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# On model reduction of coupled systems

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## I. Introduction

In the problem of model reduction moment matching techniques represent an efficient tool, see e.g. [13], [7], [2], [21], [11], [16], [4] for a complete overview for linear systems. Since the complexity of the models used for system analysis and simulation as well as controller design has increased significantly, model reduction is strongly called for. There exists a large variety of model reduction techniques, see e.g., [2] for an extensive overview. Among them, balanced truncation and moment matching are the most widely used. In the problem of model reduction, moment matching techniques represent an efficient tool, see e.g. [10], [3] and [2] for an overview for linear systems. In such techniques the (reduced order) model is obtained by constructing a lower degree rational function that approximates the original transfer function (assumed rational). The low degree rational function matches the original transfer function and its derivatives at various points in the complex plane. There are several possible (equivalent) notions of moments for a linear system. Recently in [5], [15] (see also [12], [18]) alternative descriptions of moments in a time-domain framework have been given. Therein moments are in relation with the (well-defined) steady-state response (if it exists) of the system driven by a signal generator. The reduced order model that achieves moment matching at $\nu$ points is a parametric model, the extra parameter being tuned such that certain properties are preserved. Swapping the interconnection, it has been shown that moments are in one-to-one relation with the (well-defined) steady-state response of the output of a signal generator driven by the system. The reduced order model that achieves moment matching at $\nu$ points is also a parametric one. Furthermore in [6] a connection between the different families of models is established.

The problem of model reduction of interconected systems as controller reduction has long been studied in the literature in different model order reduction frameworks. Most of the earlier results are in the framework of stability preserving balanced truncation, e.g., posing the problem as a frequency weighted $L_\infty$, approximation problem. Many different controller representations were yielded, giving different performance of the various reduction algorithms, see [1]. Variations of the problem and the results were later introduced in e.g., [20] where it is shown that closed-loop balanced truncation with preservation of the closed-loop properties is actually equivalent to frequency weighted balanced truncation with certain selections of the weighting systems. In [23] the solution of the balancing-related frequency-weighted model and controller reduction problems using accuracy enhanced numerical algorithms has been considered. New stability-enforcing choices of the frequency-weighted Grammians which can guarantee the stability of reduced models for two-sided frequency weights have been obtained. Then it has been shown that for the frequency-weighted controller reduction problems with standard stability and performance-enforcing frequency weights the computation of the frequency-weighted Grammians can be done by solving reduced order Lyapunov equations. For both frequency-weighted model and controller reduction problems it has been indicated how to compute the Grammians directly in terms of their Cholesky factors. This has allowed the extension of the square-root and balancing-free accuracy-enhancing techniques to the frequency-weighted case.

A different approach based on interpolation approaches have been taken recently in e.g., [22] where the problem of model reduction of interconnected system has been studied. The purpose of the paper has been to present both Krylov and Grammian-based model reduction techniques that preserve the structure of the interconnections between different subsystems. Furthermore, in [14] a computationally efficient controller reduction approach using a rational Krylov method has been introduced. It has been shown that a reduced-order controller obtained via a Krylov projection is guaranteed to match the desired full-order closed loop system response at shifts used in the Krylov reduction of the controller. Another type of model reduction problem of block structured systems has been studied in [19], [8] with respect to the class of second order systems.

In this abstract we study the general problem of model reduction of interconnected systems, see, e.g., [17] in the framework of time-domain moment matching. Using recently developed moment matching techniques, we compute *families* of reduced order models that achieve moment matching and partly, or completely preserve the block structure of the given system. In particular, we address the structure of the controller implemented in [24, Section 12.6]. We adapt the problem of moment matching to obtain a family of reduced order blocks that approximate the subsystems of the given system, preserving its properties. Furthermore, we study the problem of reducing the number of blocks of the given system. Our reduction strategy, consists of interpolating each subsystem at one point and then prerform balanced truncation, in order to reduce the number of subsystems. The result is a low order controller that approximates the full order controller and preserves its properties.

