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# Fault Tolerant Control Using PID Structured Optimal Technique Against Actuator Faults in a Quadrotor UAV

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*Abstract*— **This paper proposes and implements a proportional-integral-derivative (PID) structured fault tolerant controller. The fault tolerant controller is designed using linear quadratic (LQ) technique against the partial loss of control effectiveness due to the actuator fault. Compared with the conventional PID controller, the PID structure optimal controller has the ability to manipulate the multiple-input and multiple-output system (MIMO), accommodate the fault, respond fast, and eliminate the steady-state error. Performance of the proposed controller is tested based on the platform of an unmanned quadroter helicopter (known as Qball-X4).**

*Index Terms*— **PID, LQR, Optimal control, Fault tolerant control, Actuator faults**

## I. INTRODUCTION

Challenges for improving the safety exist in an unmanned aerial vehicle (UAV), as a UAV is increasingly becoming a topic of interest [1–4], not only in academic but also in industrial applications. Quadrotor helicopter has often been chosen by researchers because of their efficiency in performing tasks [4, 5]. The real system of quadrotor is definitely a non-linear system, which brings great inconvenience to design controller due to the complexity of the model and lack of a well developed theory. In addition to many investigations and researches on nonlinear control design techniques, people have also tried to use much more convenient model such as the linearizion model and the fuzzy logic model to approximate the real system.

"Fault tolerant control systems (FTCS) are control systems which possess the ability to accommodate component failures automatically" [6]. "A fault is defined as an unpermitted deviation of at least one characteristic property of a variable from an acceptable behavior" [7, 8]. Fault tolerant control is usually classified into two categories: active fault tolerant control (AFTC) and passive fault tolerant control (PFTC). Although active fault tolerant control is more powerful than passive fault tolerant control theoretically, the design complexity and requirement of fault detection and diagnosis (FDD) capability limits the application of AFTC system in engineering application. Therefore, passive fault tolerant control is still attractive towards practical application of fault tolerant control systems.

LQR (linear quadratic regulator) has been extensively studied [9–11], which is one of the widely applied modern control techniques. Algebraic Riccati equation has been used to conduct LQR controller gain. Some properties of the LQR system depend on the selection of weighting matrix $Q$ and $R$, such as the trade-off between the performance and the energy cost of system input. By appropriate selection of the weighting matrices $Q$ and $R$, it is possible for multiple-input and multiple-output (MIMO) systems to have fault-tolerant features with a set of known faults or failures. Proportional-integral-derivative (PID) controller is the most widely used controller in the industry. Its simple structure is easily understood by engineers. However, PID controller is not effective to deal with multiple variable systems. Papers [12–14] try to establish connections between LQR and PID by tuning the PID parameter using LQR technique. But they are still coping with the single-input and single-output (SISO) system. Paper [15] adopts PI structure to fulfill the fault tolerant control with the help of a two-stage Kalman filter for adjusting the control gain. From the practical point of view and considering the great success of PID controller in various industrial appllications, the idea of designing a fault tolerant control with PID structure has been investigated to accommodate the loss of control effectiveness fault in actuators of Qball-X4 (a quadrotor at the Networked Autonomous Vehicles Lab (NAVL) of Concordia University) [16].

The motivation to design the controller is that it is hard for an engineer to tune the parameters when the performance is not the same as the expected due to the Qball-X4's parameter uncertainty and change. The objective of this paper is to design a fault tolerant controller which exploits and combines the great benefits and advantages of LQ technique and PID controller for addressing actuator faults. The contents are organized as follows: Section 2 presents the system dynamic model; Section 3 describes the fault model of actuators; Section 4 adopts the simulation results to illustrate the control performance of the designed controller, and the last section concludes this paper.

## II. MATHEMATICAL MODEL OF QBALL-X4 IN HEIGHT

### A. Non-linear Mathematical Model of Qball-X4 in Height

The dynamics of a real-life physical system can be represented in state-space model in the following general form

$$\begin{cases} \dot{x}(t) &= f(x(t), u(t)) \\ y(t) &= h(x(t), u(t)) \\ x(0) &= x_0 \end{cases} \quad (1)$$

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Linearize the Eq. (1) around the trim point $(x_t, u_t)$,

$$\dot{x} \approx A(t)x(t) + B(t)u(t) \tag{2}$$

where

$$A(t) = \frac{\partial f}{\partial x}|_{x_t,u_t} \tag{3}$$
$$B(t) = \frac{\partial f}{\partial u}|_{x_t,u_t}$$

Note that $x_t$ and $u_t$ are the solution of the equilibrium function in trim condition. Literature [17–19] depict the process of model derivative for a quadrotor. This paper takes the dynamic of Qball-X4 in height to illustrate the process of designing a PID structured optimal fault tolerant controller.

