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Link between dissipativity expressed in Riemann coordinates and the Small Gain Theorem, using the Hamiltonian formulation

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***Abstract*— This paper aims at providing some synthesis between two alternative representations of systems of two conservation laws and to make a link between the notions of dissipativity and the Small-Gain theorem. The first one, based on the invariance of its coordinates, is the representation in Riemann coordinates which has been applied successfully for the stabilization of linear and non-linear hyperbolic systems of conservation laws. The second representation is based on physical modeling and leads to port Hamiltonian systems which are extensions of infinite-dimensional Hamiltonian systems defined on Dirac structure encompassing pairs of conjugated boundary variables [11].**

We propose in this paper to link the passivity of the Hamiltonian functional, expressed in Riemann coordinates, with the Small Gain Theorem.

I. INTRODUCTION

In this paper we shall be concerned with the stabilization via boundary control of hyperbolic systems of two conservation laws. The stabilization by boundary control of irrigation channels has been intensively studied for instance in [3], [9], [10] for both linear and non linear cases. The stability of hyperbolic partial differential equations on a one-dimensional spatial domain is widely studied in the literature [2], [4]. One of the most often suggested approaches, uses Riemann invariants to derive a stabilizing boundary control [12]. In recent publications, some extensions are suggested and based on the suitable choice of control Lyapunov function expressed in terms of Riemann's coordinates of the system [2], [4], [6], [7].

The use of physically motivated control Lyapunov function for the derivation of stabilizing control laws for non-linear finite-dimensional systems has proven to be very efficient and has lead to a great variety of results [5], [21], [22]. Very often, when the system stems from physical modeling, one may derive dissipation inequalities related to energy balance equations and energy dissipating phenomena [28]. Using the dissipative port-Hamiltonian formulation for controlled physical systems [5], [19], [22], one may go one step further and assign in closed loop not only some dissipation inequality for some suitable control Lyapunov function but also assign the dynamic behavior by the structure matrices of the Hamiltonian system in closed loop [18], [20]. For infinite-dimensional systems very similar techniques, based on dissipation inequalities, which in terms of PDE's amounts to consider some entropy function [2], [7], have been used for the stabilization of boundary control systems [8], [15]. Recent works have used a boundary port-Hamiltonian formulation of systems of conservation laws [17], [23] in order to derive stabilizing boundary control for a class of linear systems defined on one-dimensional spatial domains [14], [16], [26], [27].

The notion of dissipativity is generally linked with the Small-Gain Theorem. This link has been mentioned (e.g. [2], [3], [6], [7]) several times, but not proved for Greenberg & Li's theorem [12] and the generalized version of this theorem given by [7]. Some recents papers [24](more recently, works of H. Zwart) tends to put in light the Cayley transformation for PDE's control problems.

The sketch of the paper is the following. In a first instance the port Hamiltonian formulation is recalled with respect to a Stokes-Dirac structure [17], [23] in Riemann's coordinates, like the conditions on the boundary feedback relations derived with respect to the Riemann invariants; they are expressed in terms of the port boundary variables of the Hamiltonian formulation and interpreted in terms of the dissipation inequality of the Hamiltonian functional. The links described in Fig. 1 are then established in the third part.

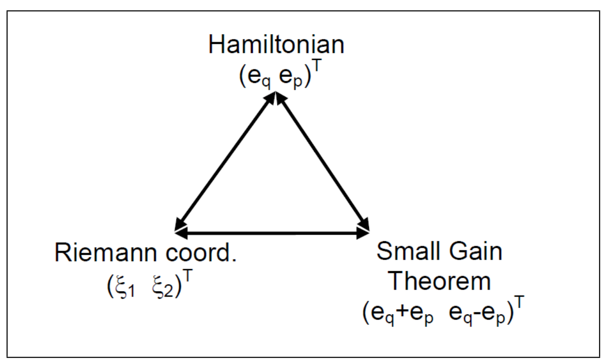

Fig. 1. Relations between Hamiltonian-Riemann-Small Gain Theorem

II. PORT HAMILTONIAN FORMULATION OF A HYPERBOLIC SYSTEM OF TWO CONSERVATION LAWS

A. Recall of the Riemann invariants for an hyperbolic system

In this section, we shall very briefly recall the main result on the stabilization of a hyperbolic system of two conservation laws suggested by Greenberg & Li [12]. Consider a spatial domain consisting of the finite interval $[0, L] \ni x$ with $L \in \mathbb{R}_+^*$ and time domain being the real interval $[0, +\infty) \ni t$. The state space is a non-empty connected

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open set in $I\!R^2$, denoted by Ω. Consider the system of two conservations laws:

$$\partial_t Y + \partial_x f(Y) = 0, \tag{1}$$

where

- $Y = (y_1 \ \ y_2)^T : [0, +\infty) \times [0, L] \to \Omega$ is the vector of the two dependent variables;
- $f : \Omega \to I\!R^2$ is a C^1-function called *the flux vector*.

