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Robustness Analysis of a Nominally String-Stable Irrigation Channel Control System

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Abstract— **This paper considers the robustness of a recently proposed distributed distant-downstream control architecture for irrigation channels. The components of this can be designed in a scalable fashion (i.e. pool-by-pool) to nominally achieve an L_∞ string-stability property that concerns the spatial propagation of transient flow peaks. The robustness of this property is investigated via known LMI based analysis conditions for bounding the L_∞ induced norm of systems with uncertain transfer functions. Application of the conditions, which are only sufficient, does not confirm string-stability robustness for the channel example presented. However, the robust induced-norm bounds obtained for substantial pool-delay parameter uncertainty are such that the degree to which transient flow peaks could be amplified remains reasonable at worst. Illustrative simulations are presented.**

I. INTRODUCTION

Irrigation networks are used to distribute water from primary sources (e.g. rivers, lakes) to farms. Within a network, each open-water channel is divided into sections called pools, which are linked by gates that locally set the flow. The automation objectives for each pool are to provide (i) matching between water in-flow and out-flow including the local off-takes to farms or secondary channels and the downstream flow load and (ii) regulation of the water-level at the downstream end of pools, which corresponds to the capacity to supply flow. So-called distant-downstream control schemes, as considered in [1], [2] for example, confine the propagation of water-level and flow transients to the upstream direction, which is desirable in that it corresponds to demand-driven water release from primary sources, while achieving the aforementioned local objectives for step water-level reference and off-take load changes. For the distributed distant-downstream controllers considered in [1], [3], it can be shown that in the case of a homogeneous channel, transient flow peaks are amplified as they propagate upstream.

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In [4], a nominal spatially-stable propagation of the transient peaks is achieved with a new distributed distant-downstream control architecture at the expense of steady-state water-level errors for step changes in off-take load and constant water-level set-points. In particular, focusing on the flow interactions between pools, the new scheme involves the augmentation of each decentralized local feedback controller with a feed-forward path from the downstream flow to the controller input. This translates to an adjustment of the local water-level reference on the basis of downstream flow, in a vein similar to time-headway schemes in automated vehicle platoons. The resulting non-amplification of the transient flow peaks is called "peak-to-peak string stability" or simply "string stability". The focus on bounding flow transient peaks is motivated by the limited authority of the control gates, which saturate in terms of the flow delivered when in the fully open or fully closed position. Within the context of vehicle platoons, L_∞ norm based characterization of string stability has been considered in various papers; see, for example, [5], [6], [7], [8], [9], [10], [11], [12].

Like the decentralized and ad-hoc inter-pool feed-forward scheme described in [1], the process of designing a controller with the distributed distant-downstream control architecture from [4] is scalable in the sense that the local components of the distributed controller are chosen on the basis of local pool model parameters alone. While this design process can nominally achieve L_∞ string-stability, the robustness of this property to model parameter uncertainty is of practical concern. Robustness of the peak-to-peak gain of the automated flow interactions between neighbouring pools is investigated here via LMI based analysis conditions from [13]. While the application of these does not confirm robustness of the string-stability property, the bounds obtained show that the degree to which the property is violated remains mild in the face of substantial uncertainty. That is, transients may be spatially amplified, but only slightly at worst, compared to other distributed distant-downstream control schemes.

The paper is structured as follows: Section II describes

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how string-stability is achieved under distributed distant-downstream control. L_∞ robust performance analysis for a general system with parametric uncertainty is discussed in Section III. String-stability robustness of the distributed control of an irrigation channel is analyzed as an example in Section IV.