$$\ddot{x} = -g + (cos\theta cos\phi)\frac{U}{M} \tag{4}$$

with

$$U = \sum_{i=1}^{4} T_i$$

where $g$ is the gravity acceleration, $\theta$ is the pitch angle, $\phi$ is the roll angle, $U$ is the lift generated by the four actuators and $M$ is the mass of the Qball, $T_i (i = [1,2,3,4])$ is the lift generated by a rotor. As Qball-X4 is equipped with 4 rotors, $i \in$ [1,2,3,4], $\ddot{x}$ is the second derivative of the height, which means acceleration in longitudinal direction. The transfer function of an actuator is

$$v_i = \begin{cases} 0 & PWM_i < 0.05 \\ \frac{\omega}{s+\omega}(PWM_i - 0.05) & 0.05 < PWM_i \leq 0.1 \end{cases} \tag{5}$$

As can be seen, there is a dead zone in actuators, this dead zone leads to the nonlinearity of the actuator. However, the dead zone can be solved by changing a coordinator. Let $u_{PWM_i} = PWM_i - 0.05$, substitute the term in Eq. (5), the dynamic of the actuator changes to linear dynamic system.

$$v_i = \frac{\omega}{s+\omega} u_{PWM_i} \tag{6}$$

The transfer function between the lift generated by each actuator and corresponding PWM input is

$$T_i = \frac{K \times \omega}{s+\omega} u_{PWM_i} \tag{7}$$

where $v_i$ is the $i^{th}$ channel of actuator, $T_i$ is the $i^{th}$ lift generated by the $i^{th}$ channel of actuator, $K$, $\omega$ are parameters of the actuators. For the longitudinal motion only, $\theta$ as well as $\phi$ are equal to zero; thereby, Eq. (4) is simplified to

$$\ddot{x} = -g + \frac{U}{M} \tag{8}$$

### *B. Linearized Model of Qball-X4*

Using a small perturbation method around the trim point $(x_t, u_t)$ to deal with Eq. (8).

$$\delta\ddot{x} = \frac{\delta U}{M} \tag{9}$$

To simplify the expression, equation (9) is represented as:

$$\ddot{x} = \frac{U}{M} \tag{10}$$

It means that the height of the helicopter is only related to the gravity, its mass and the lift generated by the four propellers from the height dynamic equation. One can use

$$v = \frac{\omega}{s+\omega} u_{PWM} \tag{11}$$

to express the transfer function of an actuator. To rewrite Eq. (10) in state-space model, one obtains

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{K}{M} \\ 0 & 0 & -\omega \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} [1111] \begin{bmatrix} u_{PWM_1} \\ u_{PWM_2} \\ u_{PWM_3} \\ u_{PWM_4} \end{bmatrix} \tag{12}$$

where $\ddot{x}$ and $\omega$ are the same as the definition in Eq. (4) and Eq. (5) respectively, $\dot{x}$ is the first order derivative about height. The Eq. (12) is presented in the form

$$\dot{x_0}(t) = Ax_0(t) + Bu(t) \tag{13}$$

where $x_0(t) = [x_1(t), x_2(t), x_3(t)] = [x(t), \dot{x}(t), v]$, $u(t) = [u_{PWM_1}, u_{PWM_2}, u_{PWM_3}, u_{PWM_4}]^T$ .
As $x_t$ is assumed to be a trim point value, hence $x_t$ is treated as a constant. It is defined as $x_t = 0$ without losing generality in the following. $u_t$ is the solution of Eq. (4).

## III. FAULT TOLERANT CONTROL STRUCTURE AND MATHEMATICAL MODEL WITH FAULTS

Once an actuator encounters faults, to maintain the same performance of the actuator, the controller tries to compensate the handicap caused by the faulty actuator. It forces the remaining actuators to work in the state of over their normal condition, which may damage the remaining actuators due to the physical limitations of the actuators. Hence, it is necessary to put the limitation of actuator into consideration. Fault tolerant control structure about actuators failure and the expression of Qball-X4 model with faults are given as follows.

The structure of fault tolerant control is showed in Fig. 1.

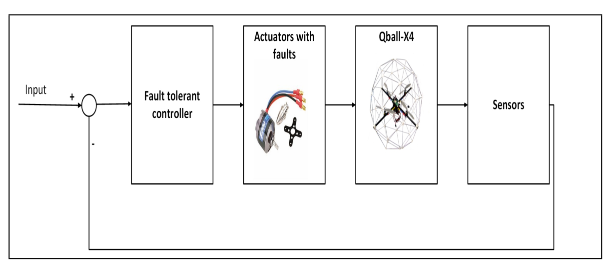


Fig. 1. Fault tolerant control system in closed control loop

The fault addressed in this paper is the partial loss of control effectiveness in actuators. Here vector $[r_1, r_2, r_3, r_4]$ is used to express the partial loss of control effectiveness in actuators, which is a value $\in$ [0 1]. A value $r_i = 0$ (i $\in$ [1,2,3,4]) means the actuator is in normal condition, while a value $r_i = 1$ stands for the total loss of effectiveness of $i^{th}$

actuator. The actuator fault is conveniently represented in the form of multiplicative modelling [20], so the actuator fault $u_k^f$ at time instant $k$ is presented in Eq. (14) based on the normal actuator input $u_k$ at time instant $k$.

