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Robust Sensor Fault Diagnosis and Tracking Controller for a UAV Modelled as LPV System

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***Abstract*— This work is dedicated to the robust fault detection and tracking problem for a UAV system with external disturbances. A quadrotor modelled as a Linear Parameter Varying system (LPV) is considered as a target to design and to illustrate recent advanced control methods. Firstly, the fault detection problem is addressed by considering the design of a robust fault detection observer with H_∞ performance. The challenge is to attenuate external disturbances and a to generate useful residual signals to detect and isolate faults in sensors. Second, a feedback controller is designed by considering a comparator integrator control scheme to stabilize the system and to reach the tracking signal. In both cases the Lyapunov theory and $\mathscr{L}_2$-gain technique are used to obtain sufficient stability conditions in LMIs (linear matrix inequalities) formulation. Finally, some simulations in fault-free and faulty cases are done on the quadrotor system.**

I. Introduction

In the recent years, Quadrotor helicopter has become a popular Unmanned Aerial Vehicle (UAV), specially in military applications [1]. Nevertheless, quadrotor systems have been proved to be efficient in other tasks such as surveillance, search, rescue, remote sensing, geographic studies, recognition, aerial transportation, inspection and maintenance, among others [2]. Comparing to a conventional helicopter, a quadrotor is essentially simpler to build. In contrast, the differentials equation describing the dynamics of the quadrotor are high nonlinear, unstable and constantly affected by aerodynamic disturbances [3].

An attractive alternative to represent nonlinear dynamics is through Linear Parameter Varying models (LPV) approach. LPV systems are mathematical models that are able to exactly represent or to approximate to an arbitrary degree of accuracy a large class of nonlinear systems in a compact set of LTI models [4]. There are many works for LPV systems involving different topics of control as observer design [5], feedback control [6] and, fault diagnosis [7], [8]. In addition, some LPV models of quadrotor have been proposed with good results on observer design, stabilization and control [9], [10], [11].

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Beyond the difficult of design control systems, there is also the demand of reliability and safety. As reported by the Office of the Secretary of Defence of USA, identifies the development of self-repairing, smart flight control systems as a crucial step in the overall advancement of UAV autonomy [12]. To deal with this problem, fault diagnosis (FD) system is designed to identify malfunctions at any time of the flight envelop. In FD systems, the generation of residual signals is the core element of diagnosis. There are many ways to generate residuals by observer design, parity space, adaptive observers, among others. More detailed information can be consulted in survey [13]. In this paper, H_∞ approach to generate the residuals and to design the tracking controller system is adopted.

The main contribution of this paper is to develop a robust fault diagnosis residual generator and tracking controller for a quadrotor system modelled as LPV system. The observer and controller are designed based on Lyapunov and $\mathscr{L}_2$-gain theory in order to minimize the effect of disturbances. In addition, the tracking controller is designed by considering an integrator-comparator block and the LPV system. A bank of residual generators based on Generalized Observer Scheme (GOS) is built to perform robust fault detection and isolation. Finally, the fault diagnosis and the tracking-controller systems are applied to the quadrotor to illustrate the proposed method.

Notations: The notations used in this article are standard. For a matrix $A \in \mathbb{R}^{m\times n}$, A^T, A^{-1} and $A^\dagger$ denote its transpose, inverse and pseudoinverse respectively. The symbol $*$ denotes the transposed element in the symmetric positions of a matrix. He$\{A\}$ is a shorthand notation for $A + A^T$.