The following notation is used throughout. $\mathbb{R}$ is the field of real numbers. The convex hull of real matrices $\delta_1, \cdots, \delta_H$ is denoted by $\mathrm{Co}\{\delta_1, \delta_2 \cdots, \delta_H\}$. Given matrices X and Λ with compatible dimensions, $*^T X\Lambda$ is short-hand for $\Lambda^T X\Lambda$. For measurable signals, $\|u\|_\infty := \sup_t |u(t)|$. The space L_∞ comprises those signals u such that $\|u\|_\infty$ is finite. The Laplace transform of a signal u is denoted by U. $T_{V\to U}(s)$ denotes the transfer function of a system that maps from v to u in the time-domain. The L_∞ induced norm of a system with stable transfer function $G(s)$ is denoted by $\|G\|_{\infty\to\infty} = \|g(\cdot)\|_1 := \int_0^\infty |g(t)|dt$, where $g(t)$ is the impulse response. $\mathscr{F}_u(M,\Delta)$ represents the upper LFT interconnection of transfer functions M and Δ.

II. Achieving string-stability Under Distributed Distant-Downstream Control

The side-view of an irrigation channel decomposed into pools and the control structure considered in this paper are depicted in Figure 1. Let Y_i, U_i, D_i, V_i denote the Laplace transforms of the water-level at the downstream gate $y_i(t)$, the flow over the upstream gate $u_i(t)$, the off-take flow onto farms and secondary channels which are typically drawn from the downstream end $d_i(t)$, and the flow over the downstream gate $v_i(t)$ of pool i, respectively. Note that a channel of N pools (i.e. the path-graph interconnection of pools) is considered in this paper where the pools are indexed from downstream towards upstream, with the bottom and top pools indexed by 0, and N, respectively, and with flow coupling $U_{i-1} = V_i$. The dynamics of a pool in an irrigation channel can be described by [14], [15]

$$Y_i = \frac{1}{\alpha_i s}[U_i e^{-\tau_i s} - (V_i + D_i)], \qquad (1)$$

which captures mass balance and transport delay, denoted by τ_i of pool i. The parameter α_i is a measure of pool surface area. This model is suitable for control design purposes in terms of the objectives introduced earlier. Given a constant water-level reference set-point r_i, the local control objectives for each pool are $\lim_{t\to\infty} e_i(t) := r_i - y_i(t) = 0$ and $\lim_{t\to\infty} u_i(t) = \lim_{t\to\infty}(v_i(t) + d_i(t))$ in response to step changes in flow load $d_0(t) = d_0$. These can be satisfied with two integrators in the local closed-loop, i.e. by using a local feedback controller containing an integrator and

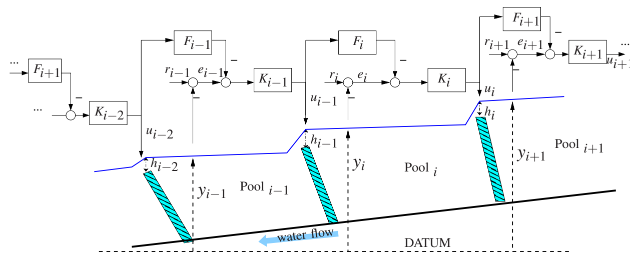

Fig. 1. Decentralised feedback with feedforward control scheme of a channel.

treating the downstream flow-load as a disturbance in the same way as the off-take. However, such a completely decentralized control architecture results in a spatially unstable amplification of flow transient peaks towards upstream end of the channel [3], [1]. The H_∞ loop-shaping based distributed controller considered in [1], [16] also exhibits such string instability.

*Definition 1 (L_∞ **string stability**):* A channel under distributed distant-downstream control is said to be L_∞ string stable if there exists a scalar $0 < B < \infty$ such that, with $d_i = 0$ for $i = 1,2,\ldots$ and d_0 uniformly bounded across time, $\|u_i\|_\infty \le B\|d_0\|_\infty$ for all $i = 0,1,2,\ldots$.

Remark 1: By virtue of the steady-state flow matching requirement, sub-linear growth of $\|u_i\|_\infty$ is not possible with all d_i set to unit steps, for example. This is why the definition above specifies $d_i = 0$ for $i = 1,2,\ldots$.