$$u_k^f = u_k + (I - \Sigma_A)(\bar{u} - u_k) \tag{14}$$

where $I$ is the unity matrix in proper dimensions, $\Sigma_A = diag\{\delta_1^a, \delta_2^a, \ldots, \delta_m^a\}$, $\delta_i^a$ is the loss of control effectiveness, which value is $\in [0,1]$, $\delta_i^a = 0$ means a total fault of the $i^{th}$ actuator of the controlled system, $m$ is the number of actuators, and $\bar{u}$ is not a manipulated value. Based on the fault expression in Eq. (14), mathematical model of Qball-X4 with actuator faults can be expressed in the following equation with $\bar{u} = 0$.

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{K}{M} \\ 0 & 0 & -\omega \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix}$$
$$[\delta_1^a \ \delta_2^a \ \delta_3^a \ \delta_4^a] \begin{bmatrix} u_{PWM_1} \\ u_{PWM_2} \\ u_{PWM_3} \\ u_{PWM_4} \end{bmatrix} \tag{15}$$

## IV. PID Structure Based Optimal Fault Tolerant Control Design

This section presents the procedures of designing PID structure based optimal fault tolerant controller. The basic linear quadratic (LQ) problem is briefly introduced for the PID structure based optimal FTC design. The PID structure is illustrated and connected to the LQ technique.

### A. The Linear Quadratic Regulator

**LQR Theorem**

Let the system $(A,B)$ be reachable. Let $R$ be positive definite and $Q$ be positive definite (semi-definite), then the closed-loop system $(A-Bk)$ is asymptotically stable.

The controller is to solve a regulator problem. Supposing the initial condition of a real system is non-zero, the controller is to take the plant from the non-zero state to a zero state. The design of the controller is to minimize the objective function Eq. (16) subjected to the model in Eq. (12).

$$J = \frac{1}{2}\int_0^T (x^T Qx + u^T Ru)\ dt + \frac{1}{2}x^T(T)P_1 x(T) \tag{16}$$

The Hamiltonian H is defined in Eq.(17).

$$H(x(t),u(t),\lambda,t) = L(x(t),u(t)) + \lambda^T f(x(t),u(t)) \tag{17}$$

where $L(x(t),u(t))$ is $x^T Qx + u^T Ru$, $\lambda$ is the Lagrangian multiplier vector adjoined to the constraints of the system dynamics, while $\lambda^T$ is the transpose of $\lambda$. According to the Pontryagin's minimum principle:

$$H = x^T Qx + u^T Ru + \lambda^T(Ax + Bu) \tag{18}$$
$$\dot{x} = (\frac{\partial H}{\partial \lambda}) = Ax + Bu \qquad x(0) = x_0$$
$$-\dot{\lambda} = (\frac{\partial H}{\partial x}) = Qx + A^T\lambda \qquad \lambda(T) = P_1 x(T)$$
$$0 = \frac{\partial H}{\partial u} = Ru + \lambda^T B$$

where

$$u(k_i) = \begin{bmatrix} u_{PWM_1} \\ u_{PWM_2} \\ u_{PWM_3} \\ u_{PWM_4} \end{bmatrix}$$

$Q = Q^T \geq 0$, $R = R^T > 0$. The objective function establishes the trade-off between the convexity speed of the control speed and the energy used.

Suppose $\lambda = P(t)x(t)$, combined with Pontryagin's minimum principle in Eq. (18) and H-J-B (Hamilton-Jacobi-Bellman) equation, Riccati equation can be obtained as shown in Eq. (19)

$$-\dot{P} = PA + A^T - PBR^{-1}B^T P + Q \tag{19}$$

subject to $P(T) = P_1$, where $P_1$ is defined in the cost-to-go function in Eq. (16). The optimal control efforts is denoted in the form

$$u(t) = -R^{-1}B^T P(t)x \tag{20}$$

The cost-to-go function is subject to the system dynamic Eq. (1) around the trim point $(x_t, u_t)$. The control efforts turns out in the form $u = -Kx$, where $K$ is solved by the Eq. (21) when $T = \infty$.