II. Dynamic model and problem definition

The quadrotor is a helicopter composed of four input forces provided by each propeller and six outputs representing its pose as shown in Fig. 1. The system under consideration is the nonlinear model adopted from [14], [11], and the LPV system given in [10]. From the parameters shown in Table I, the quad-rotor states and the control inputs are defined as

$$x = [x_0, y_0, z_0, \psi, \theta, \phi, u_0, v_0, \omega_0, p, q, r_o]^T \quad (1)$$

$$u = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T, \quad (2)$$

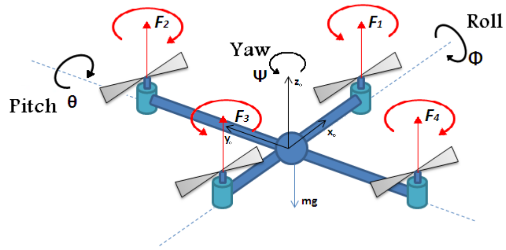

Fig. 1. Quadrotor configuration

TABLE I
PARAMETERS FOR THE QUADROTOR NONLINEAR MODEL

Parameter (unit)	Symbol	Value
Position (m)	$(x_o,\, y_o,\, z_o)$	-
Velocity (m/s)	$(u_0,\, v_0,\, \omega_0)$	-
Angular velocity expressed in a body reference frame (rad/s)	$(p,\, q,\, r_o)$	
Euler angles yaw, pitch and roll (rad)	$(\psi,\, \theta,\, \phi)$	-
Resulting thrust of the four rotors	u_1	-
Difference of thrust between the left rotor and the right rotor	u_2	-
Difference of thrust between the front rotor and the back rotor	u_3	-
Difference of torque between the two clockwise turning rotors and the two counter-clockwise turning rotors	u_4	-
Mass (kg)	m	0.7
Aerodynamic forces and moments acting on the UAV.	$(A_x,\, A_y,\, A_z)^T$ and $(A_p,\, A_q,\, A_r)^T$	-
Moments of inertia along the $x, y,$ and z directions $(kg.m^2)$	$\begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix}$	$\begin{bmatrix} 1.241 \\ 1.241 \\ 1.241 \end{bmatrix}$
Gravity of earth (m/s^2)	g	9.81
Distance from the motors to the centre of gravity (m)	d	0.3

The nonlinear model for the quadrotor is given by the following equations

$$\dot{x} = f(x) + \sum_{i=1}^{4} g_i(x) u_i \tag{3}$$

where

$$f(x) = \begin{bmatrix} u_0 \\ v_0 \\ \omega_0 \\ q\,\mathsf{sin}\phi\,\mathsf{sec}\theta + r_o \mathsf{cos}\phi\,\mathsf{sec}\theta \\ q\mathsf{cos}\phi - r_o\,\mathsf{sin}\phi \\ p + q\mathsf{sin}\phi\,\mathsf{tan}\theta + r_o\mathsf{cos}\phi\,\mathsf{tan}\theta \\ \frac{A_x}{m} \\ \frac{A_y}{m} \\ \frac{A_z}{m} + g \\ \frac{I_y - I_z}{I_x} q r_o + \frac{A_p}{I_x} \\ \frac{I_z - I_x}{I_y} p r_o + \frac{A_p}{I_y} \\ \frac{I_x - I_y}{I_z} q r_o + \frac{A_r}{I_z} \end{bmatrix}$$

and,

$$\begin{aligned} g_1(x) &= [0,\, 0,\, 0,\, 0,\, 0,\, 0,\, g_{1,7},\, g_{1,8},\, 0,\, 0,\, 0]^T \\ g_2(x) &= \left[0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, \frac{d}{I_x},\, 0,\, 0\right]^T \\ g_3(x) &= \left[0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, \frac{d}{I_y},\, 0\right]^T \\ g_4(x) &= \left[0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, \frac{I}{I_z}\right]^T, \end{aligned}$$

with

$$\begin{aligned} g_{1,7} &= -\frac{1}{m}\left(\mathsf{cos}\phi\,\mathsf{cos}\psi\,\mathsf{sin}\theta + \mathsf{sin}\phi\,\mathsf{sin}\psi\right) \\ g_{1,8} &= -\frac{1}{m}\left(\mathsf{cos}\phi\,\mathsf{sin}\theta\,\mathsf{sin}\psi - \mathsf{cos}\psi\,\mathsf{sin}\phi\right) \\ g_{1,9} &= -\frac{1}{m}\left(\mathsf{cos}\theta\,\mathsf{cos}\phi\right). \end{aligned}$$

