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Discrete Optimal Control for a Quadrotor UAV: Experimental Approach

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Abstract— **In this paper we propose a discrete time optimal control law to stabilize the four-rotor rotorcraft in attitude and position. The main objective of this kind of control law is to save energy and therefore increase the effective time in takeoff and hover flight phases for this robotic platforms. The optimal control law is synthesized considering a infinite horizon combined with an exact linearization by applying a nonlinear control law over nonlinear equations describing the robot dynamic model. The control law obtained is simple, easy and better adapted to be programmed in a micro-controller. Both simulation and experimental test and results show a satisfactory UAV behavior.**

I. INTRODUCTION

Aerial robotics is a very attracted area from the point of view of applications and research topics. Many robot configurations and control laws have been developed and synthesized in order to provide to aerial robotic systems the ability to fly in autonomous way with larger time of flight. One of the most popular Unmanned Aerial Vehicles (UAV's) is the quadrotor, it is robust with respect to crashes and it is easy to repair because it has a simple mechanic which does not include swash-plates and linkages found in the conventional configuration helicopters. Four-rotor flying robots are versatile platforms capable to develop many kind of tasks. At the beginning, they are only designed, developed, built and used by the defense area, however in recent years they have been applied specifically to replace to the human being in dangerous tasks, such as inspection of nuclear, toxic, volcanic areas.

Linear and nonlinear control techniques have been applied to drive the quadrotor, those techniques have considered approaches like robust, adaptive, optimal and many more. However, there is still the opportunity to achieve an improvement in the quardrotor dynamic performance by applying a new control strategy or improving one of the previously proposed. Energy consumption is a crucial issue in any dynamic system, this issue has a major relevance in the UAV systems, mainly in the quadrotor where the lift force is only provided by the couple motor-propeller. In order to deal with this problem, in this paper we propose to apply a discrete time optimal control strategy in order to stabilize the quadrotor in attitude and position. This discrete time optimal control law is synthesized applying the Linear Quadratic Regulator (LQR) approach and also using the nonlinear dynamical model by an exact linearization, making it less conservative than other optimal controller previously proposed. This control law presents some advantages:

- We can penalize the convergence time of the state.
- We can optimize the use of the battery at hover and takeoff phases.

Furthermore, according to optimal control philosophy it allows to save energy to increase the effective time of flight for the mini helicopter. LQR is an optimal control approach used to synthesize a control law minimizing a cost function. LQR technique provides a matrix which is used to solve the nonlinear algebraic Riccati equation in order to obtain optimal feedback gain matrices.

Previous works have developed optimal control techniques to stabilize the quadrotor helicopter. Nevertheless, the discrete time optimal control laws have been synthesized using a linear model of four-rotor helicopter.

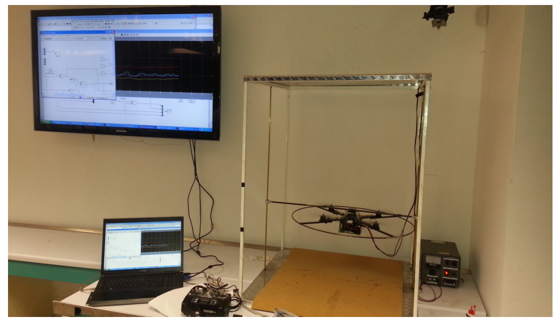

Fig. 1. Quadrotor in X-configuration performing a test flight applying a discrete time optimal control law.

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In [1] authors obtain a discrete linear model around to specific operation point. The linear optimal control law is obtained by using LQR approach and only the attitude stabilization was considered. Alexis *et al.* [2] design a Constrained Finite Time Optimal (CFTO) control scheme to perform the attitude stabilization of a mini quadrotor helicopter. They show experimental results where the helicopter is subject to wind gusts. In order to design the control scheme they assume that the flying robot flies in a bounded operation region, so a linear discrete time optimal control law is obtained.

