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Robust Controlled and Conditioned Invariant Subspaces for Uncertain Switched Linear Systems

N. Otsuka†, G. Conte‡ and A. M. Perdon‡

Abstract— **This paper concerns robust controlled invariant subspaces and robust conditioned invariant subspaces for a family of vertex systems for polytopic uncertain switched linear systems. Further, disturbance decoupling problem via static output feedback under arbitrary switching for polytopic uncertain switched linear systems is also investigated.**

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Keywords : Robust controlled invariance, Robust conditioned invariance, Disturbance decoupling, Uncertain switched linear systems, Static output feedback

I. INTRODUCTION

Since the concepts of controlled (A, B) invariant and conditioned (C, A) invariant subspaces for finite-dimensional time-invariant linear systems were studied in the framework of the so-called geometric theory (e.g., see [2], [16]), various control problems have been solved by using the concepts (e.g., see [2], [15], [16]). After that robust controlled (A, B) invariant and conditioned (C, A) invariant subspaces for parameter-dependent linear systems and for a family of linear systems were studied as extensions of the concepts of the invariant subspaces (e.g., [1], [3], [4], [7], [9]-[11]). Recently, the geometric control theory has been used to study disturbance decoupling problems via state feedback for switched linear systems (e.g., [5], [6], [12], [17]). Further, the same problems for polytopic uncertain switched linear system have also been investigated, and solvability conditions for the problems were given [13],[14].

In this paper robust controlled invariant subspaces under common input and robust conditioned invariant subspaces under common output for a family of vertex systems of polytopic uncertain switched linear systems are first investigated. Next, a disturbance decoupling problem via static output feedback under arbitrary switching for polytopic uncertain switched linear systems is also investigated. The analogous problem in the linear time invariant non-switched and non-uncertain case was studied and solved in [8], [15]. In Section II some notations and systems formulation which are needed in this study are given. In Section III the robust controlled invariant and robust conditioned invariant subspaces are studied. In Section IV disturbance decoupling problem via static output feedback is formulated, and then solvability conditions are presented. Further, a checkable necessary condition is also given. Finally, concluding remarks are given in Section V.

†N. Otsuka is with Division of Science, School of Science and Engineering, Tokyo Denki University, Hatoyama-Machi, Hiki-Gun, Saitama 350-0394, Japan. otsuka@mail.dendai.ac.jp

‡G. Conte and A. M. Perdon are with Dipartimento di Ingegneria dell′informazione-Universitẁa Politecnica delle Marche - Via Brecce Bianche, 12 - 60131 Ancona, Italy

II. PRELIMINARIES.

In this section we first give some notations which are used throughout this study. For a linear map $A: \mathcal{X} \to \mathcal{Y}$ from a vector space $\mathcal{X}$ into a vector space $\mathcal{Y}$ and a subspace $\mathcal{V}$ of $\mathcal{Y}$ the image, the kernel and the inverse image are denoted by Im(A), Ker(A) and $A^{-1}\mathcal{V} := \{x \in \mathcal{X} \mid Ax \in \mathcal{V}\}$, respectively.

Next, we consider a family of linear systems $\{\Sigma_i; i = 1, \cdots, N\}$ given by

$$\Sigma_i : \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t),\ x(0) = x_0, \\ y(t) = C_i x(t), \end{cases}$$