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \tag{21}$$

and the feedback control gain is $K = R^{-1}B^T P$. Hence, the feedback control efforts is $u(t) = -Kx$, which makes the controlled system change to

$$\dot{x} = (A - BK)x \tag{22}$$

Note that the equation is a linearized equation around the trim point $(x_t, u_t)$. Therefore, the actual input $u_{ac}$=$-Kx + u_t$.

### B. PID Structure of Optimal Fault Tolerant Control

To design a PID structure of optimal fault tolerant controller for a system expressed in Eq. 12, recall the structure of PID controller

$$u_{PID}(t) = K_p e(t) + K_i \dot{e}(t) + K_d \int_0^T e(t)\ dt \tag{23}$$

where $e(t)$ is defined as $e(t) = y_r - y$. $y_r$ is the reference trajectory, $y$ denotes the output of the controlled system. PID controller is to use the error between the reference input and the actual output through proportional, integral and derivative part to achieve the task of desired performance.

Follow the structure of PID controller, define new parameters for proportional part $x_4(t) = e(t)$, derivative part

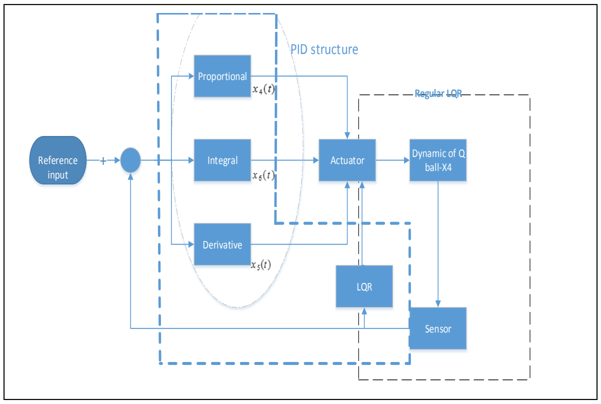


Fig. 2. PID structure of optimal feedback fault tolerant controller

$x_5(t) = e(\dot{t})$ and the integral part $x_6(t) = \int_0^t e(t)dt$, then PID structure feedback optimal controller is showed in Fig 2.

where $x_4(t), x_5(t), x_6(t)$ is represented in general form in Eq. (24).

$$\begin{cases} \dot{x_4}(t) = f_1(x_1, x_2, \cdots, x_6, u(t)) \\ \dot{x_5}(t) = f_2(x_1, x_2, \cdots, x_6, u(t)) \\ \dot{x_6}(t) = f_3(x_1, x_2, \cdots, x_6, u(t)) \end{cases} \tag{24}$$

According to the model presented in Eq. (12),

$$\begin{bmatrix} \dot{x_4}(t) \\ \dot{x_5}(t) \\ \dot{x_6}(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -K/M & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t). \end{bmatrix} \tag{25}$$

The augmented system is formed as:

$$\begin{aligned} \begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \\ \dot{x_3}(t) \\ \dot{x_4}(t) \\ \dot{x_5}(t) \\ \dot{x_6}(t) \end{bmatrix} = & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & K/M & 0 & 0 & 0 \\ 0 & 0 & -\omega & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & K/M & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \end{bmatrix} \\ & + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \omega & \omega & \omega & \omega \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{PWM_1} \\ u_{PWM_2} \\ u_{PWM_3} \\ u_{PWM_4} \end{bmatrix} \end{aligned} \tag{26}$$

However, there is a problem with the Eq. (26) that some of the parameters are not controllable independently. Recall Eq. (23), PID structure takes the advantages of proportional, integral and derivative by the reference and actual output, while in the design of LQR

$$u(t) = -Kx(t) \tag{27}$$

$$= -[k_1 \; k_2 \; k_3 \; k_4] \begin{bmatrix} x - x_r \\ \dot{x} - \dot{x_r} \\ x_3 \\ x_6 \end{bmatrix}$$

$$= [k_p \; k_d \; -k_3 \; -k_i] \begin{bmatrix} e \\ \dot{e} \\ x_3 \\ \int_0^t e(t)dt \end{bmatrix} \tag{28}$$

Let's suppose the reference signal is the trim point $x_t$. This assumption simplifies the design process without losing the genetic as the reference signal should be obtained after the transient which is another trim point. The above designed controller with LQ technique has the PID structure, which can be shown as:

$$\begin{aligned} k_p &= -k_1 \\ k_d &= -k_2 \\ k_i &= k_4 \end{aligned} \tag{29}$$

In this case, if the designed controller can make Eq. (27) stable, then the controlled system is stable and $\lim_{t \to \infty} e(t) = 0$.

$$\begin{aligned} \begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \\ \dot{x_3}(t) \\ \dot{x_6}(t) \end{bmatrix} = & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & K/M & 0 \\ 0 & 0 & -\omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_6(t) \end{bmatrix} \\ & + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \omega & \omega & \omega & \omega \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{PWM_1} \\ u_{PWM_2} \\ u_{PWM_3} \\ u_{PWM_4} \end{bmatrix} \end{aligned} \tag{30}$$

The controller based on state feedback achieves the correct steady-state response to the reference trajectory. The integral feedback makes the model of the system uncertainty free. In other words, if there is a fault in the system, it can still accommodate the fault without influencing the performance significantly.