In other hand, there exist some works where the optimal control law is obtained to continuous time systems. In [6] authors have tested a linear LQR algorithm on a four-rotor helicopter in order to do a comparison with respect to nested saturation algorithm. This paper uses a linear dynamic model as the work above described. Meanwhile, Santoso *et al.* in [3] describe a continuous linear optimal control for a fix wing UAV. This paper uses a linear dynamic model described as transfer function to synthesize this control law. Finally in [4] a continuous suboptimal control is applied to a quadrotor, this control strategy is based on Control Lyapunov Functions (CLF). Furthermore, sufficient conditions are obtained to ensure the asymptotic stability of the closed loop system. This work uses a nonlinear affine dynamic model, which is a difference with respect to all works previously mentioned.

In this contribution we use an exact linearization combined with a LQR, unlike other approaches, the exact linearization avoid the use of a bounded operation region in the plant. Additionally the exact linearization allows penalize with some easiness the matrices Q and R associated with the LQR problem, despite to considered plant is a nonlinear process. In the experimental validation, we calculated the saving energy as a consequence of use the optimal controller compared with a proportional derivative (PD) controller heuristically tuned. Note that the tuning of a PD controller for a nonlinear system could be a not easy task.

This article is organized as follows: Section II presents the discrete time optimal control law with finite horizon applied to stabilize the quadrotor helicopter. The experimental setup platform is described in Section III, while the Simulation and experimental results are shown in Section IV. Finally in Section V the conclusions and discussions are presented.

II. Optimal Stabilization

Energy consumption is an relevant topic for the UAV's field. A four rotor helicopter energized by a typical LiPO battery has a time of flight around 20 minutes, so it is important that UAV converge to reference quickly as it is possible with low consumption of energy. This problem is the optimal control problem with infinite horizon, which it is not easy to solve for nonlinear systems, such as the dynamic model of the mini helicopter.

As is proposed in [7], we can stabilize the quadrotor by a subsystems as follows. Consider the following reduced model proposed in [referencia1]:

$$\begin{aligned} m\ddot{x} &= -u\sin\theta \\ m\ddot{y} &= u\cos\theta\sin\phi \\ m\ddot{z} &= u\cos\theta\cos\phi - mg \\ \ddot{\varphi} &= \tau_\phi \\ \ddot{\theta} &= \tau_\theta \\ \ddot{\psi} &= \tau_\psi \end{aligned} \tag{1}$$

This model could be optimally stabilized using the classical result of Linear Quadratic Regulator (LQR) combined with a nonlinear control law by exact linearization. In [ref1] is exposed that an LQR could have some problems concerning to region where it is valid, because it is obtained from a linear approximation of the dynamic model (1). In this paper we solve this problem considering subsystem in the model (1) which are obtained after to apply an exact linearization control law. The first subsystem considered is the equation for the dynamic z.

A. Optimal Stabilization of subsytem z

We first stabilize the subsystem z by an exact linearization of the model given in (1). So, consider the subsystem:

$$m\ddot{z} = u\cos\theta\cos\phi - mg$$

which has the following companion form in space state representation ($x_z = [\ x_{1,z} \quad x_{2,z}\ ]^T$):

$$\begin{aligned} \dot{x}_{1,z} &= x_{2,z} \\ \dot{x}_{2,z} &= -g + \left(\frac{\cos\theta\cos\phi}{m}\right)u \end{aligned}$$

Then using Euler approximation for derivative terms, it has the following discrete time representation:

$$\begin{aligned} x_{1,z}(k+1) &= Tx_{2,z}(k) - x_{1,z}(k) \\ x_{2,z}(k+1) &= T\left(\tfrac{\cos\theta(k)\cos\phi(k)}{m}\right)u(k) \\ &\quad -x_{2,z}(k) - Tg \end{aligned} \tag{2}$$

where T denotes the sampling time. Now, we observe that the system (2) could be exactly linearized with the control law:

