{\theta} &= \tilde{\tau}_\theta \end{aligned} \tag{46}$$

Continuing in the same manner, it can be seen that

$$\tilde{\tau}_\theta = -k_1 e_2 - k_2 e_3 - k_3 e_4 - \rho\ sign\,(s) \tag{47}$$

As in the previous case, this control law (47) will drive the mini-Quadrotor UAV to the desired reference signal $r(t)$ in a finite time, if and only if

$$\rho > L \tag{48}$$

likewise L denotes the upperbound of the external perturbation. This condition ensures a stable performance of the trajectory-tracking for our mini-Quadrotor UAV onto $x - axis$.

V. SIMULATION RESULTS

In this section, simulation examples are provided to illustrate efficiency of the robust controller. The parameters of the quadrotor are chosen according to a quadrotor employed in [5]. For simplicity the dynamics of DC motor is ignored. The controller parameters in simulation are briefly described in Table I.

TABLE I

THE SLIDING MODE CONTROLLER PARAMETERS FOR TRAJECTORY TRACKING.

Parameters	Value
Mass of the Quad-rotor aircraft, (m) $[kg]$	1.00
Gravitational acceleration, (g) $[m/s^2]$	9.81
Reaching law parameter, (ρ)	1.2
Positive constant, (k_1)	0.8
Positive constant, (k_2)	0.8
Positive constant, (k_3)	0.8

Simulation results of the mini-Quadrotor UAV using the sliding mode for trajectory tracking are presented below. In these simulations, a external disturbances in the input has been added to test stability robustness in the sliding mode for the trajectory-tracking of the mini-Quadrotor UAV. Moreover, we present two cases to demonstrate when the system can be stable or unstable depending of the value of gain in the controller to achieve the trajectory-tracking onto the reference signal $r(t)$.

A. Case I: Trajectory-tracking stable: $\rho > L$

The results of the simulation for the trajectory-tracking based on sliding modes technique are shown in Figures 4 to 6, while Figure 3 shows the random signal with a flat as a bounded disturbance added. We can observer that the Quadrotor UAV follows the trajectory proposed $r(t)$ successfully in a reasonable time with small oscillations around the desired reference signal. This statement can be observed much better in Figure 5.

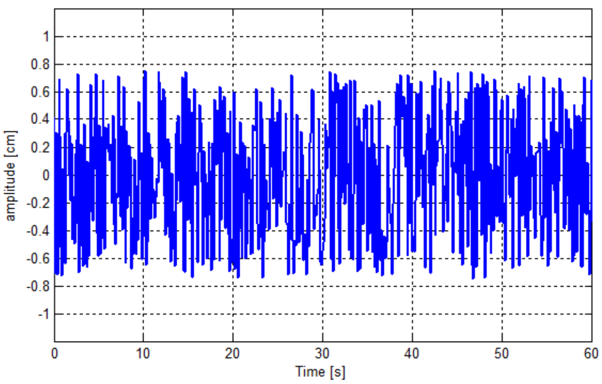

Fig. 3. Plot of the random signal injected into the system to test the performance of the controller.

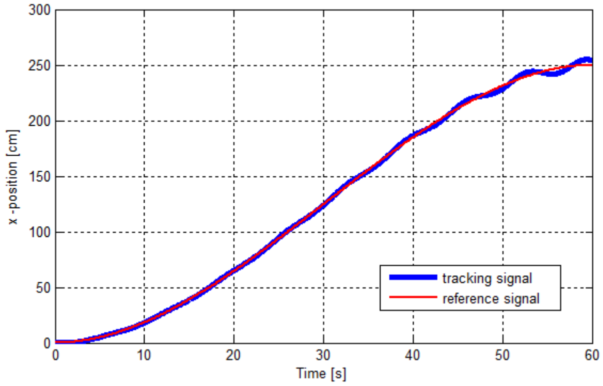

Fig. 4. *Position-trajectory* behavior (in this case: $x(t)$) and desired reference signal $r(t)$. The trajectory-tracking converges faster.