To obtain the LPV equations of the quadrotor, the nonlinear model is linearised around different operation points. Then, by considering the different sub-models the following representation is obtained

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{h} \rho_i(x(t)) \left[A_i x(t) + B_i u(t) + R_i d(t)\right] \\ y(t) &= C x(t) \end{aligned} \tag{4}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $d(t) \in \mathbb{R}^q$, and $y(t) \in \mathbb{R}^p$ are the state vector, the control input, the disturbances, and the measured vector respectively. A_i, B_i, R_i and, C and are constant matrices of appropriate dimensions. $\rho_i(x(t))$ are scheduling functions which depend on $x(t)$. The scheduling functions of the h sub-models satisfy the following convex set sum property:

$$\forall i \in [1, 2, ..., h],\ \rho_i(x(t)) \geq 0,\ \sum_{i=1}^{h} \rho_i(x(t)) = 1,\ \forall t. \tag{5}$$

By assuming observable outputs and to generate the residuals, a robust fault diagnosis observer of (4) described by the following equations is considered

$$\dot{z}(t) = \sum_{i=1}^{h} \rho_i(x(t)) \left[N_i z(t) + G_i u(t) + L_i y(t)\right] \tag{6}$$

$$\begin{aligned} \hat{x}(t) &= z(t) + T_2 y(t) \\ r(t) &= W(y(t) - C\hat{x}(t)), \end{aligned} \tag{7}$$

where $z(t)$ represents the state vector of the observer, $\hat{x}(t)$ the estimated state vector. N_i, G_i, L_i, and T_2 are the gain matrices of (6) to be synthesized. $r(t)$ is the residual signal and W the residual weighting matrix to determine. The gain matrices of the fault diagnosis observer (6) must be designed in order to guarantee the convergence of the state estimation error and maximize the robustness against disturbances $d(t)$.

The second objective is to design a feedback controller such that the steady-state response tends to $\lim_{t\to\infty} y(t) := w(t)$, where $w(t)$ is the desired position. To reach the desired position an integrator comparator is added as shown in Fig. 2, such that

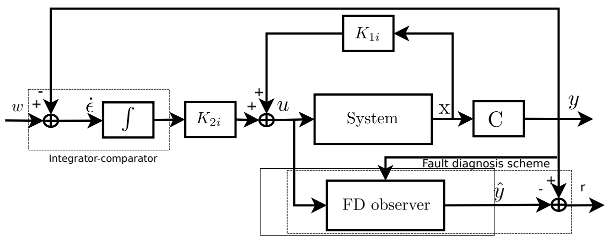

Fig. 2. Fault diagnosis and Tracking controller scheme

$$\dot{\epsilon}(t) = w(t) - y(t) = w(t) - Cx(t). \tag{8}$$

The control law $u(t)$ is given by the following feedback controller

$$u(t) = K_{1i}x(t) + K_{2i}\epsilon(t), \tag{9}$$

where K_{1i} and K_{2i} are the state feedback gains matrices to be synthesised. Then, the problem is reduced to determine optimal values of the controller gains.

III. Fault diagnosis system

Under the presence of sensor faults, system (4) can be represented by

$$\dot{x}(t) = \sum_{i=1}^{h} \rho_i(x(t)) \left[A_i x(t) + B_i u(t) + R_i d(t)\right] \tag{10}$$
$$y(t) = Cx(t) + D_f f(t)$$

where $f(t)$ denotes the sensor fault vector. Clearly, from Fig. (2) it is easy to see that the fault diagnosis observer is decoupled from the controller, it means that the fault diagnosis observer can be designed separately. The estimation error is defined as

$$e(t) = x(t) - \hat{x}(t)$$
$$e(t) = (I - T_2C)x(t) - z(t),$$

Under the assumption that there exists $T_1 \in \mathbb{R}^{n\times n}$ matrix such that

$$T_1 = I - T_2C, \tag{11}$$

then, a particular solution of matrices T_1 and T_2 is computed as:

$$\begin{bmatrix} T_1 & T_2 \end{bmatrix} = \begin{bmatrix} E \\ C \end{bmatrix}^{\dagger}. \tag{12}$$

