21st International Symposium on
Mathematical Theory of Networks and Systems
July 7-11, 2014. Groningen, The Netherlands

A Dynamic Output Feedback Controller for Practical Coordinated Tracking of Second Order Uncertain Heterogeneous Agents

Jung-Su Kim[1] and Juhoon Back[2]

Abstract— **This paper presents an output feedback controller for practical coordinated tracking of multi-agent systems. It is assumed that the agents under consideration have large parameter uncertainties, that they are affected by external disturbance, and that the agent dynamics can be different from one another. To handle the uncertainty, external disturbance, and heterogeneity effectively, a reduced-order version of disturbance observer has been employed. Compared to the recent result which relies on the conventional disturbance observer, the proposed controller has the following advantages. Firstly, the order of controller is reduced. Secondly, the stability proof as well as the design procedure becomes simpler. Simulation results are included to demonstrate that the proposed controller achieves practical coordinated tracking successfully.**

Index Terms— **Multi-agent systems, disturbance observer, output feedback control.**

I. INTRODUCTION

Due to increasing demand on multi-agent systems (MAS), e.g., multiple vehicles and robots, and satellites to name a few, coordinated control has received much attention in the literature. In particular, since synchronization and consensus unify many coordinated control problems, much research effort has been directed to them [4], [13], [15], [16], [17], [21], [18]. These results have focused mainly on reaching information agreement among agents for the case where there is no leader. Besides, those previous results are usually concerned with an identical and nominal model for the agent.

However, for practical applications, it is necessary to be able to assign a desired group behavior to the MAS, which can be formulated as a coordinated tracking problem [5], [6], [7], [11]. There can be several problem settings regarding the coordinated tracking. If the leader has no input, the problem is almost the same as the leaderless case. In addition, if the follower agents are aware of the model which is used for generating the input to the leader, the internal model principle based approaches provide controller design methods [20], [10]. Note that the input to the leader can be used for the purpose of giving the MAS a specific mission. When the follower agents do not know anything about the input to the leader, the problem becomes more difficult. Recently, an adaptive control is devised for this case in [11] but models of follower agents are identical and there is no external disturbances.

In this paper, we consider the coordinated tracking problem of MAS which is composed of second order liner systems. It is assumed that the parameters of this systems are uncertain but bounded and they can be different from one agent to another. Moreover, each agent is affected by external disturbances. These assumptions are realistic since many dynamic systems are involved is the MAS. For instance, for two vehicles with identical structure, it is much more realistic to suppose that their parameters can be different but the parameters of the agents belong to known compact sets.

The main tool to solve the problem is the disturbance observer (DOB) [8], [12], [19], which is known to be a very effective tool to deal with plant uncertainties and external disturbances. It is noted that many types of disturbance observers are available in the literature; see [14], and the one used in this paper is a reduced-order disturbance observer introduced in [3]. Since the DOB is a dynamic output feedback controller which approximately (rather than asymptotically) cancels the effect of disturbances and parameter uncertainties, the controller developed in this paper ensures practical tracking in the sense that any prescribed level of steady state error can be achieved by tuning the controller parameters.

The contribution of the paper is twofold: 1) a relative output measurement based dynamic controller for practical coordinated tracking of MAS consisting of uncertain heterogeneous second order agents is proposed when the input to the leader is unknown to the followers. 2) The controller structure, the procedure to obtain the controller parameter, and the stability proof are simpler than those of [1]. They are mainly from the fact that the order of the proposed controller is two, while that of [1] is four.

The remainder of the paper is organized as follows. In Section II, preliminaries and the problem setup are described. Section III proposes a reduced-order disturbance observer based dynamic control for multi-agent systems consisting of uncertain heterogeneous second order agents. In Section IV, the simulation results show that the proposed controller successfully results in coordinated tracking in spite of parameter uncertainties and external disturbances in the agents.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. Overview of Disturbance Observer

This section briefly introduces a disturbance observer for second order linear systems. Figure 1 describes the basic structure of a control system with a DOB. In Figure 1, $P(s)$

*This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2012R1A1A2006923)

[1]Jung-Su Kim is with the Department of EIE, Seoul National University of Science and Technology, Seoul 139-743, Korea. jungsu@seoultech.ac.kr

[2]Juhoon Back is with the School of Robotics, Kwangwoon University, Seoul 139-701, Korea. backhoon@kw.ac.kr