In [4], a distributed distant-downstream controller is proposed comprising decentralized feedback compensators $K_i(s) = \frac{\kappa_i(1+\phi_i s)}{s(1+\rho_i s)}$, each locally augmented with a feed-forward filter $F_i(s)$ as shown in Figure 1. The following holds in the frequency domain for each pool locally:

$$U_i = \frac{L_i(s)}{1 + L_i(s)e^{-\tau_i s}}[(1 - \frac{F_i(s)}{P_i(s)})V_i + D_i], \qquad (2)$$

where $P_i(s) := \frac{1}{\alpha_i s}$ and $L_i(s) := K_i(s)P_i(s)$ is the open-loop transfer function without the delay. From (2) it can be seen that the feed-forward filter $F_i(s)$ affects the local transfer function $T_{V_i\to U_i}(s)$, without modifying the local load disturbance rejection performance. Let

$$G_i(s) := \frac{L_i(s)}{1 + L_i(s)e^{-\tau_i s}}(1 - \frac{F_i(s)}{P_i(s)}), \qquad (3)$$

and suppose that $F_i(s)$ is chosen so that both it and $G_i(s)$ are stable transfer functions. Denoting the impulse response of G_i by g_i, the following lemma holds.

Lemma 1: [4] The distributed distant-downstream control scheme shown in Figure 1 is L_∞ string stable in the sense of Definition 1 if $\|g_i\|_1 \le 1$ for $i = 1,2,\ldots$.

Proof: The result follows by virtue of the relation $\|u_i\|_\infty \leq \|g_i\|_1 \|u_{i-1}\|_\infty$, with u_0 the bounded response to a bounded off-take d_0 at the bottom pool; i.e. $U_0(s) = L_0(s)/(1+L_0(s)e^{-\tau_0 s})D_0(s)$. ∎

A channel with a feed-forward path filter chosen as

$$F_i(s) = \frac{-1}{K_i(s)(1+T_{c,i}s)} + P_i(s)(1 - \frac{e^{-\tau_i s}}{1+T_{c,i}s}) \quad (4)$$

gives

$$G_i(s) = (\frac{L_i(s)}{1+L_i(s)e^{-\tau_i s}})(1 - \frac{F_i(s)}{P_i(s)}) = \frac{1}{1+T_{c,i}s}, \quad (5)$$

where $T_{c,i} > 0$. This achieves steady-state flow matching along the channel (i.e. $\lim_{s\to 0} G_i(s) = 1$) and L_∞ string-stability for a nominal channel model with parameters τ_i and α_i. Clearly, other choices for $G_i(s)$ are possible.

If the condition in Lemma 1 cannot be verified, but instead $\|g_i\|_1 < \gamma$ for some $\gamma > 1$ and $i = 1, 2, \ldots$, then at worst the peak of any flow transient can be amplified by a factor of γ as it propagates from one pool to the next. That is, $\|u_i\|_\infty \leq \gamma^i \|u_0\|_\infty$. So provided γ is sufficiently close to 1 and the number of pools is sufficiently small, the amplification along the channel can remain acceptable.

In the next section, sufficient LMI conditions are provided to characterize a robust bound on the L_∞ induced norm (i.e. L_1 norm of the impulse response) of an uncertain transfer function. These are then applied to assess the robustness of $\|G_i\|_{\infty\to\infty} = \|g_i\|_1$ to delay parameter uncertainty for each pool of an irrigation channel example, including simulations in Section IV.