Note that the system (1) may also be written:

$$\partial_t Y + F(Y)\partial_x Y = 0 \tag{2}$$

where F is the Jacobian of the flux vector f. Assuming that the system is hyperbolic, implies that this system can be diagonalised using the Riemann invariants (see for instance [13, pages 34 - 35]). This means that there exists a change of coordinates $\xi(Y)$ whose Jacobian matrix is denoted $D(Y)$,

$$D(Y) = \frac{\partial \xi}{\partial Y}, \tag{3}$$

and diagonalises $F(Y)$ in Ω:

$$D(Y)F(Y) = \Lambda(Y)D(Y) \qquad Y \in \Omega.$$

In the coordinates ξ, the system (1) can then be rewritten in the following (diagonal) characteristic form:

$$\partial_t \xi + \Lambda(\xi)\partial_x \xi = 0 \tag{4}$$

with ξ: $[0, L] \times [0, +\infty) \to I\!R^2$, $(x, t) \mapsto \xi(x, t)$, and $\Lambda(\xi) = \text{diag}(\lambda_1(\xi), \lambda_2(\xi))$, with $\lambda_1(\xi), \lambda_2(\xi)$ satisfying the conditions:

- the λ_i's are continuously differentiable functions on a neighborhood of the origin;
- $\lambda_2(0) < 0 < \lambda_1(0)$.

In this paper we shall consider the following result of Greenberg and Li [12] which may be recalled as follows.

Theorem 2.1: Consider the hyperbolic system of conservation laws in Riemannian coordinates (4) with the following relations on the boundary variables:

$$\xi_2(0) = \mathbf{K_1}(\xi_2(0)),\ \xi_1(L) = \mathbf{K_2}(\xi_2(L)) \tag{5}$$

with the functions K_1 and K_2 being C^1 and satisfying:

$$\mathbf{K_1}(0) = \mathbf{K_2}(0) = 0 \text{ and } |\mathbf{K_1}'(0)\mathbf{K_2}'(0)| < 1. \tag{6}$$

Consider initial values:

$$\lim_{t \to 0^+} (\xi_1, \xi_2)(x, t) = (\xi_{1,0}, \xi_{2,0})(x),\ 0 < x < L, \tag{7}$$

being C^1 and satisfying the assumption that to be small in the C^1 norm and the compatibility conditions:

$$\xi_{2,0}(0) = \mathbf{K_1}(\xi_{1,0}(0)) \tag{8}$$

$$\xi_{1,0}(L) = \mathbf{K_2}(\xi_{2,0}(L)) \tag{9}$$

$$\begin{aligned}&\lambda_2(\xi_{1,0}, \xi_{2,0})(0)\, \partial_x \xi_{2,0}(0) \\ &\quad = \lambda_1(\xi_{1,0}, \xi_{2,0})(0)\, \mathbf{K_1}'(0)\, \partial_x \xi_{1,0}(0)\end{aligned} \tag{10}$$

$$\begin{aligned}&\lambda_1(\xi_{1,0}, \xi_{2,0})(L)\ \partial_x \xi_{1,0}(L) \\ &\quad = \lambda_2(\xi_{1,0}, \xi_{2,0})(L)\, \mathbf{K_2}'(L) \partial_x \xi_{2,0}(L)\end{aligned} \tag{11}$$

Then the initial value problem, for this system, has a unique C^1 solution. Moreover, its solution decays to zero in the C^1 norm with an exponential rate.