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## II. Preliminaries

In this section we recall the idea of moment matching for linear, single-input, single-output systems, from a time domain point of view as presented in [6]. Let

$$K(s) = C(sI - A)^{-1}B, \tag{1}$$

$K : \mathbb{C} \to \mathbb{C}$, with $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^n$ and $C \in \mathbb{R}^{1\times n}$. Consider the system

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx, \end{aligned} \tag{2}$$

with $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$. Assume that the system (2) is a minimal realization of the transfer function $G(s)$. The moment of (1) is defined as follows.

**Definition 1.** [5], [2] The 0-moment at $s_1 \in \mathbb{C}$ of system (1) is the complex number $\eta_0(s_1) = C(s_1 I - A)^{-1}B$. The $k-$moment of system (1) at $s_1$ is the complex number $\eta_k(s_1) = \dfrac{(-1)^k}{k!} \cdot \dfrac{d^k[C(sI - A)^{-1}B]}{ds^k}\Bigg|_{s=s_1}$, $k \geq 1$ and integer. □

Consider the linear system (2) and let the matrices $S \in \mathbb{R}^{\nu\times\nu}$, $L \in \mathbb{R}^{1\times\nu}$ and $Q \in \mathbb{R}^{\nu\times\nu}$, $R \in \mathbb{R}^{\nu}$ be such that the pair $(L, S)$ is observable and the pair $(Q, R)$ is controllable, respectively. Based on Definition 1, moments can be characterized using the solution of the Sylvester equation

$$A\Pi + BL = \Pi S, \tag{3}$$

in the unknown $\Pi \in \mathbb{C}^{n\times\nu}$ and its "dual"

$$Q\Upsilon = \Upsilon A + RC, \tag{4}$$

in the unknown $\Upsilon \in \mathbb{C}^{\nu\times n}$. Assume that $\sigma(A) \cap \sigma(S) = \emptyset$, where $\sigma(A)$ is the spectrum of the matrix $A$. Since $\Sigma$ is minimal, equation (3) has a unique solution $\Pi$ and $\mathrm{rank}\ \Pi = \nu$. Assuming $\sigma(A) \cap \sigma(Q) = \emptyset$, then equation (4) has a unique solution $\Upsilon$ and $\mathrm{rank}\ \Upsilon = \nu$ (see e.g., [9]).

**Definition 2.** [5], [6]

1) Let $\phi = [\phi_1\ \phi_2\ ...\ \phi_\nu] \in \mathbb{C}^{1\times\nu}$ be such that

$$\phi = C\Pi. \tag{5}$$

We call the moments of system (2) at $\sigma(S)$ the elements $\phi_i$, $i = 1, ..., \nu$. The interpolation points are the eigenvalues of $S$, i.e., $\{s_1, s_2, ..., s_\nu\} = \sigma(S)$.

2) Let $\varphi = [\varphi_1\ \varphi_2\ ...\ \varphi_\nu]^T \in \mathbb{C}^{\nu}$ be such that

$$\varphi = \Upsilon B. \tag{6}$$

We call the moments of system (2) at $\sigma(Q)$ the elements $\varphi_i$, $i = 1, ..., \nu$. The interpolation points are the eigenvalues of $Q$, i.e., $\{s_1, s_2, ..., s_\nu\} = \sigma(Q)$. □

Note that the moments as in Definition 1 are equivalent to the notions depicted in Definition 2. Selecting $(L, S)$ and $(Q, R)$ in canonical forms, easy computations yield $[\eta(s_1)\ ...\ \eta(s_\nu)] = \phi = \varphi$.

Based on Definition 2, we define a family of parameterized models of order $\nu$ that achieve moment matching at the interpolation points $\{s_1, ..., s_\nu\} = \sigma(S)$.

**Theorem 1.** *[5], [6]*

1) Let the pair $(L, S)$ be observable and assume $\sigma(A) \cap \sigma(S) = \emptyset$. Let $\xi(t) \in \mathbb{R}^\nu$ and consider the family of linear models

$$\Sigma_G : \begin{cases} \dot{\xi} = (S - GL)\xi + Gu, \\ \eta = C\Pi\xi, \end{cases} \tag{7}$$

parameterized by $G \in \mathbb{C}^\nu$, where $\Pi$ is the unique solution of (3). Assume $\sigma(S - GL) \cap \sigma(S) = \emptyset$. Let $\hat{\phi} \in \mathbb{C}^{1\times\nu}$ be the moments of (7) at $\sigma(S)$. Then (7) describes a family of reduced order models of (2), parameterized in $G$ and achieving moment matching at $\sigma(S)$, i.e., $\phi = \hat{\phi}$.