III. Assessing L_∞ Robust Performance

To establish L_∞ induced norm performance bounds for parametrically uncertain transfer functions, an LFT modelling framework can be employed [17]. Given a

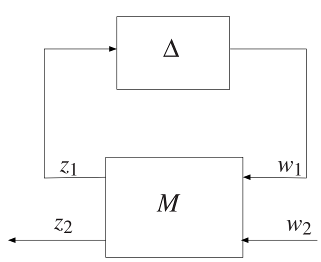

Fig. 2. Upper LFT representation of $\mathscr{F}_u(M,\Delta)$.

transfer function $M(s) = [\begin{smallmatrix} M_{11}(s) & M_{12}(s) \\ M_{21}(s) & M_{22}(s) \end{smallmatrix}]$ and a real matrix Δ that represents parametric uncertainty, the transfer function for the upper LFT interconnection that maps from w_2 to z_2 in Figure 2, is given by

$$\mathscr{F}_u(M,\Delta) = M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12}. \quad (6)$$

If $M(s)$ holds a state-space realization

$$\left[\begin{array}{c|cc} A & B_0 & B_1 \\ \hline C_0 & D_{00} & D_{01} \\ C_1 & D_{10} & D_{11} \end{array}\right], \quad (7)$$

then a state-space realization of $\mathscr{F}_u(M,\Delta)$ is given by

$$\left[\begin{array}{c|c} A(\Delta) & B(\Delta) \\ \hline C(\Delta) & D(\Delta) \end{array}\right], \quad (8)$$

where $A(\Delta) := A + B_0(I - \Delta D_{00})^{-1}\Delta C_0$, $B(\Delta) := B_1 + B_0(I - \Delta D_{00})^{-1}\Delta D_{01}$, $C(\Delta) := C_1 + D_{10}(I - \Delta D_{00})^{-1}\Delta C_0$ and $D(\Delta) := D_{11} + D_{10}(I - \Delta D_{00})^{-1}\Delta D_{01}$. Following [18], Proposition 3.14 in [13], and Theorem 3.9 in [19], Lemma 2 below provides sufficient LMI conditions that characterize a robust bound on the L_∞ induced norm of the system with the uncertain transfer function $\mathscr{F}_u(M,\Delta)$, given a set in which Δ can reside.

Lemma 2: Given an uncertainty set $\mathbf{\Delta}$ of appropriately dimensioned real matrices and a scalar $\gamma > 0$, if the interconnection (6) is well-posed (i.e. the inverse exists as a proper transfer function for all $\Delta \in \mathbf{\Delta}$) and $\exists\ X = X^T,\ \lambda > 0,\ \mu$ such that

$$\left[\begin{array}{cc} I & 0 \\ A(\Delta) & B(\Delta) \\ \hline 0 & I \end{array}\right]^T \left[\begin{array}{cc|c} \lambda X & X & 0 \\ X & 0 & 0 \\ \hline 0 & 0 & -\mu I \end{array}\right] \left[\begin{array}{cc} I & 0 \\ A(\Delta) & B(\Delta) \\ \hline 0 & I \end{array}\right] < 0, \quad (9)$$

$$\left[\begin{array}{cc} I & 0 \\ \hline 0 & I \\ C(\Delta) & D(\Delta) \end{array}\right]^T \left[\begin{array}{c|cc} -\lambda X & 0 & 0 \\ \hline 0 & (\mu - \gamma)I & 0 \\ 0 & 0 & \frac{I}{\gamma}I \end{array}\right] \cdot \left[\begin{array}{cc} I & 0 \\ \hline 0 & I \\ C(\Delta) & D(\Delta) \end{array}\right] \leq 0, \quad (10)$$

for all $\Delta \in \mathbf{\Delta}$, then the poles of $A(\Delta)$ are in the left half plane and $\|\mathscr{F}_u(M,\Delta)\|_{\infty\to\infty} \leq \gamma\ \forall \Delta \in \mathbf{\Delta}$.