Corresponding author: Juhoon Back

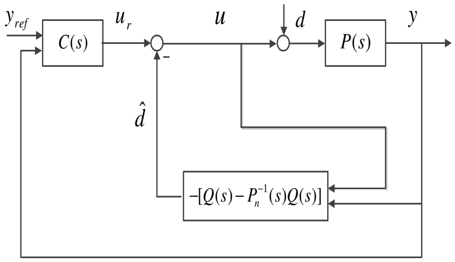

Fig. 1. Reduced-order disturbance observer based control system..

represents an uncertain plant and $C(s)$ denotes the controller designed for the nominal model $P_n(s)$ given by

$$P(s): \begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \phi_1 x_1 + \phi_2 x_2 + g(u+d) \\ \ \ y = x_1, \end{cases}$$

$$P_n(s): \begin{cases} \dot{\bar{x}}_1 = \bar{x}_2, \\ \dot{\bar{x}}_2 = \bar{\phi}_1 \bar{x}_1 + \bar{\phi}_2 \bar{x}_2 + \bar{g} u_r \\ \ \ \bar{y} = \bar{x}_1 \end{cases}$$

where ϕ_1, ϕ_2 are uncertain parameters in the plant while $\bar{\phi}_1, \bar{\phi}_2$ are nominal values of ϕ_1 and ϕ_2, respectively. y_r denotes the reference, u_r the control input generated by $C(s)$, u the control input, d the external disturbance, y the plant output, and $\hat{d}$ the estimate of the disturbance. The component $Q(s)$ known as Q-filter is designed such that the DOB estimates the disturbance as well as the uncertainties in the system and compensates its effect on the closed-loop system on top of the controller output u_r designed for the nominal model $P_n(s)$. The Q-filter used in this paper has the following form

$$Q(s) = \frac{\alpha_0}{(\tau s)^2 + \alpha_1(\tau s) + \alpha_0},$$

where α_0, α_1, and τ are positive design parameters. From Fig. 1, we have

$$\begin{aligned} \hat{d}(s) &= -Q(s)u(s) + Q(s)P_n^{-1}(s)y(s) \\ &= -[Q(s) \quad -Q(s)P_n^{-1}(s)] \begin{bmatrix} u(s) \\ y(s) \end{bmatrix}, \end{aligned}$$

from which one has

$$u(s) = \frac{1}{1-Q(s)}u_r(s) - \frac{1}{1-Q(s)}Q(s)P_n^{-1}(s)y(s).$$

Based on this relation, a reduced-order state space realization of DOB is introduced in [3], which is shown below.

$$\begin{aligned} \dot{q}_1 &= -\frac{\alpha_1}{\tau}q_1 + q_2 + \frac{\alpha_0}{\tau^2 \bar{g}}\left(\bar{\phi}_2 + \frac{\alpha_1}{\tau}\right)y \\ \dot{q}_2 &= \frac{\alpha_0}{\tau^2}u_r + \frac{\alpha_0 \bar{\phi}_1}{\tau^2 \bar{g}}y \\ u &= q_1 + u_r - \frac{\alpha_0}{\tau^2 \bar{g}}y. \end{aligned}$$

It is noted that the inner-loop consisting of the uncertain plant $P(s)$, the Q-filter, and the inverse of the nominal model behaves like the nominal model $P_n(s)$ in the low frequency region in terms of the input-output relation. Therefore, the DOB can compensate not only the external disturbances d but also the model uncertainty in the plant in order to enhance robustness of the existing controller $C(s)$ designed for the nominal model. The systematic DOB design methods in [3], [2] are adopted for the coordinated tracking controller in this paper.

B. Problem Formulation

The MAS under consideration consists of N uncertain second order linear systems given by

$$\begin{aligned} \dot{x}_1^i &= x_2^i \\ \dot{x}_2^i &= \phi_1^i x_1^i + \phi_2^i x_2^i + g^i(u^i + d^i) \\ y^i &= x_1^i \end{aligned} \tag{1}$$

where i, $i = 1, \ldots, N$, denotes the agent index, $x^i = [x_1^i \ x_2^i]^T \in \mathbb{R}^2$ the state vector, $u^i \in \mathbb{R}$ the control input, $y^i \in \mathbb{R}$ the output, and $d^i \in \mathbb{R}$ the disturbance entering into the ith agent. The constants ϕ_1^i, ϕ_2^i, and g^i represent system parameters which are uncertain and maybe different from one another. In what follows the agents with index $1, \ldots, N$ are called followers.

For the uncertain parameters and disturbances in this MAS, the following assumption is made.