$$u(k) = m(u_1(k) + g)\left(\cos\theta(k)\cos\phi(k)\right)^{-1} \tag{3}$$

whit $\cos\theta(k)\cos\phi(k)\neq 0$, if $\theta,\phi\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ which is a reasonable assumption according to operating conditions proposed for the quadrotor in this work. In fact, substituting the control law (3) on the subsystem (2) we arrive to:

$$x_z(k+1)=A_z x_z(k)+B_z u_1(k) \tag{4}$$

where $A_z=\begin{bmatrix}1 & T\\ 0 & 1\end{bmatrix}$, $B_z=\begin{bmatrix}0\\ T\end{bmatrix}$ and the pair (A_z,B_z) is controllable. Therefore, we have selected an infinite horizon optimal control law in order to control the system (4) which minimizes the performance index:

$$J_z=\sum_{k=1}^{\infty}\left(x_z^T(k)Q_z x_z(k)+u_1^2(k)R_z\right) \tag{5}$$

where $Q_z\geq 0$, $R_z>0$ are given and they penalize the state convergence and the energy consumption respectively. Then, we want to obtain a control law $u_1(k)$, which minimizes J_z subject to (4). As it is very well known, if the pair (A_z,B_z) is controllable, then the discrete algebraic Ricati equation [5] is given as follows:

$$P_z=A_z^T P_z A_z-A_z^T P_z B_z\left(R_z+B_z^T P_z B_z\right)^{-1}B_z^T+Q_z \tag{6}$$

which has an unique solution P_z defining the optimal sequence:

$$u_1^*(k)=-(R_z+B_z^T P_z B_z)^{-1}B_z^T P_z A_z x_z^*(k),\ \forall\ k\geq 0 \tag{7}$$

According with the optimal control theory, the system (4) in closed loop with the control law (7) is stable and minimizes the performance index (5).

B. Stabilization of subsytem ψ

Now, consider the yaw dynamic subsystem:

$$\ddot{\psi}=\tau_\psi,$$

with state space representation given by

$$\begin{aligned}\dot{x}_{1,\psi}&=x_{2,\psi}\\ \dot{x}_{2,\psi}&=\tau_\psi,\end{aligned}$$

this continuous model could be represented in the discrete domain as:

$$x_\psi(k+1)=A_\psi x_\psi(k)+B_\psi\tau_\psi(k)$$

where $A_\psi=\begin{bmatrix}1 & T\\ 0 & 1\end{bmatrix}$, $B_\psi=\begin{bmatrix}0\\ T\end{bmatrix}$ is a controllable pair. Defining the performance index as follows

$$J_\psi=\sum_{k=1}^{\infty}\left(x_\psi^T(k)Q_\psi x_\psi(k)+\tau_\psi^2(k)R_\psi\right), \tag{8}$$

where $Q_\psi\geq 0$ and $R_\psi>0$ are the appropriate dimensions, so the optimal control law is given by

$$\tau_\psi^*(k)=-(R_\psi+B_\psi^T P_\psi B_\psi)^{-1}B_\psi^T P_\psi A_\psi x_\psi^*(k),\forall k\geq 0$$

where the matrix P_ψ satisfies a discrete algebraic Riccati equation (DARE) similar to (6).

C. Stabilization of subsytem $y-\phi$

Consider the subsystem $y-\phi$ as:

$$\begin{aligned}m\ddot{y}&=u\cos\theta\sin\phi\\ \ddot{\phi}&=\tau_\phi,\end{aligned}$$

We consider the state space representation ($x_{1y}=y$, $x_{2y}=\dot{y}$, $x_{3\phi}=\phi$, $x_{4\phi}=\dot{\phi}$):

$$\begin{aligned}\dot{x}_{1y}&=x_{2y}\\ \dot{x}_{2y}&=\frac{1}{m}u\cos\theta\sin x_{3\phi}\\ \dot{x}_{3\phi}&=x_{4\phi}\\ \dot{x}_{4\phi}&=\tau_\phi\end{aligned}$$

its discrete time representation is given as follows

$$\begin{aligned}x_{1y}(k+1)&=x_{1y}(k)+Tx_{2y}(k)\\ x_{2y}(k+1)&=\frac{T}{m}u(k)\cos\theta(k)\sin x_{3\phi}(k)+x_{2y}(k)\\ x_{3\phi}(k+1)&=Tx_{4\phi}(k)+x_{3\phi}(k)\\ x_{4\phi}(k+1)&=T\tau_\phi(k)+x_{4\phi}(k)\end{aligned}$$