With x_{cl} defined as the state vector of (8), it follows from (9) that

$$x_{\mathrm{cl}}^T X x_{\mathrm{cl}} \leq \frac{\mu}{\lambda}\|w_2\|_\infty^2,$$

which in combination with (10) yields [19]:

$$\|z_2\|_\infty^2 \leq \gamma^2 \|w_2\|_\infty^2.$$

It is difficult to verify conditions (9) and (10) as they involve functions of the uncertainty $\Delta \in \mathbf{\Delta}$. Applying the so-called Full Block S-Procedure, equivalent conditions can be derived [19], [20]:

Lemma 3: Given the set of real matrices $\mathbf{\Delta}$, if the interconnection (6) is well-posed with all $\Delta \in \mathbf{\Delta}$ and

$\exists\ X = X^T,\ \lambda > 0,\ \mu,\ Q_1 = Q_1^T,\ S_1,\ R_1 = R_1^T,\ Q_2 = Q_2^T,\ S_2,\ R_2 = R_2^T,\ \gamma > 0$ such that

$$\begin{bmatrix}\Delta\\ I\end{bmatrix}^T \begin{bmatrix}Q_1 & S_1\\ S_1^T & R_1\end{bmatrix}\begin{bmatrix}\Delta\\ I\end{bmatrix} > 0, \begin{bmatrix}\Delta\\ I\end{bmatrix}^T \begin{bmatrix}Q_2 & S_2\\ S_2^T & R_2\end{bmatrix}\begin{bmatrix}\Delta\\ I\end{bmatrix} \geq 0, \tag{11}$$

$$*^T \left[\begin{array}{cc|cc|c} \lambda X & X & 0 & 0 & 0\\ X & 0 & 0 & 0 & 0\\ \hline 0 & 0 & Q_1 & S_1 & 0\\ 0 & 0 & S_1^T & R_1 & 0\\ \hline 0 & 0 & 0 & 0 & -\mu I \end{array}\right] \left[\begin{array}{ccc} I & 0 & 0\\ A & B_0 & B_1\\ \hline 0 & I & 0\\ C_0 & D_{00} & D_{01}\\ \hline 0 & 0 & I \end{array}\right] < 0,$$

$$*^T \left[\begin{array}{c|cc|cc} -\lambda X & 0 & 0 & 0 & C_1^T\\ \hline 0 & Q_2 & S_2 & 0 & D_{10}^T\\ 0 & S_2^T & R_2 & 0 & 0\\ \hline 0 & 0 & 0 & (\mu-\gamma)I & D_{11}^T\\ C_1 & D_{10} & 0 & D_{11} & -\gamma I \end{array}\right] \cdot \left[\begin{array}{cccc} I & 0 & 0 & 0\\ \hline 0 & I & 0 & 0\\ C_0 & D_{00} & D_{01} & 0\\ \hline 0 & 0 & I & 0\\ 0 & 0 & 0 & I \end{array}\right] \leq 0, \tag{12}$$

then the poles of $A(\Delta)$ are in the left half plane and $\|\mathscr{F}_u(M,\Delta)\|_{\infty\to\infty} \leq \gamma,\ \forall\Delta\in\mathbf{\Delta}$.

Conditions in (11) involve the uncertainty $\Delta \in \mathbf{\Delta}$, yielding an infinite number of inequalities that need to be checked. Following [19], these can be equivalently converted to tractable conditions by taking $\mathbf{\Delta}$ to be the convex hull of finite number of matrices if $Q_1 < 0$ and $Q_2 < 0$ are assumed for convexity, introducing conservatism in general [19]:

Lemma 4: Given real matrices $\delta_1,\ \cdots,\ \delta_H$, if the interconnection (6) is well-posed with all $\Delta = \delta_n$ and $\exists\ X = X^T,\ \lambda > 0,\ \mu,\ Q_1 = Q_1^T,\ S_1,\ R_1 = R_1^T,\ Q_2 = Q_2^T,\ S_2,\ R_2 = R_2^T,\ \gamma > 0$ such that