Assumption 1: The system parameters ϕ_1^i, ϕ_2^i, g^i belong to known compact sets. Precisely, $\phi_j^i \in [\phi_j^-, \phi_j^+]$ where $\phi_j^- \le \phi_j^+$, $j = 1, 2$, and $g^i \in [g^-, g^+]$ where $0 < g^- \le g^+$. The disturbance is continuously differentiable and $d^i(t)$ as well as $\dot{d}^i(t)$ is uniformly bounded with a known bound. $\diamond$

We assume that our MAS contains a leader, with index $N+1$, given by

$$\begin{aligned} \dot{\bar{x}}_1 &= \bar{x}_2 \\ \dot{\bar{x}}_2 &= \bar{\phi}_1 \bar{x}_1 + \bar{\phi}_2 \bar{x}_2 + \bar{g}\bar{u} \\ \bar{y} &= \bar{x}_1. \end{aligned} \tag{2}$$

where $\bar{x} = [\bar{x}_1 \ \bar{x}_2]^T \in \mathbb{R}^2$ represents the state of the leader, $\bar{u} \in \mathbb{R}$ the input, $\bar{y} \in \mathbb{R}$ the output, $\bar{\phi}_1$ and $\bar{\phi}_2$ negative constants, and $g^- \le \bar{g} \le g^+$. The parameters $\bar{\phi}_1$, $\bar{\phi}_2$, and $\bar{g}$ are assumed to be known and they are used when we design the controller.

For the leader, it is assumed that $\bar{u}(t)$ is continuously differentiable, and that $\bar{u}(t)$ as well as $\dot{\bar{u}}(t)$ is uniformly bounded. Note that the trajectories of (2) are bounded since the system is asymptotically stable and $\bar{u}(t)$ is uniformly bounded. We assume that only a limited number (but at least one) of followers exchange information (relative output in this paper) with the leader, while no followers can access the input $\bar{u}$ applied to the leader.

In order to consider the interaction among $N+1$ agents, a weighted directed graph $\mathcal{G}$ is used.[1] Besides, we consider a subgraph of $\mathcal{G}$, denoted by $\mathcal{G}^F$, which represents the interaction among followers. The weighted adjacency matrix and its Laplacian associated with $\mathcal{G}$ are denoted by $\mathcal{A} \in \mathbb{R}^{(N+1)\times(N+1)}$ and $L \in \mathbb{R}^{(N+1)\times(N+1)}$, respectively. It is

[1]In this paper, standard notions of graph theory are used. See, e.g., [15] for more details.

assumed that the leader does not get any information from followers and that at least one follower gets information from the leader, which is stated in the following assumption.

Assumption 2: The graph $\mathcal{G}$ is fixed and contains a directed spanning tree, and the graph $\mathcal{G}^F$ is undirected and connected. ◇

This assumption implies that the leader gives information to at least one follower and thus the last column of $\mathcal{A}$ is not zero. Moreover, it ensures that L has one simple zero eigenvalue with associated eigenvector 1_{N+1}. As a result, it follows that L is of the form

$$L = \begin{bmatrix} L^\star & L_{12} \\ 0 & 0 \end{bmatrix}, \; L^\star \in \mathbb{R}^{N\times N}, L_{12} \in \mathbb{R}^N.$$

Note that $L^\star$ is a symmetric positive definite matrix.

The ith agent receives the weighted sum of relative measurements $y^j - y^i$, $j \in \mathcal{N}_i$, where $\mathcal{N}_i$ represents the set of the neighbors of agent i, namely

$$z^i = \sum_{j=1}^{N+1} a_{ij}(y^j - y^i). \tag{3}$$

With this problem setup in mind, this paper aims at designing a dynamic controller, parametrized by τ, of the form

$$\begin{aligned} \dot{q}^i &= F_\tau(q^i, z^i), \; q^i \in \mathbb{R}^2 \\ u^i &= H_\tau(q^i, z^i). \end{aligned} \tag{4}$$

Note that the order of the controller is the same as that of the agent.