These Figures illustrates that the proposed controller renders the mini-Quadrotor UAV follow the designated trajectory asymptotically in presence of the external disturbances in the input, although the sliding mode control signal is not necessarily convergent to zero. The control law also can improve the trajectory-tracking combining and selecting different values of gain in the sliding mode controller to get better tracking of the reference signal.

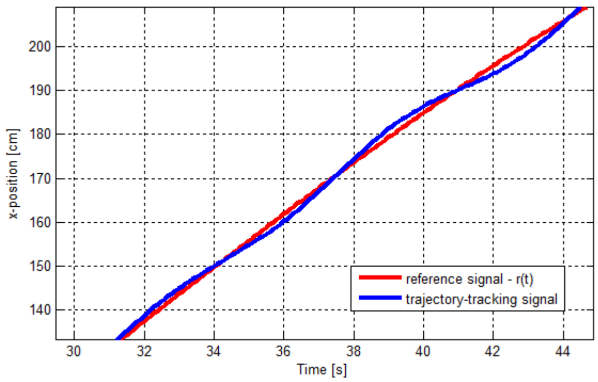

Fig. 5. Zoom of the trajectory-tracking behavior towards the reference signal $r(t)$.

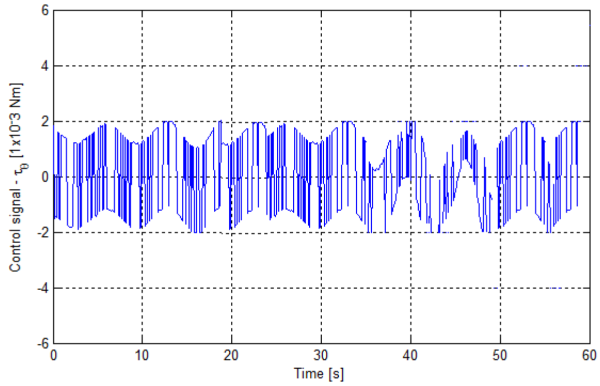

Fig. 6. Sliding mode control signal applied to the Quadrotor UAV to reach the desired reference signal $r(t)$.

B. Case II: Trajectory-tracking unstable: $\rho < L$

The simulation results for this case are presented in the following Figures which shows that the trajectory-tracking of the mini-Quadrotor UAV diverges from the reference signal $r(t)$ when t tends to infinity. This result is due to the gain controller (ρ) is not big enough to compensate the external disturbance (L) that affects the vehicle on its mission to track trajectory. As a result the trajectory-tracking is unstable.

Clearly, the trajectory-tracking (see Figure 7) is unstable as t tends to infinity while the control signal (see Figure 8) diverges along time and consequently the system (Quadrotor UAV) is unstable and never reaches the desired trajectory $r(t)$.

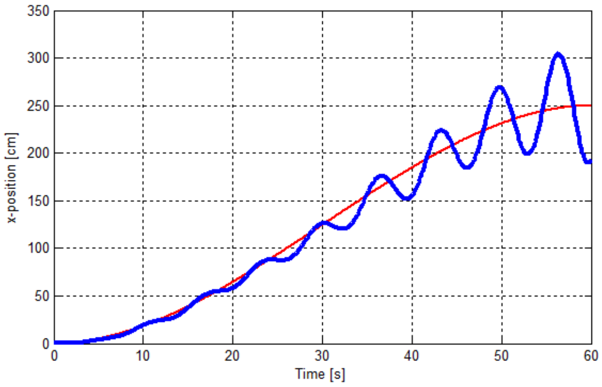

Fig. 7. *Position-trajectory* behavior (in this case: $x(t)$) and desired reference signal $r(t)$. The trajectory-tracking signal diverges from the desired reference and it is unstable.

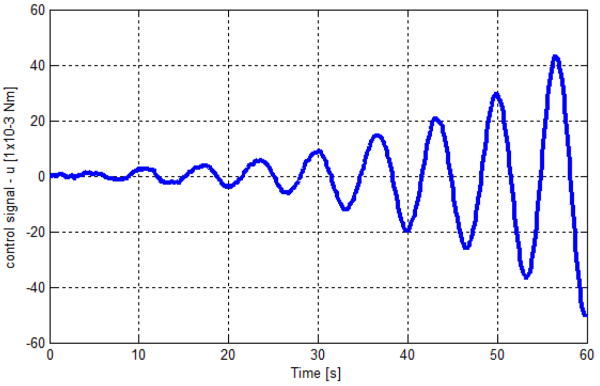

Fig. 8. Sliding mode control signal unstable. The Quadrotor UAV never reaches the desired reference signal $r(t)$.