B. Boundary port Hamiltonian systems and Riemann coordinates

1) Hamiltonian operator expressed in the Riemann coordinates: In this section we shall consider a hyperbolic system of two conservation laws (1) which admits a Hamiltonian representation, that is such that vector of flux variables may be written following

$$\partial_x\, e(Y) = \mathcal{J} \begin{pmatrix} \delta_{y_1} H \\ \delta_{y_2} H \end{pmatrix}. \tag{12}$$

with the canonical Hamiltonian operator

$$\mathcal{J} = \epsilon \begin{pmatrix} 0 & \partial_x \\ \partial_x & 0 \end{pmatrix} \tag{13}$$

Hence the system is written:

$$-\partial_t Y = \begin{pmatrix} 0 & \partial_x \\ \partial_x & 0 \end{pmatrix} (\delta_Y H). \tag{14}$$

In the sequel we express the Hamiltonian system in terms of the Rieman's invariants and give the expression of the Hamiltonian operator as well as of the boundary port variables. Denote by $\tilde{H}(\xi)$ the Hamiltonian expressed in the Riemann invariants: $\tilde{H}(\xi) = H \circ Y(\xi)$ where Y denotes, with an abuse of notation, the inverse change of coordinates to the Riemann coordinates.

Let us define $D_\xi = D \circ Y(\xi)$. One obtains by multiplying both terms of (14) by D:

$$\begin{aligned} & -D(Y)\partial_t Y = D(Y) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \partial_x(\delta_Y H) \\ \Leftrightarrow \quad & -\partial_t \xi = D_\xi(\xi) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \partial_x(D^T(\xi)\delta_\xi \tilde{H}(\xi)) \\ \Leftrightarrow \quad & -\partial_t \xi = D_\xi \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \partial_x(D_\xi^T)\delta_\xi \tilde{H}(\xi) \\ & +D_\xi \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} D_\xi^T \partial_x(\delta_\xi \tilde{H}(\xi)). \end{aligned}$$

Hence in terms of the Riemann invariants the system is written:

$$-\partial_t \xi = (B\partial_x + C)\delta_\xi \tilde{H}(\xi), \tag{15}$$

where $B = D_\xi \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} D_\xi^T$ and $C = D_\xi \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \partial_x(D_\xi^T)$. The following properties may be noted: firstly the matrix B is symmetric and secondly it is related with the matrix C by:

$$\partial_x B = C^T + C. \tag{16}$$

2) Boundary port variables: In this section we shall check the formal skew-symmetry of the differential operator $(B\partial_x + C)$ and then define port boundary variables associated with it. Let us define the following bracket on smooth functions on the spatial domain $[0,\ L]$:

$$\{e_1, e_2\} = \int_0^L e_1^T (B\partial_x + C) e_2 \, dx \tag{17}$$

and consider the symmetric product [11]:

$$\{e_1, e_2\} + \{e_2, e_1\} = \int_0^L \partial_x (e_1^T B e_2) = \left[(e_1^T B e_2)\right]_0^L . \tag{18}$$

The product (18) corresponds to Stokes theorem applied to the equation for the differential operator $(B\partial_x + C)$. Furthermore the second member of (18) vanishes for all functions e_1, e_2 with compact support strictly included in the domain $[0,\ L]$ and hence for these functions the bracket is skew-symmetric.

However considering functions which do not vanish on the boundary of the domain, the bracket (17) is no more skew-symmetric. In this case the time variation of the Hamiltonian becomes:

$$\frac{d\tilde{H}(\xi)}{dt} = \left[(\delta_{\xi_1} \tilde{H}^T B \, \delta_{\xi_2} \tilde{H})\right]_0^L . \tag{19}$$

The definition of the boundary port variables follows strictly the construction suggested in [14]. Now the differential operator $B\partial_x + C$ completed with the definition defines the following vector space:

$$\tilde{\mathcal{D}} = \left\{ \left(\left(\begin{array}{c} f \\ f_\partial \end{array} \right), \left(\begin{array}{c} e \\ e_\partial \end{array} \right) \right) \in \mathcal{F} \times \mathcal{E} \ / \ f = (B\partial_x + C) e \right.$$
$$\left. \begin{pmatrix} e_\partial(L) \\ f_\partial(L) \\ e_\partial(0) \\ f_\partial(0) \end{pmatrix} = \mathbf{diag}\,(1,\ 1,\ -1,\ 1) \begin{pmatrix} e_1(L) \\ e_2(L) \\ e_1(0) \\ e_2(0) \end{pmatrix} \right\} \tag{20}$$

Adapting the proofs in [14], one may prove that the vector space $\tilde{\mathcal{D}}$ is a Dirac structure with respect to the pairing defined on $\left(C^\infty[0,L] \times C^\infty[0,L] \times I\!R^2\right) \times \left(C^\infty[0,L] \times C^\infty[0,L] \times I\!R^2\right) \ni ((f,\ f_\partial), (e,\ e_\partial))$:

$$\langle (f,\ f_\partial), (e,\ e_\partial) \rangle = \int_0^L e^T f \, dx + e_\partial^T f_\partial$$

which is canonical in the sense that it does not depend on the differential operator anymore.