2) Let the pair $(Q, R)$ be controllable and assume $\sigma(A) \cap \sigma(Q) = \emptyset$. Let $\xi(t) \in \mathbb{R}^\nu$ and consider the family of linear models

$$\Sigma_H : \begin{cases} \dot{\xi} = (Q - RH)\xi + \Upsilon Bu, \\ \eta = H\xi, \end{cases} \tag{8}$$

parameterized by $H \in \mathbb{R}^{1\times\nu}$, where $\Upsilon$ is the unique solution of (4). Assume $\sigma(Q - RH) \cap \sigma(Q) = \emptyset$. Let $\hat{\varphi} \in \mathbb{C}^{1\times\nu}$ be the moments of (8) at $\sigma(Q)$. Then (8) describes a family of reduced order models of (2), parameterized in $H$ and achieving moment matching at $\sigma(Q)$, i.e., $\varphi = \hat{\varphi}$. □

**Remark 1.** Any system

$$\begin{aligned} \dot{\xi} &= F\xi + G\xi, \\ \eta &= H\xi, \end{aligned} \tag{9}$$

with $F \in \mathbb{R}^{\nu\times\nu}$, $G \in \mathbb{R}^{\nu}$, $H \in \mathbb{R}^{1\times\nu}$ matches the moments of (2) at $\sigma(S)$ if and only if it is equivalent[1] to (7), i.e., $HP = C\Pi$, where the coordinate transformation $P \in \mathbb{C}^{\nu\times\nu}$ is the unique solution of the Sylvester equation $FP + GL = PS$ (see, e.g., [5] for more details). Similar arguments hold for the case of (8) (see, e.g., [6] for more details). □

## III. Moment matching for block systems

Consider the linear system (2) with the following structure:

$$A = [A_{ij}]_{i,j=1,\ldots,N}, A_{ij} \in \mathbb{C}^{n_i\times n_j} \tag{10}$$

$$B = \begin{bmatrix} B_1 \\ \vdots \\ B_N \end{bmatrix}, B_i \in \mathbb{C}^{n_i}, C = \begin{bmatrix} C_1 & \ldots & C_N \end{bmatrix}, C_j \in \mathbb{C}^{1\times n_j}.$$

We adapt the notions presented in Section II to the structure given in (10), i.e., we perform model reduction on each diagonal component of (10), which automatically triggers the reduction of the other components. Let $S_1, ..., S_N$, $S_i \in$

[1] Two minimal systems described by state-space equations are equivalent if they have the same transfer functions, i.e., the same input-output behaviour.

$\mathbb{C}^{\nu_i \times \nu_i}$ and $L_1$, ..., $L_N$ of appropriate dimension, such that the pairs $(L_i, S_i)$ are observable. The moments of the diagonal subsystems of (2) with the block structure (10) at $(L, S)$ are described by $C_i \Pi_i$, $i = 1, ..., N$, where $\Pi_i$satisfy

$$A_{ii}\Pi_i + B_i L_i = \Pi_i S_i, \tag{11a}$$
$$A_{ij}\Pi_j + B_i L_j = 0, \tag{11b}$$

$i, j = 1, ..., N$. Note that in practice, these equations are never solved. Information about the moments is usually obtained via Krylov projections or other computationally efficient schemes. Furthermore, equations (11b) are useful in the choice of $L_j$, as projecting directions in cases such as tangential interpolation for MIMO systems. A reduced order model (9) that has a structure similar to (10), is given by equations of the form (9), with

$$F = [F_{ij}]_{i,j=1,...,N}, G = \left[G_1^T ... G_N^T\right]^T, H = [H_1 ... H_N], \tag{12}$$

with $F_{ij} \in \mathbb{C}^{\nu_i \times \nu_j}, G_i \in \mathbb{C}^{\nu_i}$ and $H_j \in \mathbb{C}^{1 \times \nu_j}$. A family of reduced order models (12) parameterized in $G_i$ that matches the moments of (10) at $(L, S)$ is given by the following equations

$$F_{ii} = S_i - G_i L_i, F_{ij} = -G_j L_i, H_i = C_i \Pi_i, i, j = 1, ..., N \tag{13}$$