According with the definition for $u(k)$ given in (3) we have that second state in this subsystem become to

$$x_{2y}(k+1)=T(u_1^*(k)+g)\tan x_{3\phi}(k)+x_{2y}(k)$$

however according with the optimal control theory, $u_1^*(k)$ tends to zero when k tends to infinity. Then, we consider that $\exists\ n\in\mathbb{Z}^+$ such that for all $k\geq nT$, $|u_1^*(k)|$ is bounded and neglected, consequently we arrive to:

$$x_{2y}(k+1)=gT\tan x_{3\phi}(k)+x_{2y}(k)$$

We want to find a control $\tau_\phi^*(k)$, such that $x_{y,\phi}(k)=\begin{bmatrix}x_{1y} & x_{2y} & x_{3\phi} & x_{4\phi}\end{bmatrix}^T$ goes to zero as fast as possible and the performance index

$$J_{y,\phi}=\sum_{k=1}^{\infty}\left(x_{y,\phi}^T(k)Q_{y,\phi}x_{y,\phi}(k)+\tau_\phi^{*2}(k)R_\phi\right)$$

is minimized. If there exist an optimal control $\tau_\phi^*(k)$ which does this task, then $\tan x_{3\phi}(k)\rightarrow x_{3\phi}(k)$ and we can design the optimal control $\tau_\phi^*(k)$ for the approximated system:

$$x_{y,\phi}(k+1)=A_{y,\phi}x_{y,\phi}(k)+B_{y,\phi}\tau_\phi^*(k),$$

where

$$A_{y,\phi} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & gT & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & T \end{bmatrix}, \; B_{y,\phi} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ T \end{bmatrix} \quad (9)$$

it is a easy task to verify that the pair $(A_{y,\phi},\ B_{y,\phi})$ is controllable, then the optimal control $\tau_\phi^*(k)$ is

$$\tau_\phi^*(k) = -H_{y,\phi}^{-1} B_{y,\phi}^T P_{y,\phi} A_{y,\phi} x_{y,\phi}^*(k),\ \forall\ k \geq 0,$$

where $H_{y,\phi} = R_\phi + B_{y,\phi}^T P_{y,\phi} B_{y,\phi}$ and $P_{y,\varphi}$ is the unique solution of the DARE expressed as follows:

$$P_{y,\phi} = A_{y,\phi}^T P_{y,\phi} A_{y,\phi} - A_{y,\phi}^T P_{y,\phi} B_{y,\phi} H_{y,\phi}^{-1} B_{y,\phi}^T + Q_{y,\phi}$$

D. Stabilization of subsytem $x - \theta$

Consider the subsystem $x - \theta$ defined as follows:

$$\begin{aligned} m\ddot{x} &= -u \sin\theta \\ \ddot{\theta} &= \tau_\theta, \end{aligned}$$

The state space representation ($x_{1x} = x$, $x_{2x} = \dot{x}$, $x_{3\theta} = \theta$, $x_{4\theta} = \dot{\theta}$) is

$$\begin{aligned} \dot{x}_{1x} &= x_{2x} \\ \dot{x}_{2y} &= -\frac{1}{m} u \sin x_{3\theta} \\ \dot{x}_{3\theta} &= x_{4\theta} \\ \dot{x}_{4\theta} &= \tau_\theta. \end{aligned}$$