where $x(t) \in \mathcal{X} := \mathbf{R}^n$ is the state, $u(t) \in \mathcal{U} := \mathbf{R}^m$ is the input, $y(t) \in \mathcal{Y} := \mathbf{R}^q$ is the measurement output and $A_i : \mathcal{X} \to \mathcal{X}$, $B_i : \mathcal{U} \to \mathcal{X}$, $C_i : \mathcal{X} \to \mathcal{Y}$ are matrices. For simplicity we use the notation $I := \{1, 2, \cdots, N\}$. For a set of subspaces $\{\mathcal{V}_1, \cdots, \mathcal{V}_N\}$ of $\mathcal{X}$, $(\mathcal{V}_1 \oplus \cdots \oplus \mathcal{V}_N) := \{(x_1, \cdots, x_N) \mid x_i \in \mathcal{V}_i\ (i \in I)\}$ is the direct sum. For simplicity $(\mathcal{V} \oplus \cdots \oplus \mathcal{V})$ for a subspace $\mathcal{V}$ of $\mathcal{X}$ is denoted by $\mathcal{V}^{\oplus N}$. Define the sum of two subspaces $(\mathcal{V}_1 \oplus \cdots \oplus \mathcal{V}_N)$ and $(\mathcal{W}_1 \oplus \cdots \oplus \mathcal{W}_N)$ of $\mathcal{X}^{\oplus N}$ by

$$\begin{aligned}(\mathcal{V}_1 \oplus \cdots \oplus \mathcal{V}_N) + (\mathcal{W}_1 \oplus \cdots \oplus \mathcal{W}_N) \\ := (\mathcal{V}_1 + \mathcal{W}_1) \oplus \cdots \oplus (\mathcal{V}_N + \mathcal{W}_N).\end{aligned}$$

Let $(A_1 \oplus \cdots \oplus A_N)$ be a linear map from $\mathcal{X}$ into $\mathcal{X}^{\oplus N}$ given by

$$\mathcal{X} \ni x \ \mapsto (A_1 x, \cdots, A_N x) \in \mathcal{X}^{\oplus N}$$

and then we use a shorter notation $\overset{N}{\underset{i=1}{\oplus}} A_i$.

Further, let $[A_1, \cdots, A_N]$ be a linear map from $\mathcal{X}^{\oplus N}$ into $\mathcal{X}$ given by

$$\mathcal{X}^{\oplus N} \ni (x_1, \cdots, x_N) \ \mapsto A_1 x_1 + \cdots + A_N x_N \in \mathcal{X}.$$

If we apply a state feedback of the form $u(t) = F_i x(t)$ to the system Σ_i, then we have a family of closed-loop systems given by

$$\Sigma_i^{F_i} : \begin{cases} \dot{x}(t) = (A_i + B_i F_i) x(t),\ x(0) = x_0, \\ y(t) = C_i x(t). \end{cases}$$

In this study we assume that the following matrices have polytopic uncertainties described as

$$A_i = \sum_{j_i=1}^{N_i} \mu_{i,j_i} A_{i,j_i}, B_i = \sum_{j_i=1}^{N_i} \mu_{i,j_i} B_{i,j_i}, C_i = \sum_{j_i=1}^{N_i} \mu_{i,j_i} C_{i,j_i},$$

where $\sum_{j_i=1}^{N_i} \mu_{i,j_i} = 1$ $(i = 1, \cdots, N)$, A_{i,j_i}, B_{i,j_i} and C_{i,j_i} are the vertex matrices of A_i, B_i and C_i, respectively. N_i is the number of vertex matrices for each subsystem Σ_i and $\mu_{i,j_i} (\geq 0)$ $(j_i = 1, \cdots, N_i)$ are uncertain parameters. For convenience we use the notations $I_i := \{1, 2, \cdots, N_i\}$ and $\mu := (\mu_1, \cdots, \mu_N) \in \Omega := \Omega_1 \times \cdots \times \Omega_N$ which means a cartesian product, where

$$\Omega_i := \{\mu_i = (\mu_{i,1}, \cdots, \mu_{i,N_i}) | \sum_{j_i=1}^{N_i} \mu_{i,j_i} = 1,\ \mu_{i,j_i} \geq 0\}.$$

Then, a closed-loop subsystem $\Sigma_i^{F_i}$ becomes the polytopic system as follows.