### *C. Cost Function for PID Structure of Optimal Fault Tolerant Control*

Let $x_0 = [x_1(t), x_2(t), x_3(t)], x_e = [x_4(t), x_5(t), x_6(t)]$. The cost function is

$$J = \int_0^\infty x_0^T Q_0 x_0 + x_e^T Q_e x_e + u^T R u \; dt \tag{31}$$

Compared to Eq. (16), the PID structured optimal controller takes the error part into consideration in the design of the controller. It has the same structure as classical PID in taking care the error—proportional, integral and derivative, which can be tuned by engineers for MIMO system conveniently. It means that this controller solves the augmented stability problem and the tracking problem at the same time. $Q_0$, $Q_e$ and $R$ are chosen carefully at the designing process considering the relevant parameters and property of actuators. A reasonable simple choice for the matrices $Q_e$ and $R$ is by the Bryson's Rule [21].

$$q_{ii} = \frac{1}{x_{im}^2} \qquad (32)$$
$$r_{jj} = \frac{1}{u_{jm}^2}$$

where $q_{ii}$ and $r_{jj}$ are the $i^{th}$ and $j^{th}$ diagonal value; $x_{im}$ represents the maximum acceptable value of the $x_i$ state, $u_{jm}$ denotes the maximum acceptable value of the input $u_i$. Note that $Q$ and $R$ are

$$Q = \begin{bmatrix} q_{11} & 0 & \cdots & 0 \\ 0 & q_{22} & \cdots & 0 \\ \vdots & \cdots & & \\ 0 & 0 & \cdots & q_{mm} \end{bmatrix} \qquad (33)$$

$$R = \rho \begin{bmatrix} r_{11} & 0 & \cdots & 0 \\ 0 & r_{22} & \cdots & 0 \\ \vdots & \cdots & & \\ 0 & 0 & \cdots & r_{mm} \end{bmatrix} \qquad (34)$$

$\rho$ is to adjust the relative amplitude of the $Q$ and $R$. The performance and physical limitation is always the trade-off considered in the design. Additionally, Compared to the pure integral control, the PID structured LQR controller can also overcome the windup problem by just adjusting the penalization of $Q_e$. Overall, the PID structured LQR combines the popular LQR design with PID structure with the convenience of the controller design using optimal method, instead of ad-hoc tuning of PID controller gains.

## V. SIMULATION AND PERFORMANCE ANALYSIS

This section depicts the fault tolerant ability of PID structure based optimal fault tolerant contoller. The fault scenario adopted in this paper is the 10% loss of control effectiveness. The type of fault is incipient fault for 4 actuators. The height response performance is presented in the form of simulation.