The discrete representation of this model is:

$$\begin{aligned} x_{1x}(k+1) &= x_{1x}(k) + T x_{2x}(k) \\ x_{2x}(k+1) &= \frac{T}{m} u(k) \sin x_{3\theta}(k) + x_{2x}(k) \\ x_{3\theta}(k+1) &= T x_{4\theta}(k) + x_{3\theta}(k) \\ x_{4\theta}(k+1) &= T\tau_\theta(k) + x_{4\theta}(k). \end{aligned}$$

By similar arguments as above we can arrive to:

$$x_{x,\theta}(k+1) = A_{x,\theta} x_{x,\theta}(k) + B_{x,\theta} \tau_\theta^*(k),$$

where $x_{x,\theta}(k) = [x_{1x}(k) \quad x_{2x}(k) \quad x_{3\theta}(k) \quad x_{4\theta}(k)]^T$, and the matrices $A_{x,\theta}$ and $B_{x,\theta}$ are the same that given in (9). Then the optimal control law $\tau_\theta^*(k)$ is given by:

$$\tau_\theta^*(k) = -H_{x,\theta}^{-1} B_{x,\theta}^T P_{x,\theta} A_{x,\theta} x_{x,\theta}^*(k),\ \forall\ k \geq 0,$$

where $H_{x,\theta} = R_\theta + B_{x,\theta}^T P_{x,\theta} B_{x,\theta}$ and $P_{x,\theta}$ is the unique solution of a DARE.

III. EXPERIMENTAL PLATFORM

The dynamics of a real flying quad-rotor has 6 degrees- of-freedom (DOF) movement, three for orientation and three more for position. The experimental setup platform used allows angular movement in roll and pitch angles (ϕ and θ) and displacements along x and z axes. The coordinated control of all four rotors will provide the desired altitude z, while yhe x movement is produced by changing $(f1 + f4) - (f2 + f3)$. The pitch torque is a function of the force difference described by $(f1 + f4) - (f2 + f3)$, and finally the roll torque is produced by the difference $(f1 + f2) - (f3 + f4)$ (see Figure 2). Therefore, currently setup allows only 4 DOF in 3D space. According to this setup we can obtain a similar result as a real aircraft evolving inside of a limited space. The quad-rotor is fixing with two metal bars which limit the movements. Two joints and pistons at the top of quad-rotor allow movement around the x and y axes in order to produced ± 15 degrees in roll and pitch angular movements respectively.

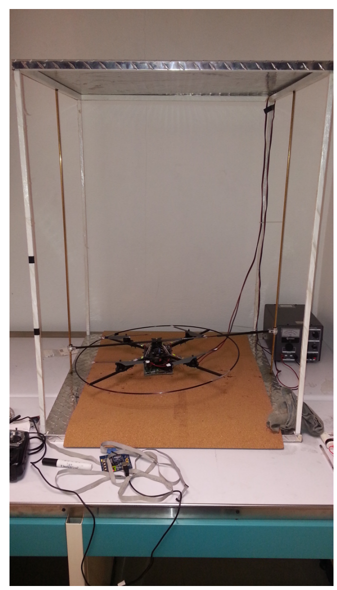

Fig. 2. Quadrotor Experimental Setup of 4DOF.

This platform is based on RabbitCore module RCM4300 (8-Bit Flash memory program), running the discrete time optimal control law to stabilize the mini helicopter. The mini-core has the following main features: operates at 58.98 Mhz (10-ns Cycle Time), with 512K bits serial I2C EEPROM memory, low-power (1.8-V Core, 3.3-V I/O), 4 PWM channels (10-bit resolution), 8 ADC channels (12-bit resolution), 5 serial ports, 2 input-capture channels, 10 timers (16-bit resolution) and I^2C port. Also this microcontroller manages the inertial measurements provided by the IMU module. The IMU module is based on Inertial Navigation System by Microstrain, it measures three angular rates $(\dot{\phi}, \dot{\theta}, \dot{\psi})$ and three angular positions (ϕ, θ, ψ). Moreover this experimental setup has external communication using RS232 protocol, then it can send and receive data from a external PC running Matlab, where more difficult control algorithm can be programmed and tested. The communication device used to have available this feature in our platform is Xbee Modem working at 2.4GHz and 115200 bauds per second.