$$\begin{cases} \dot{x}(t) = \sum_{j_i=1}^{N_i} \mu_{i,j_i} (A_{i,j_i} + B_{i,j_i} F_i) x(t),\ x(0) = x_0, \\ y(t) = \sum_{j_i=1}^{N_i} \mu_{i,j_i} C_{i,j_i} x(t). \end{cases}$$

Now, if we consider the vertex systems Σ_{i,j_i} $(j_i \in I_i)$ of each subsystem Σ_i given by

$$\Sigma_{i,j_i} : \begin{cases} \dot{x}(t) = A_{i,j_i} x(t) + B_{i,j_i} u(t),\ x(0) = x_0, \\ y(t) = C_{i,j_i} x(t), \end{cases}$$

and a state feedback $u(t) = F_i x(t)$, then we have a family of closed-loop vertex systems $\Sigma_{i,j_i}^{F_i}$ $(j_i \in I_i)$ as follows.

$$\Sigma_{i,j_i}^{F_i} : \begin{cases} \dot{x}(t) = (A_{i,j_i} + B_{i,j_i} F_i) x(t),\ x(0) = x_0, \\ y(t) = C_{i,j_i} x(t) \end{cases}$$

where F_i means a common state feedback gain for all vertex systems of each subsystem Σ_i.

III. ROBUST INVARIANT SUBSPACES.

A. Robust controlled invariant subspaces

In this section we will summarize some results in [13] on robust controlled invariant subspaces under common input for a family of linear systems.

Definition 3.1: Let $\mathcal{V}$ and $\mathit{\Lambda}$ be subspaces of $\mathcal{X}$.

$\mathcal{V}$ is said to be *robust controlled* $(A_{i,j_i \in I_i}, B_{i,j_i \in I_i})$-*invariant under common input of vertex systems* Σ_{i,j_i} $(j_i \in I_i)$ for all $i \in I$ if

$$(\overset{N_1}{\underset{j_1=1}{\oplus}} A_{1,j_1} \oplus \cdots \oplus \overset{N_N}{\underset{j_N=1}{\oplus}} A_{N,j_N}) \mathcal{V} \subset \mathcal{V}^{\oplus \sum_{i=1}^{N} N_i} + (\mathrm{Im}(\overset{N_1}{\underset{j_1=1}{\oplus}} B_{1,j_1}) \oplus \cdots \oplus \mathrm{Im}(\overset{N_N}{\underset{j_N=1}{\oplus}} B_{N,j_N})). \quad (1)$$

Further, we define

$$\mathbf{V}^{\mathbf{c}}_{\mathbf{vert}}(\mathit{\Lambda}) := \{\mathcal{V}\ (\subset \mathit{\Lambda}) \mid \mathcal{V} \text{ satisfies the condition (1)}\}. \ \blacksquare$$

The following Lemma gives equivalent conditions for the robust controlled invariant subspaces.

Lemma 3.2: [13]

$\mathcal{V}$ is robust controlled $(A_{i,j_i \in I_i}, B_{i,j_i \in I_i})$-invariant under common input of vertex systems Σ_{i,j_i} $(j_i \in I_i)$ for all $i \in I$ if and only if for all $i \in I$ there exist $F_i : \mathcal{X} \to \mathcal{U}$ such that

$$(A_{i,j_i} + B_{i,j_i} F_i) \mathcal{V} \subset \mathcal{V} \quad (2)$$

for all $j_i \in I_i$. Further, we define a set of F_i by

$$\mathbf{F}_i(\mathcal{V}) := \{F_i \mid F_i \text{ satisfies the condition (2) for all } j_i \in I_i\}.$$

$\blacksquare$

The following Lemma gives a computational algorithm to obtain the maximal element of $\mathbf{V}^{\mathbf{c}}_{\mathbf{vert}}(\mathit{\Lambda})$.

Lemma 3.3: [13]

The class $\mathbf{V}^{\mathbf{c}}_{\mathbf{vert}}(\mathit{\Lambda})$ has the unique maximal element $\mathcal{V}^*_{vert}(\mathit{\Lambda})$ and it can be computed from the following steps.