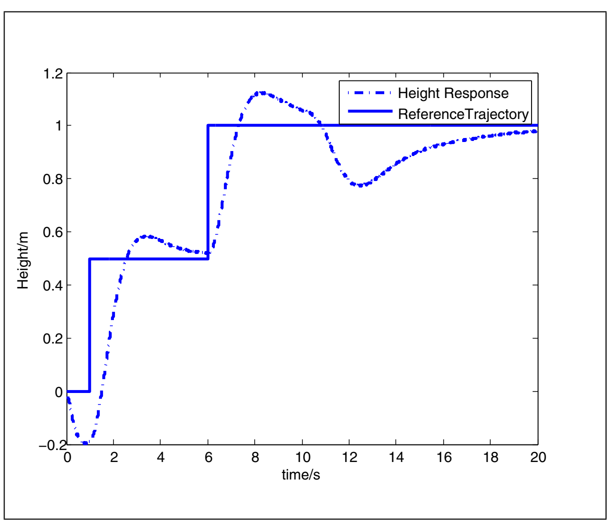


Fig. 3. Time response in height under fault-free and post-fault scenarios

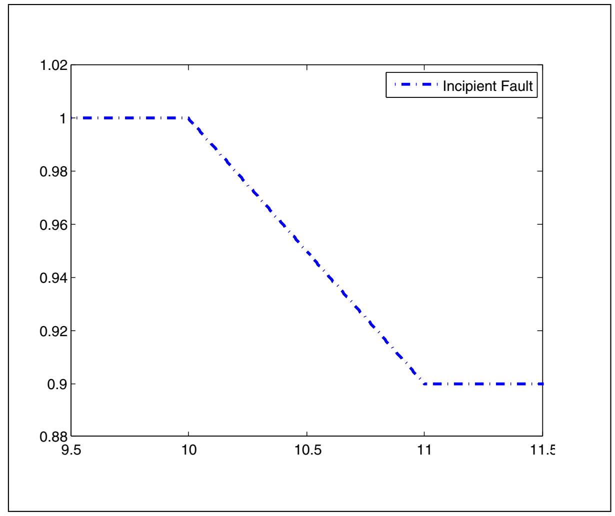


Fig. 4. 10% incipient fault injected into the actuator

Fig. 3 presents the details of time response of height in fault-free and post-fault conditions. The time response of height in first 10$s$ is to establish the trim position. The height of 0$m$ is not the ground position. It is the position to release the quadrotor. At the start of the operation, the Qball-X4 is dropping directly down in the direction of the ground because of gravity. After a second, the lift generated by rotors is increasing to overcome the effectiveness of gravity. As can be seen, Qball-X4 reaches to its trim position around 5$s$. At the time instant 10$s$, a new desired height is given to test the time response of the controller showing the good time response with the PID structured optimal controller in fault-free mode. The next to be tested is the fault tolerant ability of the designed controller. Hence, at the time instant 15$s$, the incipient fault is injected into the system, showed in Fig. 4. As showed in Fig. 3, Qball-X4 has a little perturbation after the fault and return to keep the same trim condition.

As the constraints of the actuator, the input signal cannot exceed 0.05. Therefore, it is necessary to value the control signal. Fig. 5 depicts the actuator input in fault-free and post-fault conditions. As can be seen, the controller signal is always under the limitation of 0.05. It is very clear that after the fault, the actuator needs much more control input power to compensate the loss of control effectiveness induced by a fault. Hence, for keeping the same trim point, the controller input before the fault occurrence in 10$s$ is smaller than that in post-fault condition, which can be seen in Fig. 5.

### A. Discussion

Fig. 3 presents that the designed controller can accommodate the actuator faults. To have a detailed look on how the designed controller achieves the fault tolerant capability for accommodating actuator fault, let's recall the state $x(6) = \int_0^T e(t)\, dt$, and $\dot{x_0}(t) = Ax_0(t) + Bu(t)$. The augmented matrix is based on the original state space

$$\begin{cases} \dot{x_0}(t) = Ax_0(t) + Bu(t) \\ \dot{x_6}(t) = y_r - x_1 \end{cases} \qquad (35)$$

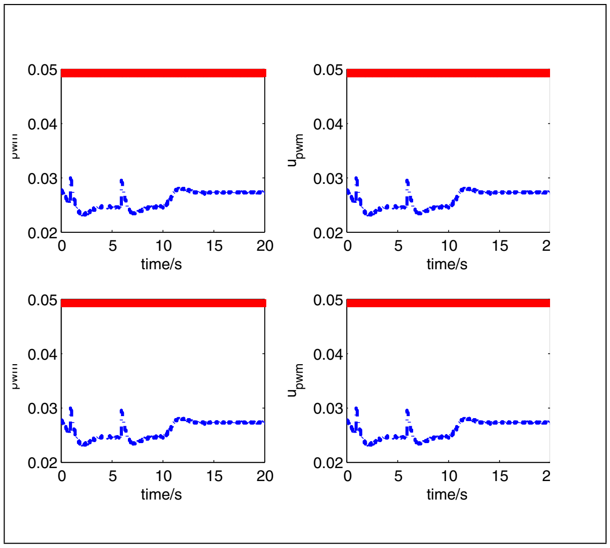


Fig. 5. Actuator input in fault-free and post-fault scenarios

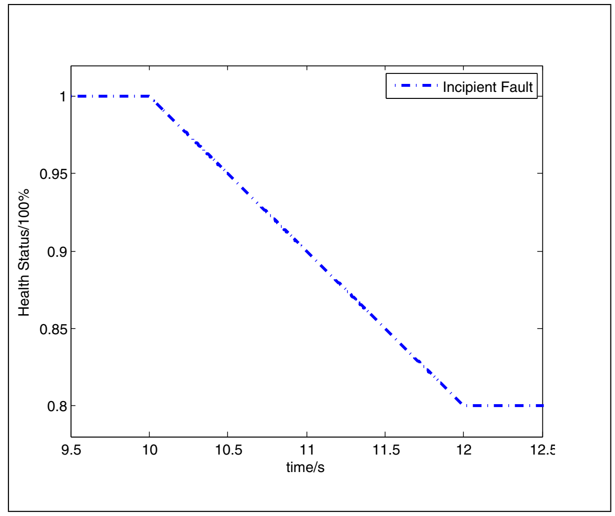


Fig. 6. 20% of fault injected to actuators

the control efforts

$$u(t) = -[k_1 \ k_2 \ k_3 \ k_4] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_5 \end{bmatrix} \tag{36}$$

the first three states are based on the model in Eq. (13)

$$\begin{aligned} u(t) &= k_0 x_0 + k_4 x_6 \\ &= k_0 x_0 + k_4 \int_0^T e(t)\ dt \\ &= u_1 + u_2 \end{aligned} \tag{37}$$

where $k_0 = [k_1 \ k_2 \ k_3]$, $u_1 = k_0 x_0$ and $u_2 = k_4 \int_0^T e(t)\ dt$.