IV. Simulation and Experimental Results

n this section we illustrate the optimal control with exact linearization by simulation and experimental results. First, we simulate the optimal control using an Euler approximation in order to discretized the nonlinear model of the quadrotor with sample period $T = 0.01$ seconds, which is the same the sample period set for the experimental results.

A. Simulation Routines

The simulation routines are developed considering the matrices $Q_{z,1}$ and $Rz, 1$ to penalize the state values and control signal respectively, they are defined by:

$$Q_{z,1} = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix} \quad R_{z,1} = \begin{bmatrix} 1000 & 0 \\ 0 & 170 \end{bmatrix} \tag{10}$$

Figure (3) shows the position in the xyz when the reference is 2m, 1m and 3m for x, y and z, respectively.

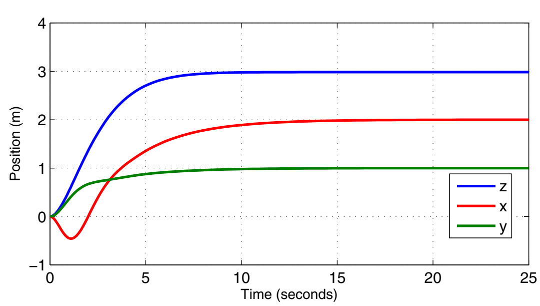

Fig. 3. Simulated position of the quadrotor applying an optimal control plus exact linearization.

Observe that the matrix $R_{z,1}$ hardly penalizes the optimal control u_1. Figure (4) shows the orientation of the quadrotor when the initial conditions for pitch, roll and yaw are $0.2rad$, $0.1rad$ and $0.3rad$ respectively (the references were set to zero).

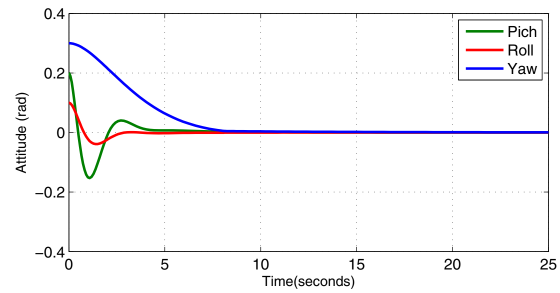

Fig. 4. Simulated orientation of the quadrotor using an optimal control plus exact linearization.

Figure (5) shows the optimal control signals computed:

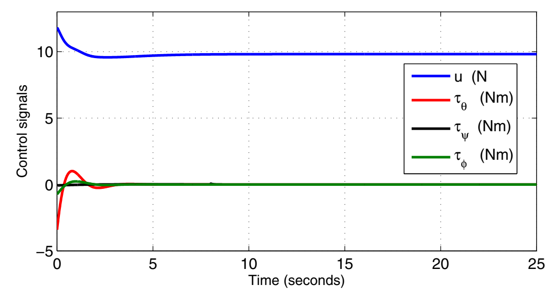

Fig. 5. Simulated control signal applied to quadrotor using an optimal control plus exact linearization.

We validate this simulated probes with experimental results tested over the altitud z and the velocity $\dot{z}$, this restriction was imposed due the limitations of our experimental platform.

B. Experimental Results

In this subsection we show the experimental results obtained to apply the discrete optimal control strategy. We are only tested the optimal control on the z-dynamics but it can be extended to any other quadrotor dynamic. In the first experiment, matrices defined in (10) have been used to synthesize the discrete optimal control law. Results obtained in this case are shown in Figures 6-8, where the z-position, z-velocity and discrete optimal control signal are respectively plotted in those Figures.