The dynamic of the error equation is given by

$$\dot{e}(t) = T_1\dot{x}(t) - \dot{z}(t)$$
$$\dot{e}(t) = \sum_{i=1}^{h} (\rho_i(x(t))) \left[T_1A_ix(t) + T_1B_iu(t) + T_1R_id(t) \right.$$
$$\left. - N_iz(t) - G_iu(t) - L_iy(t)\right] \tag{13}$$

with

$$N_i = T_1A_i - K_iC \tag{14}$$
$$K_i = L_j - N_iT_2. \tag{15}$$
$$G_i = T_1B_i \tag{16}$$

Then, by considering (13)-(15) and (7), the residual state-space error system is obtained as

$$\dot{e}(t) = \sum_{i=1}^{h} \rho_i(x(t) \left[N_ie(t) - T_1R_id(t)\right]$$
$$r(t) = WCe(t). \tag{17}$$

Sufficient conditions to guarantee asymptotically stability of (17) must be obtained despite of the disturbance signal. In order to reach this objective, the problem is reformulate in the H_∞ performance approach. The following Theorem is obtained as result:

Theorem 3.1: Given system (4), the observer (6) and let the attenuation level $\gamma > 0$. The residual state-space error system (17) is globally stable with H_∞ performance if it satisfy $\| r(t) \|_2^2 \leq \gamma^2 \| d(t) \|_2^2$ and if there exist a matrix $P = P^T \geq 0$ and gain matrices $K_j = P_1^{-1}\Phi_i$, $\forall i,j \in [1,2,...,h]$, such that:

$$\begin{bmatrix} \mathsf{He}\left(A_i^TQ - C^T\Phi_i^T\right) & T_1R_i & (WC)^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} \leq 0. \tag{18}$$

Proof: To guarantee that asymptotically convergence to zero and robustness, H_∞ performance is written as

$$J_{rd} := \dot{\Omega}(t) + J_1 \leq 0 \tag{19}$$
$$J_1 = r^T(t)r(t) - \gamma^2 d^T(t)d(t) \leq 0, \tag{20}$$

where J_{rd} represents the $\mathscr{L}_2$ gain of system (17) (from $d(t)$ to $r(t)$) bounded by γ. $\Omega(t)$ is a Lyapunov function defined as $\Omega(t) = V(x_e(t)) = e^T(t)Pe(t)$. The derivate of the Lyapunov function is obtained as

$$\dot{\Omega}(t) = \sum_{i=1}^{h} \rho_i(x(t)) \left[e(t)^T \mathsf{He}(A_i - L_iC)^TPe(t)\right]. \tag{21}$$

Then the condition (19) is rewritten as

$$\begin{bmatrix} e(t)^T \\ d(t)^T \end{bmatrix}^T \Lambda_i \begin{bmatrix} e(t) \\ d(t) \end{bmatrix} \leq 0, \tag{22}$$

with

$$\Lambda_i = \begin{bmatrix} \mathsf{He}\left(A_i^TP - C^T\Phi_i^T\right) + C^TW^TWC & PRi \\ * & -\gamma^2 I \end{bmatrix}. \tag{23}$$

Hence, if $\Lambda_i \leq 0$ implies $J_{rd} \leq 0$. Finally the Schur complement implies (18). This ends the proof. ■