We say that the practical coordinated tracking problem is solved if for any given $\epsilon > 0$ there exists $\tau^* > 0$ such that the controller guarantees that the states are bounded for all $t \geq 0$ and that for any $0 < \tau < \tau^*$, it holds that

$$\limsup_{t\to\infty} \|x^i(t) - \bar{x}(t)\| \leq \epsilon, i = 1, \dots, N. \tag{5}$$

III. Main Result

Based on the reduced-order disturbance observer introduced in [3], the proposed controller is given by

$$\begin{aligned} \dot{q}_1^i &= -\frac{\alpha_1}{\tau} q_1^i + q_2^i - \frac{\alpha_0}{\tau^2 \bar{g}} \left(\bar{\phi}_2 + \frac{\alpha_1}{\tau} \right) z^i \\ \dot{q}_2^i &= -\frac{\alpha_0 \bar{\phi}_1}{\tau^2 \bar{g}} z^i \\ u^i &= q_1^i + \frac{\alpha_0}{\tau^2 \bar{g}} z^i \end{aligned} \tag{6}$$

where $q^i \in \mathbb{R}^n$ is the state of the controller for the ith agent and $\alpha_0 > 0, \; \alpha_1 > 0$, and $\tau > 0$ are design parameters.

In what follows, we investigate the stability of the closed-loop system involving the controller (6). We start by defining new coordinates $\tilde{x}_1^i, \tilde{x}_2^i$ which represent the error between the followers and the leader, and η_1^i, η_2^i representing the output of the controller (6) and its time derivative. After rewriting the system in new coordinates, we show that the states of the followers will converge to the vicinity of the leader's trajectory and stay there. Moreover, it is shown that the parameters of the controller can be tuned so that the steady state error is maintained within a prescribed bound which can be arbitrarily small.

Firstly, define the tracking error of the ith follower $\tilde{x}_1^i = x_1^i - \bar{x}_1$, $\tilde{x}_2^i = x_2^i - \bar{x}_2$, and compute their dynamics as

$$\dot{\tilde{x}}_1^i = \tilde{x}_2^i, \tag{7a}$$
$$\dot{\tilde{x}}_2^i = \phi_1^i \tilde{x}_1^i + \phi_2^i \tilde{x}_2^i + g^i u^i + w^i \tag{7b}$$

where $i = 1, \dots, N$ and

$$\begin{aligned} \tilde{\phi}_1^i &= \phi_1^i - \bar{\phi}_1, \; \tilde{\phi}_2^i = \phi_2^i - \bar{\phi}_2, \\ w^i &= \tilde{\phi}_1^i \bar{x}_1 + \tilde{\phi}_2^i \bar{x}_2 + g^i d^i - \bar{g}\bar{u}. \end{aligned}$$

Now we consider the coordinate transform

$$\begin{aligned} \eta_1^i &= q_1^i + \frac{\alpha_0}{\tau^2 \bar{g}} z^i \\ \eta_2^i &= \tau \left(\dot{q}_1^i + \frac{\alpha_0}{\tau^2 \bar{g}} \dot{z}^i \right). \end{aligned} \tag{8}$$

Note that the control input of the ith agent is written as

$$u^i = \eta_1^i. \tag{9}$$

Lemma 1: The dynamics of η_j^i is obtained as

$$\dot{\eta}_1^i = \frac{1}{\tau} \eta_2^i \tag{10a}$$
$$\dot{\eta}_2^i = -\frac{\alpha_1}{\tau} \eta_2^i - \frac{\alpha_0}{\tau} \frac{1}{\bar{g}} \left(\bar{\phi}_1 z^i + \bar{\phi}_2 \dot{z}^i - \ddot{z}^i \right). \tag{10b}$$

□

Proof. The relation (10a) follows from the definition (8). In order to derive (10b), we compute $\ddot{q}_1^i$ first. From (6), we have

$$\begin{aligned} \ddot{q}_1^i &= -\frac{\alpha_1}{\tau} \dot{q}_1^i + \dot{q}_2^i - \frac{\alpha_0}{\tau^2 \bar{g}} \left(\bar{\phi}_2 + \frac{\alpha_1}{\tau} \right) \dot{z}^i \\ &= -\frac{\alpha_1}{\tau} \left(\dot{q}_1^i + \frac{\alpha_0}{\tau^2 \bar{g}} \dot{z}^i \right) + \dot{q}_2^i - \frac{\alpha_0}{\tau^2 \bar{g}} \bar{\phi}_2 \dot{z}^i \\ &= -\frac{\alpha_1}{\tau^2} \eta_2^i - \frac{\alpha_0}{\tau^2} \frac{1}{\bar{g}} \left(\phi_1 z^i + \phi_2 \dot{z}^i \right). \end{aligned}$$