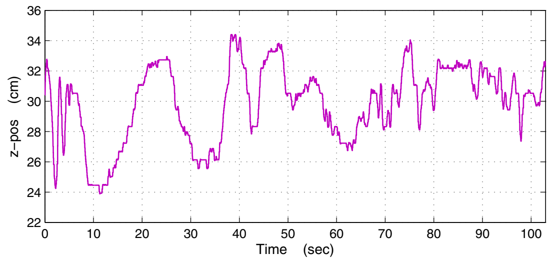

Fig. 6. Quadrotor z-position applying discrete optimal control law using $Q_{z,1}$ and $R_{z,1}$.

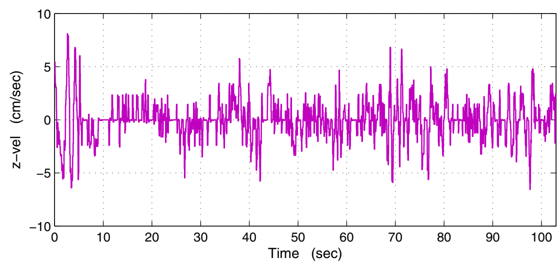

Fig. 7. Quadrotor z-velocity applying discrete optimal control law using $Q_{z,1}$ and $R_{z,1}$.

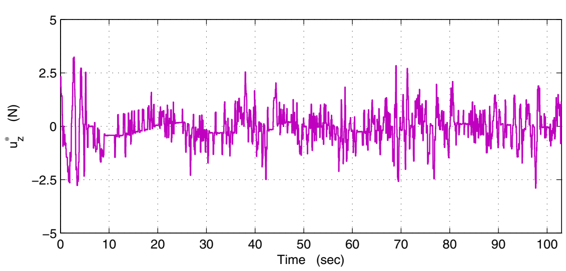

Fig. 8. Discrete optimal control law signal applied to control z-dynamics using $Q_{z,1}$ and $R_{z,1}$.

We set now the matrices $Q_{z,2}$ and $R_{z,2}$ in order to test the robustness of control law when external disturbances income to the dynamic system. With this selection of matrices the level of penalization of control signal is smaller with respect to the previously defined with matrices $Q_{z,1}$ and $R_{z,1}$. The matrices $Q_{z,2}$ and $R_{z,2}$ are defined as follows

$$Q_{z,2} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix} \qquad R_{z,2} = \begin{bmatrix} 30 & 0 \\ 0 & 170 \end{bmatrix} \tag{11}$$

For this second experiment, the z-position is shown in Figure 9, while the Figures 10 and 11 show the z-velocity and discrete optimal control signal respectively. The extarnal disturbance are applied around 32 seconds and 70 seconds.

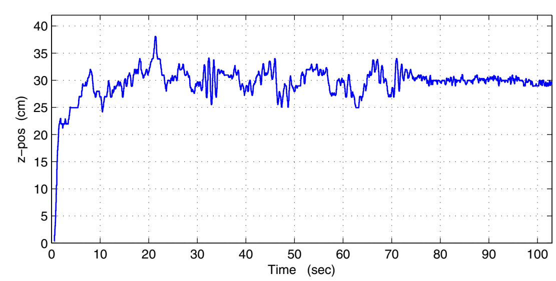

Fig. 9. Quadrotor z-position applying discrete optimal control law using $Q_{z,2}$ and $R_{z,2}$.

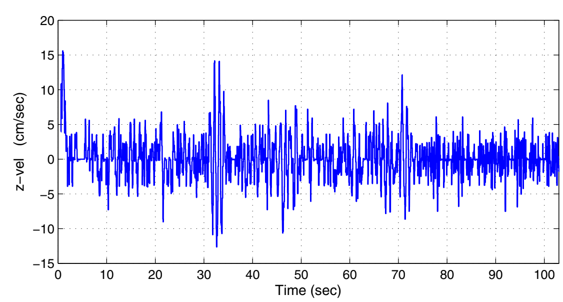

Fig. 10. Quadrotor z-velocity applying discrete optimal control law using $Q_{z,2}$ and $R_{z,2}$.