(Step 1) $\mathcal{V}_{(0)} := \mathit{\Lambda}$,

(Step 2) $\mathcal{V}_{(k)} := \mathit{\Lambda} \cap \{\overset{N_1}{\underset{j_1=1}{\oplus}} A_{1,j_1} \oplus \cdots \oplus \overset{N_N}{\underset{j_N=1}{\oplus}} A_{N,j_N}\}^{-1} \{\mathcal{V}_{(k-1)}^{\oplus \sum_{i=1}^{N} N_i} + \mathrm{Im}(\overset{N_1}{\underset{j_1=1}{\oplus}} B_{1,j_1}) \oplus \cdots \oplus \mathrm{Im}(\overset{N_N}{\underset{j_N=1}{\oplus}} B_{N,j_N})\}$ $(k = 1, 2, \cdots)$,

(Step 3) $\mathcal{V}^*_{vert}(\mathit{\Lambda}) := \mathcal{V}_{(\dim(\mathit{\Lambda}))}$. $\blacksquare$

B. Robust conditioned invariant subspaces

In this section a robust conditioned invariant subspaces under common output for a family of linear systems are studied.

Definition 3.4: Let $\mathcal{S}$ and ε be subspaces of $\mathcal{X}$.

$\mathcal{S}$ is said to be *robust conditioned* $(C_{i,j_i \in I_i}, A_{i,j_i \in I_i})$-*invariant under common output of vertex systems* Σ_{i,j_i} $(j_i \in I_i)$ for all $i \in I$ if

$$[A_{1,1}, \cdots, A_{1,N_1}, \cdots, A_{N,1}, \cdots, A_{N,N_N}] \cdot ((\mathcal{S}^{\oplus N_1} \cap \mathrm{Ker}[C_{1,1}, \cdots, C_{1,N_1}]) \oplus \cdots \oplus (\mathcal{S}^{\oplus N_N} \cap \mathrm{Ker}[C_{N,1}, \cdots, C_{N,N_N}])) \subset \mathcal{S}. \quad (3)$$

Further, we define

$$\mathbf{S}^{\mathbf{c}}_{\mathbf{vert}}(\varepsilon) := \{\mathcal{S}(\supset \varepsilon) \mid \mathcal{S} \text{ satisfies the condition (3)}\}. \ \blacksquare$$

The following Lemma gives equivalent conditions for the robust conditioned invariant subspaces.

Lemma 3.5:

$\mathcal{S}$ is robust conditioned $(C_{i,j_i \in I_i}, A_{i,j_i \in I_i})$-invariant under common output of vertex systems Σ_{i,j_i} $(j_i \in I_i)$ for all $i \in I$ if and only if for all $i \in I$ there exist $G_i : \mathcal{Y} \to \mathcal{X}$ such that

$$(A_{i,j_i} + G_i C_{i,j_i}) \mathcal{S} \subset \mathcal{S} \quad (4)$$

for all $j_i \in I_i$. Further, we define a set of G_i by

$$\mathbf{G}_i(\mathcal{S}) := \{G_i \mid G_i \text{ satisfies the condition (4) for all } j_i \in I_i\}.$$

$\blacksquare$

The following Lemma gives a computational algorithm to obtain the minimal element of $\mathbf{S}^{\mathbf{c}}_{\mathbf{vert}}(\varepsilon)$.

Lemma 3.6:

The class $\mathbf{S}^{\mathbf{c}}_{\mathbf{vert}}(\varepsilon)$ has the unique minimal element $\mathcal{S}^*_{vert}(\varepsilon)$ and it can be computed from the following steps.