Using this result, the relation (10b) is derived from

$$\begin{aligned} \dot{\eta}_2^i &= \tau \left(\ddot{q}_1^i + \frac{\alpha_0}{\tau^2 \bar{g}} \ddot{z}^i \right) \\ &= -\frac{\alpha_1}{\tau} \eta_2^i - \frac{\alpha_0}{\tau} \frac{1}{\bar{g}} \left(\phi_1 z^i + \phi_2 \dot{z}^i \right) + \frac{\alpha_0}{\tau \bar{g}} \ddot{z}^i. \end{aligned}$$

This completes the proof. □

Before proceeding, we define

$$\begin{aligned} &\tilde{x}_1 = \begin{bmatrix} \tilde{x}_1^1 & \cdots & \tilde{x}_1^N \end{bmatrix}^T, \; \tilde{x}_2 = \begin{bmatrix} \tilde{x}_2^1 & \cdots & \tilde{x}_2^N \end{bmatrix}^T \\ &\eta_1 = \begin{bmatrix} \eta_1^1 & \cdots & \eta_1^N \end{bmatrix}^T, \; \eta_2 = \begin{bmatrix} \eta_2^1 & \cdots & \eta_2^N \end{bmatrix}^T \\ &\Phi_1 = \text{diag}\left\{ \phi_1^1, \dots, \phi_1^N \right\}, \; \Phi_2 = \text{diag}\left\{ \phi_2^1, \dots, \phi_2^N \right\} \\ &\tilde{\Phi}_1 = \text{diag}\left\{ \tilde{\phi}_1^1, \dots, \tilde{\phi}_1^N \right\}, \; \tilde{\Phi}_2 = \text{diag}\left\{ \tilde{\phi}_2^1, \dots, \tilde{\phi}_2^N \right\} \\ &G = \text{diag}\left\{ g^1, \dots, g^N \right\}. \end{aligned}$$

Similarly, we define u, d, w, and z by N dimensional vectors by stacking N components related to the followers. It is noted that z^i, $\dot{z}^i$, z, and $\dot{z}$ can be written as

$$\begin{aligned} z^i &= -L_i^\star \tilde{x}_1, \quad \dot{z}^i = -L_i^\star \tilde{x}_2 \\ z &= -L^\star \tilde{x}_1, \quad \dot{z} = -L^\star \tilde{x}_2. \end{aligned} \tag{11}$$

where $L_i^\star$ denotes the ith row of $L^\star$.

With these variables and the relation (11), we compactly rewrite the whole closed-loop system as

$$\begin{aligned}\dot{\tilde{x}}_1 &= \tilde{x}_2\\ \dot{\tilde{x}}_2 &= \Phi_1\tilde{x}_1 + \Phi_2\tilde{x}_2 + G\eta_1 + w\\ \dot{\eta}_1 &= \frac{1}{\tau}\eta_2\\ \dot{\eta}_2 &= -\frac{\alpha_1}{\tau}\eta_2 - \frac{\alpha_0}{\tau}\frac{1}{\bar{g}}L^\star\left(\tilde{\Phi}_1\tilde{x}_1 + \tilde{\Phi}_2\tilde{x}_2 + G\eta_1 + w\right).\end{aligned} \tag{12}$$

Now, the main result of the paper is presented.

Theorem 1: Suppose that Assumptions 1 and 2 hold true and let α_0, α_1 be any positive numbers. Then, the controller (6) parameterized by τ solves the practical coordinated tracking problem. Precisely, for a given $\epsilon > 0$, there exists $\tau^* > 0$ such that for any $\tau \in (0, \tau^*)$, the controller (6) parameterized by τ ensures that $\limsup_{t\to\infty}\|x^i(t) - \bar{x}(t)\| \leq \epsilon, i = 1, \ldots, N$. □

Proof. In order to analyze the closed-loop dynamics (12), we first note that it exhibits a two-time scale behavior [9] for sufficiently small τ; the variables η_1, η_2 will rapidly converge to their quasi-steady states parameterized by $\tilde{x}_1, \tilde{x}_2$. Based on this observation, we compute the (quasi) equilibrium points of η_1, η_2, which are denoted by η_1^*, η_2^*, respectively, regarding other variables as fixed parameters. Simple computation yields

$$\eta_1^* = -G^{-1}\left(\tilde{\Phi}_1\tilde{x}_1 + \tilde{\Phi}_2\tilde{x}_2 + w\right), \quad \eta_2^* = 0. \tag{13}$$

We define $\tilde{\eta}_1 = \eta_1 - \eta_1^*$, $\tilde{\eta}_2 = \eta_2 - \eta_2^*$ and rewrite the dynamics of the closed-loop system in the coordinates $(\tilde{x}_1, \tilde{x}_2, \tilde{\eta}_1, \tilde{\eta}_2)$ as follows.