C. Stabilizing boundary relations with respect Riemann invariants and boundary port variables:

Consider a hyperbolic system of two conservation laws (1) which admits a Hamiltonian representation (14) with port variables.

Then, let us consider the relations on the boundary port variables defined by a C^1 function G:

$$\begin{pmatrix} e_\partial^0 \\ f_\partial^L \end{pmatrix} = G(f_\partial^0, e_\partial^L), \tag{21}$$

The energy balance equation depends on G_1, G_2 the line components of G and becomes:

$$\frac{dH}{dt} = e_\partial^L G_2(f_\partial^0, e_\partial^L) + f_\partial^0 G_1(f_\partial^0, e_\partial^L) \tag{22}$$

Using the implicit function theorem, the relations (21) on the port boundary variables, may be expressed in terms of boundary port variables (5) on the boundary values of the Riemann coordinates:

$$\begin{pmatrix} \xi_2'(0) \\ \xi_1'(L) \end{pmatrix} = \nabla K \begin{pmatrix} \xi_1'(0) \\ \xi_2'(L) \end{pmatrix}.$$

1) An abuse of notation is done all along the article in order to make easier the reading. According to the cases, the notation "f'" can stand for the derivative of f or its gradient.
2) The following notations are used,
$$\bar{G}' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} G'$$
and in the same idea
$$\hat{K}' = K \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{23}$$
3) Let us pose F the jacobian and D the matrix of changes such that
$$F = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \quad D = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$
then
$$DF \quad = \quad \Lambda D \Leftrightarrow D = \begin{pmatrix} a & \frac{-a(\alpha - \lambda_1)}{\gamma} \\ c & \frac{-c(\alpha - \lambda_2)}{\gamma} \end{pmatrix} \tag{24}$$

Finally let us note that, using the derivative of the functions K_i of the boundary conditions, (5) can be expressed as:

$$\nabla \hat{K} \quad = \quad [Id - \mathcal{A}]^{-1} [Id + \mathcal{A}] \tag{25}$$

with
$$\mathcal{A} = \begin{pmatrix} \frac{2a}{\lambda_1}(0) & 0 \\ 0 & \frac{c(\lambda_2 - \lambda_1)}{\gamma \lambda_2}(L) \end{pmatrix} \nabla \bar{G} \begin{pmatrix} \frac{\gamma \lambda_1}{-a(\lambda_2 - \lambda_1)}(0) & 0 \\ 0 & \frac{\lambda_2}{2c}(L) \end{pmatrix}$$

under the condition that $[Id - \mathcal{A}]$ is invertible.

Proposition 1: If the components of $\nabla \bar{G}$ satisfies the following conditions

$$tr(\mathcal{A}) = -G_{21}' \frac{(\lambda_2 - \lambda_1)}{2\gamma}(L) + G_{12}' \frac{2\gamma}{(\lambda_2 - \lambda_1)}(0) < 0,$$

$G(0) = 0$ and

$$\det \mathcal{A} > 0$$

then the spectral radius of $[Id - \mathcal{A}]^{-1} [Id + \mathcal{A}]$ satisfy

$$\rho([Id - \mathcal{A}]^{-1} [Id + \mathcal{A}]) < 1$$

as it is exactly a Cayley transformation of the matrix if $\mathcal{A}$ is a closed operator [1].
Note that $\mathcal{A}$ is closed if $\nabla \bar{G}$ is invertible (not the only case). If furthermore the compatibility conditions (10)-(11) are satisfied the conditions of theorem 2.1 are satisfied and

the system is exponentially stable (see extension of this theorem given by [7]).

Proof: The proof is under review, and it is not the purpose of this article. An idea is given in [11]. Let remark that the sign of $\det \nabla G$ is the same that $\det \mathcal{A}$. ■

III. Link with the Small Gain Theorem

This notion of dissipativity is generally linked with the Small-Gain Theorem. This link has been mentioned (e.g. [7]) but not proved for Greenberg & Li's theorem (2.1) and the Riemann invariants in general.