In normal condition the new equilibrium equation is

$$y_r = y_{tim} = h(x(t), u(t)) \tag{38}$$

where $u_1(t)$ is calculated based on the normal model, $u_2$ is independent of the state space model $(A, B)$, but is the function of $y_r - x_1$. However, when there are faults in actuators, the dynamic of the system is changed to

$$\begin{aligned} \dot{x_0}(t) &= Ax(t) + Bu^f(t) \\ &= Ax(t) + Bu(t) + (I - \Sigma_A)(\bar{u} - u(t)) \\ &= Ax(t) + (B + \Delta)u(t) \end{aligned} \tag{39}$$

hence

$$y_r \neq h(x(t), u(t)) \tag{40}$$

at the same time $u_2$ is working until $\dot{x_6}(t)$ is zero, which means the output is equal to the reference. This part $u_2$ makes the controller not rely on the model of the system. In other words, it can accommodate the fault as long as the fault does not affect the controllability of the system.

### B. Additional Simulation Test

In order to prove the analysis, three more experiments are done with 20% incipient fault in actuators as shown in Fig. 6.

The test is based on the same scenario with different values in parameter $k_i$ while keeping all the other parameters the same. The first controller is $k_i$ calculated in the design process. The second controller is half the value of the $k_i$ while the third one is set to be zero.

Fig. 7 shows the time response of height with a different parameter value of $k_i$. Before 10 $s$, all the parameters are the same. However, at the time instant of 10 $s$, the 20% incipient fault is injected to the 4 actuators. And the second controller $k_i$ is changed to half of its original value. The dash line presents the height response with the first controller while the solid line presents the height response with the second controller. The reference trajectory is presented by the step input in solid line.

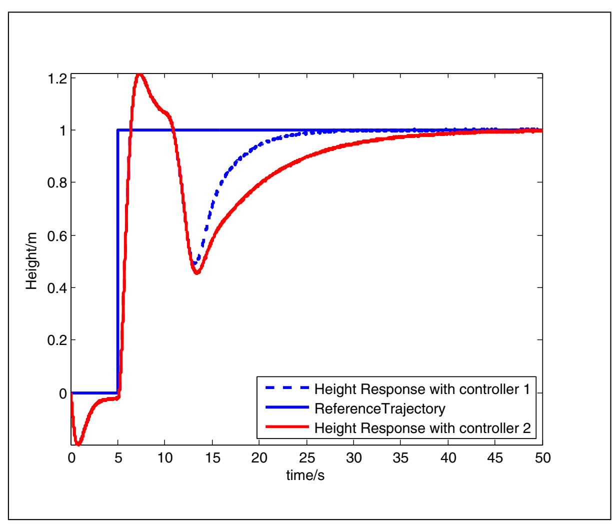


Fig. 7. Height time response with different parameters for the same scenario

Fig. 8 depicts the control input of both controllers for the 4 actuators based on the same scenario. The bold solid line denotes up boundary of an actuator input. The dash line with fluctuation is the actual control input to the actuators. Apparently, the control inputs with different parameters are

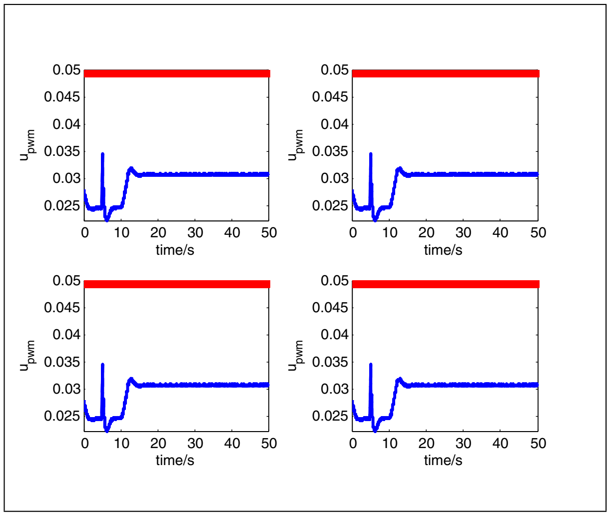


Fig. 8. Control input with different parameters for the same scenario

almost the same. Fig. 9 presents the difference between the two sets of control input of the different controller for 4 actuators. As can be seen, there is only minimal