The purpose of sensor fault diagnosis based on observers is to generate a bank of residuals. A generalized observer scheme (GOS) as proposed in [15] is considered. This classical scheme consists of p observers, where p is the number of sensor faults under consideration. A subsystem insensitive to a component f_p of the fault vector $f(t)$ is extracted for each observer by deriving the output vector $y(t)$. In order to isolate the sensor faults, a normalized residual vector is generated such that its p^{th} component is sensitive to all faults but p^{th} one. The bank of p fault diagnosis observers are given by:

$$\dot{z}^p(t) = \sum_{i=1}^{h} \rho_i(x(t)) \left[N_i^p z(t) + G_i^p u(t) + L_i^p C^p x(t)\right]$$
$$\hat{x}^p(t) = z^p(t) + T_2 C^p x(t). \quad (24)$$
$$\| r^p(t) \| = \| W^p(y^p(t) - C^p \hat{x}^p(t)) \| . \quad (25)$$

Each observer satisfy observability condition. By solving the LMI system (18) for each set of given input matrices C^p the robustness and convergence are ensured. The bank of observers generates an incidence matrix as shown in Table II. Each column is called the coherence vector associated to each fault signature. It is clear from Table II that the decoupled observer method provides an efficient FDI technique for sensor faults. In the presence of a sensor fault, the observer, insensitive to the associated fault, estimates state vector $\hat{x}(t)$ and consequently estimates the output corrupted by the fault. Decision making can be carried-out according to an elementary binary logic.

TABLE II
INCIDENT MATRIX

Fault	F_0	F_1	F_2	...	F_p
$\| r^1 \|$	0	0	1	1	1
$\| r^2 \|$	0	1	1	1	1
...	0	1	1	0	1
$\| r^p \|$	0	1	1	1	0

IV. TRACKING CONTROLLER DESIGN

By considering the comparator and integrator (8) and the system defined by (4), an augmented system $x_c = [x^T \ \epsilon^T]^T$ is obtained as

$$\dot{x}_c(t) = \sum_{i=1}^{h} \rho_i(x(t) \left[\bar{A}_{ci} x_c(t) + \bar{B}_{ci} u(t) + \bar{R}_i d(t)\right] + \bar{B}_w w(t) \quad (26)$$

with

$$\bar{A}_{ci} = \begin{bmatrix} A_i & 0 \\ -E & 0 \end{bmatrix}, \ \bar{B}_c = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \ B_w = \begin{bmatrix} 0 \\ I \end{bmatrix}, \ \bar{R}_i = \begin{bmatrix} R_i \\ 0 \end{bmatrix}.$$

By assuming that pair $(\bar{A}_{ci}, \bar{B}_{ci})$ are controllable and by considering $u(t)$ as defined in (9), the following closed-loop system is obtained

$$\dot{x}_c(t) = \sum_{i=1}^{h} \rho_i(x(t) \left[\left(\bar{A}_{ci} - \bar{B}_{ci} K_i\right) x_c(t) \right. \quad (27)$$
$$\left. + \bar{R}_i d(t)\right] + \bar{B}_w w(t). \quad (28)$$

Sufficient conditions are given through the following Theorem to stabilize and control the system by considering the $\mathscr{L}_2$-gain from $d(t)$ to $x_c(t)$.

Theorem 4.1: Given system (4), the comparator-integrator (6), the feedback controller defined by (9) and let the attenuation level $\gamma_c > 0$. The close loop system error system (28) is globally stable with H_∞ performance if $\| x_c(t) \|_2^2 \leq \gamma_c^2 \| d(t) \|_2^2$ and if there exist a matrix $X = X^T \geq 0$ and gain matrices $K_j = X_1^{-1} \Xi_i$, $\forall i, j \in [1, 2, ..., h]$, such that:

$$\begin{bmatrix} \mathsf{He}\left(X \bar{A}_{ci}^T + \Xi_i^T \bar{B}_{ci}^T\right) + \bar{R}_i \bar{R}_i^T & X \\ * & -\gamma^2 I \end{bmatrix} \leq 0. \quad (29)$$