(Step 1) $\mathcal{S}_{(0)} := \varepsilon$,

(Step 2) $\mathcal{S}_{(k)} := \mathcal{S}_{(k-1)}+$

$$[A_{11},\cdots,A_{1N_1},\cdots,A_{N1},\cdots,A_{NN_N}]\{(\mathcal{S}_{(k-1)}\cap \mathrm{Ker}C_{1,j_1})\overset{N_1}{\underset{j_1=1}{\oplus}} \oplus\cdots\oplus(\mathcal{S}_{(k-1)}\cap \mathrm{Ker}C_{N,j_N})\overset{N_N}{\underset{j_N=1}{\oplus}}\}$$

(Step 3) $\mathcal{S}^*_{vert}(\varepsilon) := \mathcal{S}_{(n-\dim(\varepsilon))}$. ■

The following lemma can be used to obtain the solvability conditions for robust disturbance decoupling problem via static output feedback for polytopic uncertain switched linear systems.

Lemma 3.7:

(i) Assume that $B_i = B_{i,1} = \cdots = B_{i,N_i}$ for all $i \in I$. If a subspace $\mathcal{V}$ is robust controlled $(A_{i,j_i\in I_i}, B_{i,j_i\in I_i})$-invariant under common input and robust conditioned $(C_{i,j_i\in I_i}, A_{i,j_i\in I_i})$-invariant under common output of vertex systems Σ_{i,j_i} $(j_i \in I_i)$ for all $i \in I$, then for all $i \in I$ there exist $K_i : \mathcal{Y} \to \mathcal{U}$ such that

$$(A_{i,j_i} + B_iK_iC_{i,j_i})\mathcal{V} \subset \mathcal{V}$$

for all $j_i \in I_i$.

(ii) Assume that $C_i = C_{i,1} = \cdots = C_{i,N_i}$ for all $i \in I$. If a subspace $\mathcal{V}$ is robust controlled $(A_{i,j_i\in I_i}, B_{i,j_i\in I_i})$-invariant under common input and robust conditioned $(C_{i,j_i\in I_i}, A_{i,j_i\in I_i})$-invariant under common output of vertex systems Σ_{i,j_i} $(j_i \in I_i)$ for all $i \in I$, then for all $i \in I$ there exist $K_i : \mathcal{Y} \to \mathcal{U}$ such that

$$(A_{i,j_i} + B_{i,j_i}K_iC_i)\mathcal{V} \subset \mathcal{V}$$

for all $j_i \in I_i$. ■

The following corollary follows from Lemma 3.7.

Corollary 3.8: Assume that $B_i = B_{i,1} = \cdots = B_{i,N_i}$ and $C_i = C_{i,1} = \cdots = C_{i,N_i}$ for all $i \in I$. Then, a subspace $\mathcal{V}$ is robust controlled $(A_{i,j_i\in I_i}, B_{i,j_i\in I_i})$-invariant under common input and robust conditioned $(C_{i,j_i\in I_i}, A_{i,j_i\in I_i})$-invariant under common output of vertex systems Σ_{i,j_i} $(j_i \in I_i)$ for all $i \in I$ if and only if for all $i \in I$ there exist $K_i : \mathcal{Y} \to \mathcal{U}$ such that

$$(A_{i,j_i} + B_iK_iC_i)\mathcal{V} \subset \mathcal{V}$$

for all $j_i \in I_i$. ■

IV. An Application to Disturbance Decoupling for Uncertain Switched Systems.

Consider the following switched linear system with disturbance $d(t)$ described as

$$\Sigma_{\sigma d} : \begin{cases} \dot{x}(t) = A_\sigma x(t) + B_\sigma u(t) + E_\sigma d(t), \ x(0) = x_0 \\ y(t) = C_\sigma x(t), \\ z(t) = D_\sigma x(t), \end{cases}$$

where $x(t) \in \mathcal{X} := \mathbf{R}^n$ is the state, $u(t) \in \mathcal{U} := \mathbf{R}^m$ is the input, $y(t) \in \mathcal{Y} := \mathbf{R}^q$ is the measurement output, $z(t) \in \mathcal{Z} := \mathbf{R}^q$ is the controlled output, $d(t)$ is the disturbance which is locally integrable function, i.e., $d \in L_1^{loc}(\mathbf{R}^+, \mathbf{R}^\eta)$, $\sigma(t) : \mathbf{R}^+ \to I$ is a switched rule which depends on time t, where $\mathbf{R}^+$ is the set of non-negative real numbers.