$$\begin{aligned} & Q_1 < 0,\ Q_2 < 0,\\ & \begin{bmatrix}\delta_n\\ I\end{bmatrix}^T \begin{bmatrix}Q_1 & S_1\\ S_1^T & R_1\end{bmatrix}\begin{bmatrix}\delta_n\\ I\end{bmatrix} > 0,\ \forall\ n\in\{1,2,\cdots\ H\}\\ & \begin{bmatrix}\delta_n\\ I\end{bmatrix}^T \begin{bmatrix}Q_2 & S_2\\ S_2^T & R_2\end{bmatrix}\begin{bmatrix}\delta_n\\ I\end{bmatrix} \geq 0\ \forall\ n\in\{1,2,\cdots\ H\}, \end{aligned} \tag{13}$$

$$*^T \left[\begin{array}{cc|cc|c} \lambda X & X & 0 & 0 & 0\\ X & 0 & 0 & 0 & 0\\ \hline 0 & 0 & Q_1 & S_1 & 0\\ 0 & 0 & S_1^T & R_1 & 0\\ \hline 0 & 0 & 0 & 0 & -\mu I \end{array}\right] \left[\begin{array}{ccc} I & 0 & 0\\ A & B_0 & B_1\\ \hline 0 & I & 0\\ C_0 & D_{00} & D_{01}\\ \hline 0 & 0 & I \end{array}\right] < 0, \tag{14}$$

$$*^T \left[\begin{array}{c|cc|cc} -\lambda X & 0 & 0 & 0 & C_1^T\\ \hline 0 & Q_2 & S_2 & 0 & D_{10}^T\\ 0 & S_2^T & R_2 & 0 & 0\\ \hline 0 & 0 & 0 & (\mu-\gamma)I & D_{11}^T\\ C_1 & D_{10} & 0 & D_{11} & -\gamma I \end{array}\right] \cdot \left[\begin{array}{cccc} I & 0 & 0 & 0\\ \hline 0 & I & 0 & 0\\ C_0 & D_{00} & D_{01} & 0\\ \hline 0 & 0 & I & 0\\ 0 & 0 & 0 & I \end{array}\right] \leq 0, \tag{15}$$

then the poles of $A(\Delta)$ are in the left half plane and $\|\mathscr{F}_u(M,\Delta)\|_{\infty\to\infty} \leq \gamma\ \forall\Delta\in \mathrm{Co}\{\delta_1,\ \delta_2,\ \cdots,\ \delta_H\}$.

The bilinear term λX in the LMIs above make the conditions non-convex. However, the problem is convex for a fixed λ. Solving the LMIs for a grid of λ, a value of λ that yields the smallest possible γ can be identified.

IV. Robustness to Uncertainty in Pool Delay Parameter

To illustrate an example of the LMI robustness analysis conditions in Section III, within the context of assessing the string-stability robustness of the distributed control architecture shown in Figure 1, this section focuses on time-delay parameter uncertainty and a homogeneous channel of three pools having the nominal model and controller parameters specified in Table I.

In order to apply the finite-dimensional state-space based conditions of Lemma 4, a Padé approximation of the delay element is used to approximate delay terms in $F(s)$ and the feedback path in Figure 1. The first order Padé approximation $e^{-\tau s} \cong \frac{1-\frac{\tau}{2}s}{1+\frac{\tau}{2}s}$ is reasonable for the delay component in both the plant model and the feed-forward filter $F(s)$ at low frequencies and suitable as long as the bandwidth of the closed-loop (i.e. cross-over of $L(s)$) and that of $G(s)$ is set small enough by $K(s)$ and $F(s)$, respectively [4]. The approximation of the feed-forward path filter is denoted by $F_p(s)$. Using $\frac{1-\frac{\tau}{2}s}{1+\frac{\tau}{2}s} = -1 + \frac{2}{1+\frac{\tau}{2}s}$, the plant model with uncertain parameter $\tau = \tau_0 + \delta_\tau$, where $\delta_\tau \in \mathbf{\Delta} \subset \mathbb{R}$, and $\mathbf{\Delta}$ is a closed interval, has the structure shown in Figure 3.