It is proposed here to link the Hamiltonian functional expressed in Riemann coordinates with the Small Gain Theorem. Indeed, one can write:

$$d_t\tilde{H}_\xi = \begin{pmatrix}\delta_\xi H^T(L) & \delta_\xi H^T(0)\end{pmatrix}\begin{pmatrix}B(L) & 0\\ 0 & -B(0)\end{pmatrix}\begin{pmatrix}\delta_\xi H(L)\\ \delta_\xi H(0)\end{pmatrix} \quad (26)$$

$$= \frac{1}{2}\begin{pmatrix}e_2(L)+e_1(L)\\ e_2(0)-e_1(0)\\ e_2(L)-e_1(L)\\ e_2(0)+e_1(0)\end{pmatrix}^T\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}\begin{pmatrix}e_2(L)+e_1(L)\\ e_2(0)-e_1(0)\\ e_2(L)-e_1(L)\\ e_2(0)+e_1(0)\end{pmatrix} \quad (27)$$

Let define

$$y = \begin{pmatrix}e_2(L)+e_1(L)\\ e_2(0)-e_1(0)\end{pmatrix} \text{ and } u = \begin{pmatrix}e_2(L)-e_1(L)\\ e_2(0)+e_1(0)\end{pmatrix}$$

then

$$d_t\tilde{H}_\xi = \frac{1}{2}\begin{pmatrix}y^T & u^T\end{pmatrix}^T\begin{pmatrix}I_2 & 0_2\\ 0_2 & -I_2\end{pmatrix}\begin{pmatrix}y\\ u\end{pmatrix}$$

$$= \frac{1}{2}y^T y - \frac{1}{2}u^T u \quad (28)$$

$$\text{if} \quad y = Mu \quad (29)$$

$$\text{then } d_t\tilde{H}_\xi = \frac{1}{2}u^T[M^2-1]u \quad (30)$$

Proposition 2: The Small Gain Theorem allows to conclude that the system (30) is stable if $\|M\| < 1$.

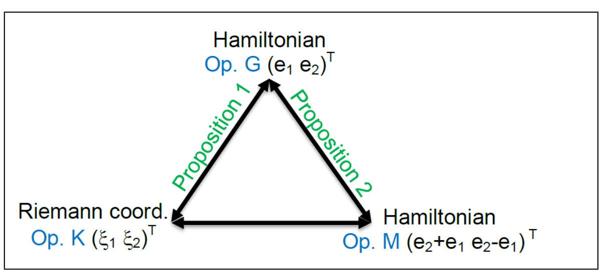

Fig. 2. Relations between Hamiltonian-Riemann-Small Gain Theorem

The main result of this paper can be stated:

Proposition 3: If the operator K established in (5) satisfies the generalized theorem (2.1) of Greenberg & Li defined in [7] then the operator M defined in (29) satisfies the condition $\|M\| < 1$ of the Small Gain Theorem.

Proof: As for the expression (25), the relation between operators M and G can be expressed as follow:

$$\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}M = \left[\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\nabla G - I_2\right]\left[I_2 + \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\nabla G\right]^{-1} \quad (31)$$

and $\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}M$ is a Cayley transformation of $\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\nabla G$.

Greenberg & Li theorem implies that the spectral radius of ∇K is less than 1. Given to the proposition 1, this implies that $det\nabla G > 0$ and $G'_{21} - G'_{12} < 0$, so

$$tr\left(\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\nabla G\right) > 0 \quad (32)$$

$$\det\left(\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\nabla G\right) = det\,(\nabla G) > 0 \quad (33)$$

By definition of the Cayley transformation, and as ∇G is invertible, the norm of $\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}M$ is less than one, and so, the norm of M is less than one, too. The condition of the Small-Gain Theorem is satisfied. Let us remark that the norm is the L^2 norm. Another norm could be chosen as the theorem proved in [7] takes the sup of all the norms. ■

IV. Conclusion

In the first part of this paper we have recalled the expression of the port Hamiltonian systems of two conservation laws and derived its expression in terms of Riemann's invariant. In these coordinates, the expression of the Dirac structure associated with this expression of the matrix differential operator, the Hamiltonian operator is derived. The expression of the boundary port variables of the Hamiltonian formulation has also been derived. In the second part of the paper, the stability conditions on the boundary values of the Riemann invariants, with some conditions on the boundary constraints on the port variables, are expressed. As a consequence we have given an interpretation of the stabilizing boundary relations in terms of the dissipation of the Hamiltonian function on the boundary of the system. The third part, the Small-Gain theorem is applied to the Hamiltionian functional. The link between the dissipativity given in Riemann's coordinates and the condition issued from the Small-Gain theorem is established in one sense.

The futurs works would to define the conditions in order to prove the inverse relation going from the Small-Gain theorem, to the dissipativity relations.

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