If we apply a static output feedback $u(t) = K_\sigma y(t)$ to the switched system $\Sigma_{\sigma d}$, then we have the following closed-loop switched system.

$$\Sigma^{K_\sigma}_{\sigma d} : \begin{cases} \dot{x}(t) = (A_\sigma + B_\sigma K_\sigma C_\sigma)x(t) + E_\sigma d(t), \ x(0) = x_0 \\ z(t) = D_\sigma x(t). \end{cases}$$

Then, the above switched system is composed as the family of closed-loop subsystems given by

$$\Sigma^{K_i}_{id} : \begin{cases} \dot{x}(t) = (A_i + B_iK_iC_i)x(t) + E_i d(t), \ x(0) = x_0 \\ z(t) = D_i x(t). \end{cases}$$

In this study, it is assumed that the following matrices have polytopic uncertainties described as

$$A_i = \sum_{j_i=1}^{N_i} \mu_{i,j_i}A_{i,j_i}, B_i = \sum_{j_i=1}^{N_i} \mu_{i,j_i}B_{i,j_i}, C_i = \sum_{j_i=1}^{N_i} \mu_{i,j_i}C_{i,j_i},$$
$$D_i = \sum_{j_i=1}^{N_i} \mu_{i,j_i}D_{i,j_i}, E_i = \sum_{j_i=1}^{N_i} \mu_{i,j_i}E_{i,j_i}$$

for $\mu := (\mu_1, \cdots, \mu_N) \in \Omega := \Omega_1 \times \cdots \times \Omega_N$.

Disturbance decoupling problem via static output feedback for polytopic uncertain switched systems is formulated as follows.

Disturbance Decoupling Problem via Static Output Feedback under Arbitrary Switching (DDPSOFAS) Given vertex matrices $A_{i,j_i}, B_{i,j_i}, C_{i,j_i}, D_{i,j_i}$ and E_{i,j_i} $(i \in I$ and $j_i \in I_i)$ for the switched linear system $\Sigma_{\sigma d}$, find (if possible) static output feedback gains $K_i : \mathcal{Y} \to \mathcal{U}$ $(i \in I)$ such that the output $z(t)$ of all subsystems to be controlled is not affected by disturbances $d(t)$ under arbitrary switching $\sigma(t)$ for all uncertain parameters $\mu = (\mu_1, \cdots, \mu_N) \in \Omega = \Omega_1 \times \cdots \times \Omega_N$. ■

Remark 4.1: Assume that $C_i = C_{i,1} = \cdots = C_{i,N_i} = I_{id}$ for all $i \in I$, where I_{id} is the identity matrix on $\mathcal{X}$. Further, define a state feedback $F_i := K_i : \mathcal{X} \to \mathcal{U}$ in DDPSOFAS. Then, the DDPSOFAS reduces to the Disturbance Decoupling Problem via State Feedback under Arbitrary Switching (DDPSFAS). ■

The following two notations are used to give the solvability conditions for DDPSOFAS.

$$\Delta := \sum_{i=1}^{N}\sum_{j_i=1}^{N_i} \mathrm{Im}E_{i,j_i} \quad \text{and} \quad \Lambda := \bigcap_{i=1}^{N}\bigcap_{j_i=1}^{N_i} \mathrm{Ker}D_{i,j_i}.$$

The following theorem gives sufficient condition for the DDPSOFAS to be solvable.