Proof: Similar to the observer design, the stability stability conditions are obtained by considering the $\mathscr{L}_2$-Gain from $d(t)$ to $x_c(t)$ such that

$$J_{x_c d} := \dot{\Gamma}(t) + J_2 \quad (30)$$
$$J_2 = x_c^T(t) x_c(t) - \gamma_c^2 d^T(t) d(t) \leq 0, \quad (31)$$

$J_{x_c d}$ represents the $\mathscr{L}_2$ gain of system (28) (from d to x_c) bounded by γ_c. $\Gamma(t)$ is a Lyapunov function defined as $\Gamma(t) = x_c^T(t) P x_c(t)(t)$. By considering the procedure described in the previous section, sufficient conditions are obtained as follows

$$\begin{bmatrix} \left(\bar{A}_{ci} - \bar{B}_{ci} K_i\right)^T P + P\left(\bar{A}_{ci} - \bar{B}_{ci} K_i\right) + I & P\bar{R}_i \\ * & -\gamma_c^2 I \end{bmatrix} \leq 0. \quad (32)$$

By considering $X = P^{-1}$ and pre and post multipliying (32) by $\begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix}$, the following LMI is rewritten as

$$\begin{bmatrix} X \bar{A}_{ci}^T + \bar{A}_{ci} X + X^T X & \bar{R}_i \\ * & -\gamma^2 I \end{bmatrix} \leq 0,$$

finally by considering the Schur complement of $X^T X$, and $\bar{R}_i$ the LMI (29) is derived. This complete the proof.

■

Remark 4.1: The minimization of γ_c may result in slow dynamics of the state estimation error. This problem can be solved by pole assignment of the matrices $(\bar{A}_{ci} - \bar{B}_{ci} K_i)$ in left half complex plane such that

$$\lambda_j(\bar{A}_{ci} - \bar{B}_{ci} K_i) \in \mathbb{D}, \ j = 1, 2, ..., n; \ i = 1, 2, ..h,$$

$\mathbb{D}$ is the α stability region as defined in [16] . Then the following Corollary (4.1) is obtained

Corollary 4.1: Given system (4), the comparator-integrator (6), the feedback controller defined by (9) and let the attenuation level $\gamma_c > 0$. The close loop system error system (28) is globally stable with H_∞ performance if $\| x_c(t) \|_2^2 \leq \gamma_c^2 \| d(t) \|_2^2$ and if there exist a matrix $X = X^T \geq 0$ and gain matrices $K_j = X_1^{-1} \Xi_i$, $\forall i, j \in [1, 2, ..., h]$ and, a positive scalar α such that:

$$\begin{bmatrix} Z_i + \bar{R}_i \bar{R}_i^T + 2\alpha P & X \\ * & -\gamma^2 I \end{bmatrix} \leq 0 \quad (33)$$

with

$$Z_i = X \bar{A}_{ci}^T + \bar{A}_{ci} X + \Xi_i^T \bar{B}_{ci}^T + \bar{B}_{ci} \Xi_i.$$

Proof: The proof is easily derived by considering the α-stability in the $\mathscr{L}_2$-gain equation (19), such that

$$J_{x_c d}: \quad = \quad \dot{\Gamma} + J_2 + 2\alpha\Omega \leq 0. \tag{34}$$

By solving (34), the conditions described in the LMI are derived. More accurate pole assignment can be obtained by defining more precise LMI regions as describe in [16]. ■

V. SIMULATION RESULTS

The proposed design approach on the Quadrotor is illustrated on this section. The matrices of (4) are not displayed here due to space limitations, however these can be consulted in the referenced paper [10]. For simulation purpose, an additional disturbance matrix $B_{di} = [1, 0, ..., 0]^T$ is considered. Convex scheduling functions are defined as:

$$\begin{aligned}
\rho_i(x(t)) &= \frac{\mu_i(x_4(t))}{\sum_{i=1}^{3} \mu_i(x_4(t))} \\
\mu_1(t) &= exp\left[\frac{1}{2}\left(\frac{x_4(t)+1.8}{0.8}\right)^2\right] \\
\mu_2(t) &= exp\left[\frac{1}{2}\left(\frac{x_4(t)-1.8}{0.8}\right)^2\right] \\
\mu_3(t) &= exp\left[\frac{1}{2}\left(\frac{x_4(t)}{0.8}\right)^2\right].
\end{aligned} \tag{35}$$