With reference to Figure 3, an upper LFT representation of $T_{V_i\to U_i}$ is given by $\mathscr{F}_u(M,\delta_\tau)$ in accordance with (6), where

$$M = \begin{bmatrix} M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix}, \tag{16}$$

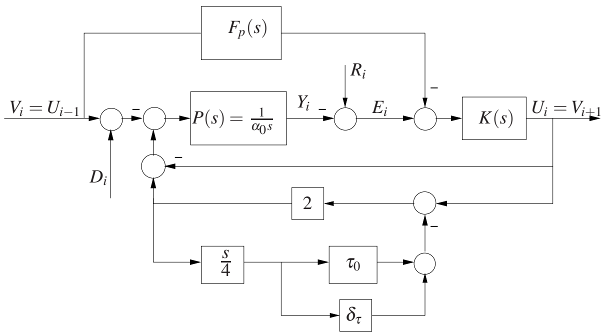

Fig. 3. Block diagram of a closed-loop with a feed-forward of the downstream flow with the delay represented with additive uncertainty.

TABLE I

POOL MODEL AND CONTROLLER PARAMETERS

Model Parameters		Controller Parameters		
nominal τ_0 (mins)	α (m^2)	κ	ρ	ϕ
8	13847	1.1198	22.4591	212.1900

with

$$M_{11}(s) = -\frac{s}{2(1+\frac{\tau_0}{2}s)} + \frac{sKP}{(1+\frac{\tau_0}{2}s)^2(1+KP\frac{1-\frac{\tau_0}{2}s}{1+\frac{\tau_0}{2}s})}$$

$$M_{12}(s) = \frac{K(P-F_p)s}{2(1+\frac{\tau_0}{2}s)(1+KP\frac{1-\frac{\tau_0}{2}s}{1+\frac{\tau_0}{2}s})}$$

$$M_{21}(s) = \frac{2KP}{(1+\frac{\tau_0}{2}s)(1+KP\frac{1-\frac{\tau_0}{2}s}{1+\frac{\tau_0}{2}s})}$$

$$M_{22}(s) = \frac{K(P-F_p)}{1+KP\frac{1-\frac{\tau_0}{2}s}{1+\frac{\tau_0}{2}s}}.$$

Table II summarizes the smallest achievable γ for which the condition of Lemma 4 can be satisfied, given the nominal parameters values specified in Table I and different uncertainty set ranges. Since the bound obtained exceeds unity for all cases considered, it is not possible to conclude robustness of the string-stability property. However, it is of note that the bounds are all very close to 1; recall that the gain in transient flow peak as it propagates upstream is bounded as such. The simulations discussed below serve to compare this with the flow peak gain associated with a H_∞ loop-shaping based distributed controller that is designed as described in [1], [16]. The simulations also reveal the conservative nature of the analysis.

TABLE II

BOUNDS ON THE PEAK TO PEAK GAIN IN PRESENCE OF UNCERTAINTY

	τ	γ
$T_c = 100$	8	1.0009
	[7,9]	1.0009
	[6,10]	1.0012
	[5,11]	1.0019
$T_c = 50$	8	1.0026
	[7,9]	1.0116
	[6,10]	1.0265
	[5,11]	1.0440

Simulations are carried out for a 3-pool channel, where a flow off-take of $17.7928\frac{m^3}{min}$ is drawn for 1000 mins from the most downstream pool. Simulations of a nominal pool model with the aforementioned H_∞ loop-shaping based controller are shown in Figure 4. Simulations of channel with uncertainty in the delay parameter under a controller designed as described in Section II are shown in Figures 5 and 6 for different values of T_c, respectively. According to Figure 4, the peak-to-peak gain is about 1.15. This is 12% larger than L_∞ induced-norm bound of the distributed controlled channel under feedforward scheme in the presence of 25% uncertainty in the nominal delay parameter in Table II. The conditions of Lemma 4 are only sufficient for L_∞ string-stability. The bounds on the flow peaks achieved are potentially loose upper bounds. The conservativeness of the method can be seen from the simulations in Figures 5 and 6; from these, it would appear that the string-stability property is actually robust to delay uncertainty.