Theorem 4.2: Assume that $B_i = B_{i,1} = \cdots = B_{i,N_i}$ or $C_i = C_{i,1} = \cdots = C_{i,N_i}$ for all $i \in I$. If there exists a subspace $\mathcal{V}$ which is robust controlled $(A_{i,j_i\in I_i}, B_{i,j_i\in I_i})$-invariant under common input and robust conditioned $(C_{i,j_i\in I_i}, A_{i,j_i\in I_i})$-invariant under common output of vertex systems Σ_{i,j_i} $(j_i \in I_i)$ for all $i \in I$ satisfying

$$\Delta \subset \mathcal{V} \subset \Lambda,$$

then DDPSOFAS is solvable. ■

The following corollary gives necessary and sufficient condition for the DDPSOFAS to be solvable.

Corollary 4.3: Assume that $B_i = B_{i,1} = \cdots = B_{i,N_i}$ and $C_i = C_{i,1} = \cdots = C_{i,N_i}$ for all $i \in I$. Then, DDPSOFAS is solvable if and only if there exists a subspace $\mathcal{V}$ which is robust controlled $(A_{i,j_i \in I_i}, B_{i,j_i \in I_i})$-invariant under common input and robust conditioned $(C_{i,j_i \in I_i}, A_{i,j_i \in I_i})$-invariant under common output of vertex systems Σ_{i,j_i} $(j_i \in I_i)$ for all $i \in I$ satisfying

$$\Delta \subset \mathcal{V} \subset \Lambda. \quad ■$$

The following corollary gives necessary conditions.

Corollary 4.4: In the same hypothesis of Corollary 4.3 necessary conditions for the solvability of DDPSOFAS are that

$$\mathcal{S}^*_{vert}(\Delta) \subset \Lambda \text{ and } \mathcal{V}^*_{vert}(\Lambda) \supset \Delta.$$

Proof. Suppose that DDPSOFAS is solvable. Then, it follows from Corollary 4.3 that there exists a subspace $\mathcal{V}$ which is robust controlled $(A_{i,j_i \in I_i}, B_{i,j_i \in I_i})$-invariant under common input and robust conditioned $(C_{i,j_i \in I_i}, A_{i,j_i \in I_i})$-invariant under common output of vertex systems Σ_{i,j_i} $(j_i \in I_i)$ for all $i \in I$ satisfying

$$\Delta \subset \mathcal{V} \subset \Lambda.$$

Then, one has $\mathcal{S}^*_{vert}(\Delta) \subset \mathcal{V} \subset \mathcal{V}^*_{vert}(\Lambda)$ and the thesis follows. ■

Remark 4.5: The important point about the above necessary condition is that it can be checked algorithmically by the constructive procedures given in Lemma 3.3 and Lemma 3.6. ■

The following corollary gives a necessary and sufficient condition for the problem via state feedback to be solvable.

Corollary 4.6: [14] Assume that $C_i = C_{i,1} = \cdots = C_{i,N_i} = I_{id}$ for all $i \in I$. Then, DDPSFAS is solvable if and only if

$$\Delta \subset \mathcal{V}^*_{vert}(\Lambda),$$

where $\mathcal{V}^*_{vert}(\Lambda)$ is the maximal robust controlled $(A_{i,j_i \in I_i}, B_{i,j_i \in I_i})$-invariant subspace under common input of vertex systems for all $i \in I$ contained in Λ. ■

V. Conclusion.

In this paper robust controlled invariant subspaces and robust conditioned invariant subspaces for a family of vertex systems for polytopic uncertain switched linear systems were firstly studied in the geometric control theory. Next, disturbance decoupling problem via static output feedback for the polytopic uncertain switched linear systems under arbitrary switching was formulated, and a sufficient condition for the problem to be solvable was given under the assumption that input or output matrices for all subsystems do not contain uncertain parameters. Further, a necessary and sufficient condition for the problem to be solvable was given under the assumption that input and output matrices for all subsystems do not contain uncertain parameters. Finally, in the same hypothesis necessary conditions which can be easily checked for the solvability of the problem were also given.

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