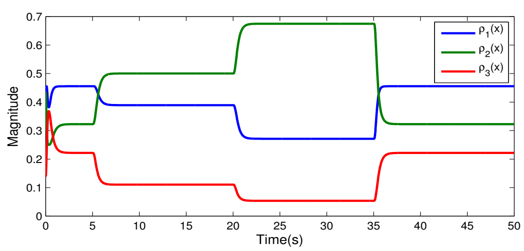

Fig. 3. Evolution of the scheduling functions.

The synthesis of the stable LPV observer with H_∞ performance (6) has been solved with Yalmip Toolbox [17]. The attenuation level obtained by solving Theorem 3.1 is $\gamma = 0.8561$ which guarantee a good attenuation of disturbances. Gain matrices are not presented here due to space limitations. For simulation purpose, the perturbation $d(t)$ is chosen as random signal uniformly distributed in $[-0.5, 0.5]$. The gains of the controller are computed by solving the Corollary 4.1 and a LMI region with $\alpha = 2$ was chosen to avoid slow dynamics. Initial conditions are considered as $x(0) = [0.2, 0.5, 0, 1.2217, 0, ..., 0]^T$ and $\hat{x}(0) = [0.2, 0.3, 0, ..., 0]^T$.In practice, initial conditions are chosen such as they correspond to the initial measured value of the states. Actuator saturation is not taking into consideration. Simulation results are displayed as follows. Fig. 3 shows the interaction between the models as defined by the scheduling functions. Fig. 4 shows the references and tracking trajectory of the positions (x, y, z) and the yaw angle ψ. Note that despite the disturbance and the set-up variations on the reference, the system reaches the reference without present any unstable behaviour. Fig. 5 shows residual in fault-free case. As result of the robust

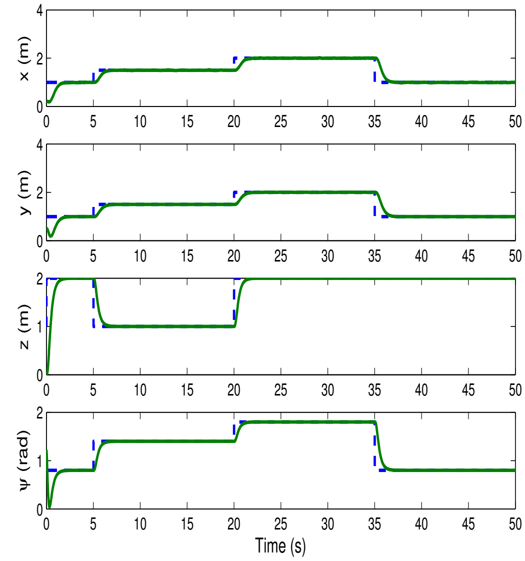

Fig. 4. Desired position (dashed line) and real positions (continuous line).

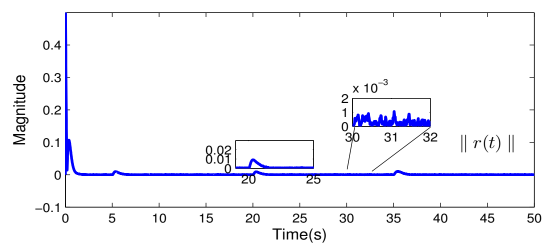

Fig. 5. Residual in fault-free case.

design the disturbance are well attenuated as displayed in the zoom from $t = 30s$ to $t = 32s$. After convergence the residual remains closer to zero. The small variations presented at $t = 5s$, $t = 20s$ and, $t = 35s$ are due to changes on the set-up positions.