Note that the free parameters $G_i$ may be used to enforce additional properties or structure on the block structured approximation (13). Just like in Section II, a dual version of the problem may be posed and solved accordingly. Hence, let $Q_1, ..., Q_N$, $Q_i \in \mathbb{C}^{\nu_i \times \nu_i}$ and $R_1^T$, ..., $R_N^T$ of appropriate dimension, such that the pairs $(Q_i, R_i)$ are controllable. The moments corresponding to the diagonal blocks of (10) at $(Q_i, R_i)$ are described by $\Upsilon_i B_i$, $i = 1, ..., N$, where $\Pi_i$satisfy

$$Q_i \Upsilon_i = \Upsilon_i A_{ii} + R_i C_i, \tag{14a}$$
$$0 = \Upsilon_j A_{ij} + R_j C_i, \tag{14b}$$

$i, j = 1, ..., N$. Note that the equations (14b) are useful in the choice of $R_j$, as projecting directions in cases such as left tangential interpolation for MIMO systems. A family of reduced order models (12) parameterized in $H_i$ that matches the moments of (10) at $(Q, R)$ is given by the following equations

$$F_{ii} = Q_i - R_i H_i, F_{ij} = -R_j H_i, G_i = \Upsilon_i B_i, i, j = 1, ..., N. \tag{15}$$

Similarlty, note that the free parameters $H$ may be used to enforce additional properties or structure on the block structured approximation (15).

## IV. Model reduction of a control law

In this section we turn our attention to a particular structure presented in [24], i.e., the rational implementation of a control law with distributed delays, for time-delay systems. Hence, we deal with a system (10) with the following particular structure

$$\begin{aligned} &A_{ii} = A - \Phi \in \mathbb{C}^{n \times n}, A_{i,i+1} = \Phi, \\ &B_i = 0, i = 1, ..., N-1, B_N = B \in \mathbb{C}^n, C_j = I_n, \end{aligned} \tag{16}$$

with $\Phi = \frac{N}{h} I_n$,where $h$ is the value of the distributed delay. Due to the sparsity of this structure, we propose a model reduction procedure adapting the results of Section III to this particular case. Let $L \in \mathbb{C}^{1 \times \nu}, S \in \mathbb{C}^{\nu \times \nu}$ be such that the pair $(L, S)$ is observable. The moments of (16) at $(L, S)$ are in one-to-one relation with the matrices $\Pi_i$, solutions of the equations

$$(A - \Phi)\Pi_i + \Phi \Pi_{i+1} = \Pi_i S, i = 1, ..., N-1, \tag{17a}$$
$$(A - \Phi)\Pi_N + BL = \Pi_N S. \tag{17b}$$

Note that the reduction procedure starts with the last block, sufficient for the reduction of all the other components.

We compute a model (12) that has the following structure

$$\begin{aligned} &F_{ii} = F - \widetilde{\Phi} \in \mathbb{C}^{\nu \times \nu}, F_{i,i+1} = \widetilde{\Phi} \in \mathbb{C}^{\nu \times \nu}, \\ &G_i = 0, i = 1, ..., N-1, G_N = G \in \mathbb{C}^\nu, H_j = \Pi_j \in \mathbb{C}^{n \times \nu}. \end{aligned} \tag{18}$$

This model achieves moment matchin at $(L, S)$ if there exist invertible matrices $P_i \in \mathbb{C}^{\nu \times \nu}, i = 1, ..., N$ such that the following conditions are satisfied

$$(F - \widetilde{\Phi})P_i = P_i S - \widetilde{\Phi} P_{i+1}, i = 1, ..., N-1, \tag{19a}$$
$$(F - \widetilde{\Phi})P_N = P_N S - GL. \tag{19b}$$