According to Table II, if T_c is chosen large enough, the bound on the peak-to-peak norm does not grow much larger than 1. Indeed, T_c is a design parameter and identifies a trade-off between attenuation of the water flow peaks and steady-state water level errors [4]. Larger T_c results in smaller bound on the peak-to-peak norms and larger steady-state water level errors, on the other hand.

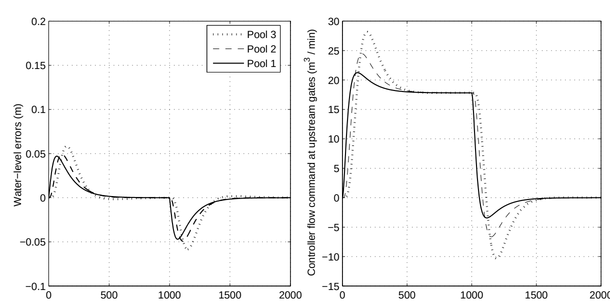

Fig. 4. Simulations with distributed control scheme.

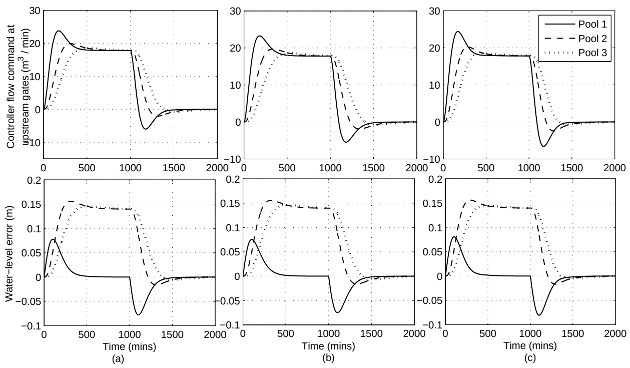

Fig. 5. Simulations with decentralized control and feed-forward scheme, $T_c = 100$. (a)$\tau = 8$(nominal), (b)$\tau = 5$, (c)$\tau = 11$

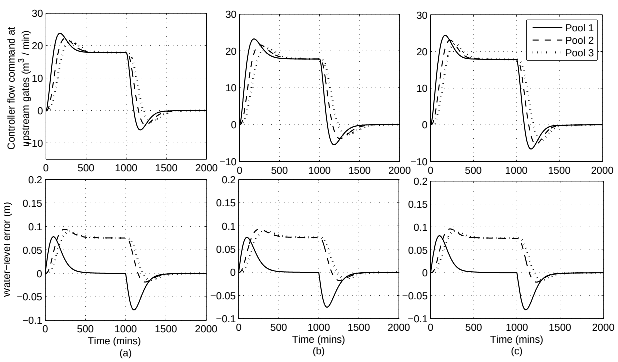

Fig. 6. Simulations with decentralized control and feed-forward scheme, $T_c = 50$. (a)$\tau = 8$(nominal), (b)$\tau = 5$, (c)$\tau = 11$

V. CONCLUSIONS

L_∞ string-stability robustness of an irrigation channel under a distributed distant-downstream control with feed-forward scheme is analyzed via some sufficient LMI conditions. The results show that the rate of amplification of the water flow peaks of a real channel is reasonably small compared to the previous design schemes of distributed distant-downstream control. It should be noted that the method is scalable spatially and the analysis can be done on a pool by pool basis for a heterogenous channel. It is desirable to derive less conservative or necessary and sufficient conditions to analyze robust L_∞ string-stability of an automated channel more precisely.

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