In order to prove the effectiveness of the proposed method under faults, a bank of 4 residual generators (one for each output) are designed as described in Section III. Two sensor faults are induced as displayed in Fig 6e. The first fault occurring on the second sensor between $t = 20s$ and $t = 25s$ is a fault with sinusoidal behaviour. The second fault from $t = 45s$ is an abrupt step fault applied to the third sensor. The normalized residual signals are shown in Fig. 6a-d. The fault detection can be done easily by comparing the residuals with the incident matrix given in Table II. For example for the first fault occurred in the sensor 2 all the residual present some changes at $t = 25s$, only the residual of the second sensor 2 remains without change. This particular signature allows to isolate the fault in sensor 2. Clearly, for all cases, the fault detection turns out to be successful. With respect to the tracking, all the positions that are not affected by faults are well reached as displayed in Fig. 7, but not the position with the faulty sensor. Nevertheless, once that the fault disappears,

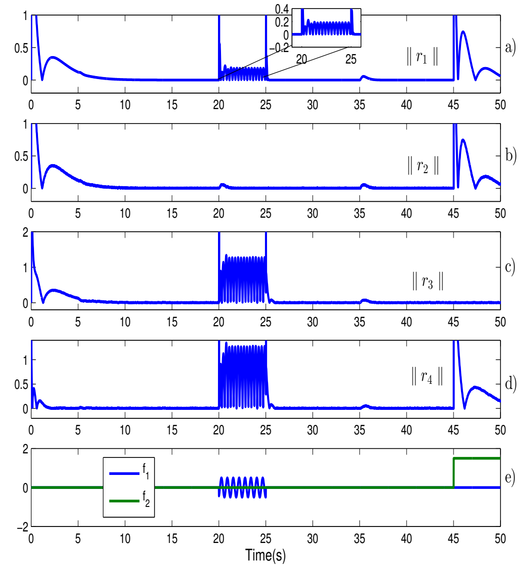

Fig. 6. a-d) Normalized residuals in faulty-case; e) induced faults.

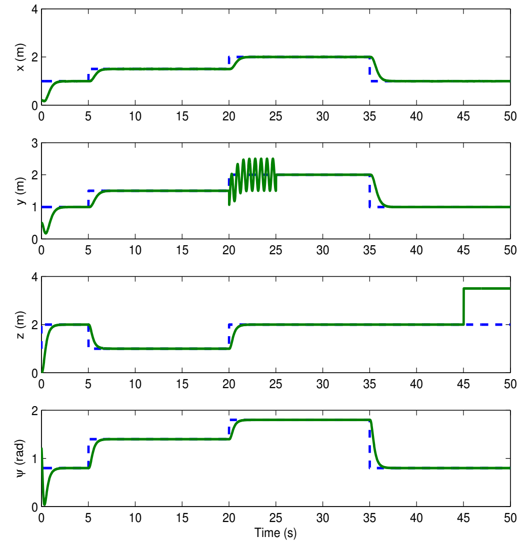

Fig. 7. Desired position (dashed line) and real positions (continuous line) in faulty-case.

the output converges again to the desired position. In order to guarantee tracking for all outputs in the presence of faults, a fault tolerant control strategy will be addressed in future work.

VI. Conclusions

In this paper, a robust fault diagnosis and tracking controller for a quadrotor modelled as LPV system was developed. Using Lyapunov and $\mathscr{L}_2$-gain theory sufficient conditions in the LMI formulation were obtained. In the same spirit, sufficient conditions were obtained to compute the gains of the controller in order to stabilize the nonlinear system and reach the tracking signal. In order to detect and isolate sensor faults, a set of residuals were generated for a bank of observers so that each residual was sensitive only to one fault. Each observer was designed to be robust against disturbances. Simulations results shows the effectiveness of the proposed method. Future research will be addressed in topic of fault tolerant control.

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