VI. CONCLUSIONS

In this paper, we present a robust control algorithm using sliding modes on the trajectory tracking tasks in a mini-Quadrotor UAV system. This control law considers external disturbances in the input. We developed several simulations to demonstrate the effectiveness and merits of the theoretical development. In these simulations, we present the stability results of the system for two cases of the variable ρ (controller gain), i.e., when $\rho > L$ the system is stable and when $\rho < L$ the system is unstable. These results show that it is necessary to know the amplitude (L) of the disturbance affecting the vehicle in order to choose the appropriate gain which is used to evaluate the sliding mode controller and achieve robustness in the trajectory-tracking to the reference signal $r(t)$ desired. We consider to implement this control algorithm in a real time application for future work.

REFERENCES

[1] R. Lozano, Ed., *Unmanned Aerial Vehicles: Embedded Control*. Hoboken, NJ: John Wiley & Sons, 2010.
[2] H. K. Khalil, *Nonlinear Systems*. Prentice Hall, 2002.

[3] Slotine, J.-J. E. and Li, W., *Applied Nonlinear Control*. Prentice-Hall, New Jersey, 1991.

[4] Zongyu Zuo, Lin Tie, Hyperbolic Tangent Function Based Adaptive Trajectory Tracking Control for Quadrotors, in *25th Chinese Control and Decision Conference (CCDC)*, pp.3721-3726 (2013).

[5] Iván González, Sergio Salazar and Rogelio Lozano, Chattering-Free Sliding Mode Altitude Control for a Quad-Rotor Aircraft: Real-Time Application, in *Journal of Intelligent & Robotic Systems*, Vol.73, pp.137-155 (2013).

[6] Z. Zuo, Adaptive trajectory tracking control design with command filtered compensation for a quadrotor, in *J. Vib. Contr.*, Vol. 19, No. 1, pp.94-108 (2013).

[7] Z. Zuo, Quadrotor trajectory tracking control: a PD control algorithm, in *Proc. 3rd Int. Conf. Computer and Electrical Engineering*, V7-361-V7-365 (2010).

[8] J. Escareno, S. Salazar-Cruz, and R. Lozano, Embedded control of a four-rotor UAV, in *Proceedings. American Control Conference*, pp.3936-3941 (2006).

[9] Salazar-Cruz, S., Palomino, A. and Lozano, R., Trajectory tracking for a four rotor mini-aircraft, in *44th IEEE Conference on Decision and Control and European Control Conference. CDC-ECC*, pp.2505-2510 (2005).

[10] P. Castillo, A. Dzul, and R. Lozano, Real-time stabilization and tracking of a four-rotor mini rotorcraft, in *IEEE Trans. Control Syst. Technol.*, Vol.12, No.4, pp.510-516 (2004).

[11] F. Kendoul, D. Lara, I. Fantoni-Coichot, and R. Lozano, Real-time nonlinear embedded control for an autonomous quadrotor helicopter, in *Journal of Guidance, Control, and Dynamics*, Vol.30, No.4, pp.1049-1060 (2007).

[12] F. Kendoul, Nonlinear hierarchical flight controller for unmanned rotorcraft: design, stability, and experiments, in *Journal of Guidance, Control, and Dynamics*, Vol.32, No.6, pp.1954-1958 (2009).

[13] F. Kendoul, Z. Yu, and K. Nonami, Embedded autopilot for accurate waypoint navigation and trajectory tracking: application to miniature rotorcraft UAVs, in *Proc. 2009 IEEE Int. Conf. Robotics and Automation*, pp.2884-2890 (2009).

[14] A.R. Teel, Global stabilization and restricted tracking for multiple integrators with bounded controls, in *Syst. and Contr. Letters*, Vol.18, No.3, pp.165-171 (1992).