A family of models parameterized in $G$,of order $N\nu < Nn$ can be obtained for $P_i = I, i = 1, ..., N$, i.e.,

$$F - \widetilde{\Phi} = S - GL, \widetilde{\Phi} = GL, H_i = \Pi_i, i = 1, ..., N. \tag{20}$$

Note that, thank to the structure of the reduced order model, the free parameter $G$ can be used to enforce additional properties. Furthermore, since the structure of the given system is preserved, the convergence of the implementation is also preserved, see, e.g., [24, Theorem 12.4]. However, the model (20) does not answer the question of reducing the number $N$ of lowpass filters used for the implementation of the control law (16). Since, in moment matching techniques there is no structured way of computing a number $\mu < N$ of low pas filters to approximate the control law, we use the balanced truncation method. We propose the following procedure:

- Let $S = s_0 \in \mathbb{C}$ and $L = 1$, i.e., interpolate at one point in the complex plane. The result will be a model (20), of order $N$ with the following structure

$$\begin{bmatrix} s_0-g & g & 0 & ... & 0 \\ 0 & s_0-g & g & ... & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & s_0-g & g \\ 0 & \dots & 0 & 0 & s_0-g \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ g \end{bmatrix}, \begin{bmatrix} \Pi_1 & \dots & \Pi_N \end{bmatrix}.$$

- Since $N$ is not extremely large, we compute a balanced realization of this model. Assuming that there exists $\mu$such that $\sigma_\mu > \sigma_{\mu+1}$, we perform truncation at order $\mu$. Note that this procedure might not preserve the bidiagonal structure of the model.
- Since $\mu$ is assumed small, there exists a coordinate transformation that transform the balanced truncation model into the bidiagonal, desired form. The final result is an implementation of the control law (16) with a number of $\mu < N$ first order low-pass filters. Again, making use of [24, Theorem 12.4], the convergence of the implementation is insured. However, the computation of an additional coordinate transformation is not desirable and not required. One may easily apply a moment matching algorithm once again, but without the reduction step. To this end, let $\bar{S}$ be a Jordan block of order $\mu$, i.e., $\bar{S}_{ii} = s^*, i = 1, ..., \mu, \bar{S}_{j,j+1}, j = 1, ..., \mu - 1$ and zero, for the rest and $\bar{L} = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix} \in \mathbb{C}^{1\times\mu}$. Note that in this particular form, the pair $(\bar{L}, \bar{S})$ is observable. Hence a model of order $\mu < N$ that has approximately the bidiagonal form of the given system is described by equations with the following parameters

$$\begin{bmatrix} s^* & g & 0 & ... & 0 \\ 0 & s^* & g & ... & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & s^* & g \\ -\bar{g} & \dots & 0 & 0 & s^* \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \bar{g} \end{bmatrix}, \begin{bmatrix} \eta_1 & \dots & \eta_\mu \end{bmatrix}.$$

Note that this model has approximately the structure of the the given system, with order $\mu < N$. The free parameters $s^*$ and $\bar{g}$ can be tuned in order to achieve the best desired performance. Mote that for very small values of $\bar{g}$ the approximation resembles the structure of the given system.

## V. CONCLUSIONS

In this abstract we have studied the general problem of model reduction of interconnected systems. Using recently developed moment matching techniques, we have computed families of reduced order models that achieve moment matching and partly, or completely preserve the block structure of the given system. In particular, we have addressed the structure of the controller implemented in [24, Section 12.6]. We have adapted the problem of moment matching to obtain a family of reduced order blocks that approximate the subsystems of the given system, preserving its properties. Furthermore, we have studied the problem of reducing the number of blocks of the given system. Our reduction strategy, consists of interpolating each subsystem at one point and then prerform balanced truncation, in order to reduce the number of subsystems. The result is a low order controller that approximates the full order controller and preserves its properties. For future work, we intend to extend the presented algorithm to the case of interpolation at multiple points in the complex plane and extend the theory to other significant cases in systems and control theory.

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