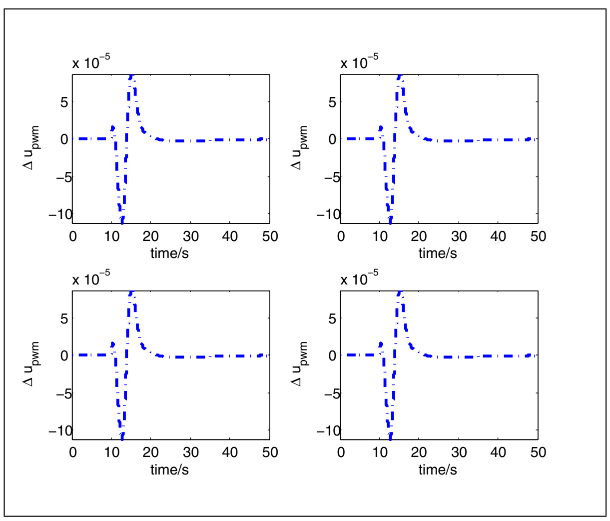


Fig. 9. Control input with different parameters for the same scenario

difference in the transit process for the different controllers. However, the control performance is dramatically changed (See Fig. 7). The first controller with the original $k_i$ has a faster response to the fault compared to the second controller. This test proves the function of the augmented state $x_6$ help to accommodate the faults. Engineers can use the property to adjust $k_i$ to change the response time to the fault, as the PID structure is familiar to the engineers in the industry.

The parameter $k_i$ is set to be zero to support the effectiveness of the $k_i$ for fault tolerance. Fig. 10 shows the performance after fault injected. At the time instant 10 second, the fault is injected to the 4 actuators, while at the same time the parameter $k_i$ is set to be zero. In this case, the augmented state is no longer working. As can be seen from

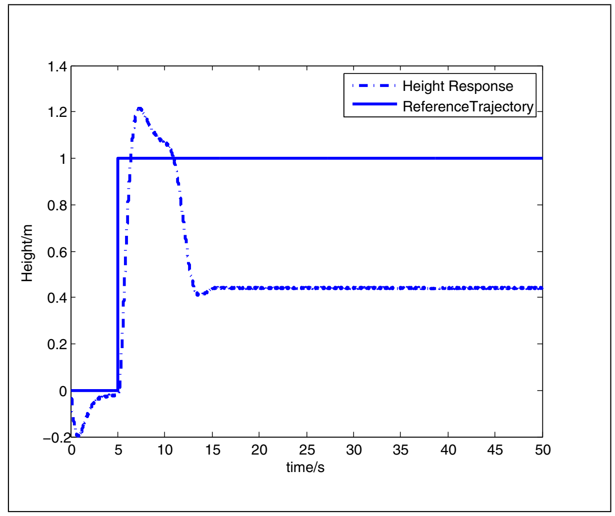


Fig. 10. Control input with different parameters for the same scenario

Fig. 10, the time response of the system cannot go back to the set-point. In other words, it cannot track the reference trajectory and lose its fault tolerant capability. However, the input effort is almost the same which is depicted in Fig. 11. This is another test to support the analysis that the augmented parameter makes the controller have the fault tolerant ability.

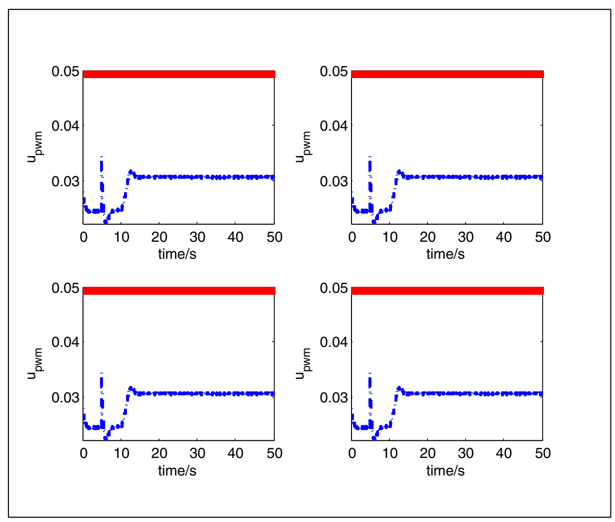


Fig. 11. Control input with different parameters for the same scenario

## VI. CONCLUSIONS

This paper proposes a PID structured optimal fault tolerant controller. The controller has the advantages of PID control in understanding and tuning, and also has the advantages to be designed using LQ optimization technique. Compared to the conventional PID controller, the designed controller using LQ technique can manipulate MISO (MIMO) system and provides the optimal output based on the objective function. From the test and analysis, the proposed controller has a good performance in accommodating loss of control effectiveness of the actuator. The performance can be improved through adjusting the weighting matrix of proportional, integral and derivative part to adapt to the various performance demands.

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