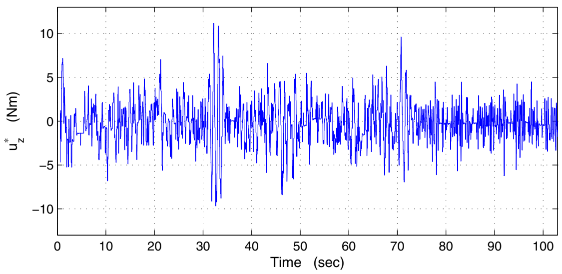

Fig. 11. Discrete optimal control law signal applied to control z-dynamics using $Q_{z,2}$ and $R_{z,2}$.

We can observe that the disturbances are well compensated by the optimal control. Finally a PD controller is tested in order to do an energy consumption comparison between it and the two discrete optimal controller described above. Experimental results for PD controller are shown in Figures 12, 13 and 14, where z-position, z-velocity and PD control signal are respectively plotted.

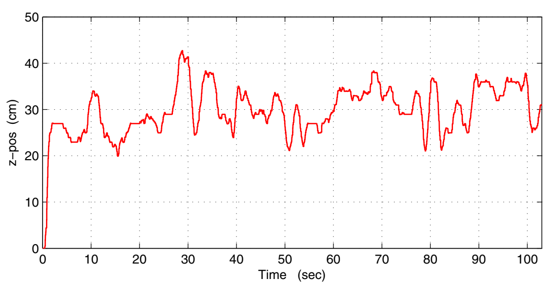

Fig. 12. Quadrotor z-position using a PD controller.

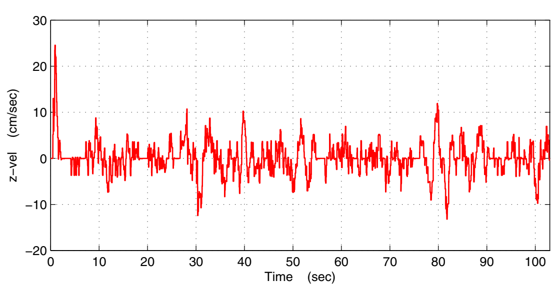

Fig. 13. Quadrotor z-velocity using a PD controller.

Controller	$J_{\|u\|}$	Time Interval (seconds)	Energy Saving (%)
PD_1	121.0	[0,103]	–
Optimal $Control_1$	102.59	[0,103]	15.21%
PD_1	34.7	[0,30]	–
Optimal $Control_2$	31.6	[0,30]	8.9%

TABLE I
PERFORMANCE INDEX EVALUATION.

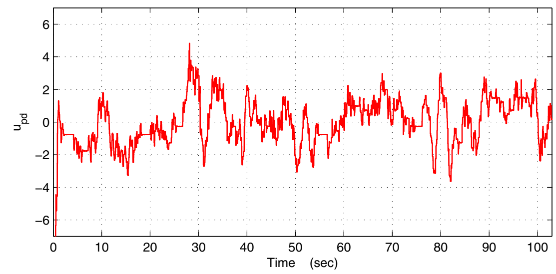

Fig. 14. PD control signal applied to control z-dynamics.

The performance index related to each control strategy applied to four rotor minihelicopter are shown in Table I. Energy was calculated using the numerical integral of absolute error $J_{|u|} = \int_{t_0}^{t_1} |u| dt$ in a finite interval of time for every controller considered.

In order to establish a coherent comparison between controllers PD_1 and Optimal $Control_2$ the performance index for both controllers are computed just before the disturbance is applied to mini helicopter, it means they are computed in $[0, 30]$ seconds. We can observe that the couple of optimal control strategies consume less energy to stabilize the flying robot.

V. CONCLUSIONS

In this article we present an optimal control law combined with a exact linearization for a quadrotor. Unlike to other approaches, our proposal does not require a bounded operation region and this advantage allows to choose the penalization matrices in an easy way. As it was showed by experimental test, different penalty level represents more time of flight of the quadrotor. However, as it well known, less energy applied to the actuators implies less robustness and vice versa. Future works include experimental validation to orientation control and optimal control law design considering the finite horizon problem.

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