$$\begin{aligned}\dot{\tilde{x}}_1 &= \tilde{x}_2\\ \dot{\tilde{x}}_2 &= \bar{\phi}_1\tilde{x}_1 + \bar{\phi}_2\tilde{x}_2 + G\tilde{\eta}_1\\ \dot{\tilde{\eta}}_1 &= \tilde{\Phi}_2\tilde{\eta}_1 + \frac{1}{\tau}\tilde{\eta}_2\\ &\quad + G^{-1}\tilde{\Phi}_2\bar{\phi}_1\tilde{x}_1 + G^{-1}\left(\tilde{\Phi}_1 + \tilde{\Phi}_2\bar{\phi}_2\right)\tilde{x}_2 + G^{-1}\dot{w}\\ \dot{\tilde{\eta}}_2 &= -\frac{\alpha_0}{\tau}\frac{1}{\bar{g}}L^\star G\tilde{\eta}_1 - \frac{\alpha_1}{\tau}\tilde{\eta}_2.\end{aligned} \tag{14}$$

The dynamics (14) is compactly rewritten as

$$\begin{bmatrix}\dot{\tilde{x}}\\ \dot{\tilde{\eta}}\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & \frac{1}{\tau}A_{22} + \tilde{A}_{22}\end{bmatrix}\begin{bmatrix}\tilde{x}\\ \tilde{\eta}\end{bmatrix} + \begin{bmatrix}0\\ B_2\end{bmatrix}G^{-1}\dot{w} \tag{15}$$

where $\tilde{x} := [\tilde{x}_1^T\ \tilde{x}_2^T]^T,\ \tilde{\eta} := [\tilde{\eta}_1^T; \tilde{\eta}_2^T]^T$,

$$A_{11} = \begin{bmatrix}0 & I_N\\ \bar{\phi}_1 I_N & \bar{\phi}_2 I_N\end{bmatrix},\ A_{12} = \begin{bmatrix}0 & 0\\ G & 0\end{bmatrix},\ B_2 = \begin{bmatrix}I_N\\ 0\end{bmatrix}$$

$$A_{21} = \begin{bmatrix}\bar{\phi}_1 G^{-1}\tilde{\Phi}_2 & G^{-1}\left(\tilde{\Phi}_1 + \bar{\phi}_2\tilde{\Phi}_2\right)\\ 0 & 0\end{bmatrix}$$

$$A_{22} = \begin{bmatrix}0 & I_N\\ -\frac{\alpha_0}{\bar{g}}L^\star G & -\alpha_1 I_N\end{bmatrix},\ \tilde{A}_{22} = \begin{bmatrix}\tilde{\Phi}_2 & 0\\ 0 & 0\end{bmatrix}.$$

The matrix A_{11} is Hurwitz since $\bar{\phi}_1$ and $\bar{\phi}_2$ are negative. Furthermore, it can be proved that the matrix A_{22} is also Hurwitz. To show the claim, we first note that A_{22} is similar to

$$\bar{A}_{22} = \begin{bmatrix}0 & I_N\\ -\frac{\alpha_0}{\bar{g}}G^{\frac{1}{2}}L^\star G^{\frac{1}{2}} & -\alpha_1 I_N\end{bmatrix},$$

where $G^{\frac{1}{2}} = \text{diag}\{\sqrt{g^1}, \ldots, \sqrt{g^N}\}$. The claim is proved since the eigenvalues of $\bar{A}_{22}$ are computed from $s^2 + \alpha_1 s + \frac{\alpha_0}{\bar{g}}\sigma_i = 0$ where σ_i denotes the ith eigenvalue of $G^{\frac{1}{2}}L^\star G^{\frac{1}{2}}$ and $\sigma_i > 0$.

Now, we take the Lyapunov function candidate

$$V = \tilde{x}^T P_1\tilde{x} + \tilde{\eta}^T P_2\tilde{\eta}.$$

where P_1 and P_2 are obtained from

$$\begin{aligned}P_1 A_{11} + A_{11}^T P_1 &= -I\\ P_2 A_{22} + A_{22}^T P_2 &= -I.\end{aligned}$$

Recall that P_1 and P_2 are symmetric positive definite since A_{11} and A_{22} are Hurwitz. Let $k_1 > 0$ and $k_2 > 0$ be constants, independent of uncertain parameters, such that

$$k_1(\tilde{x}^T\tilde{x} + \tilde{\eta}^T\tilde{\eta}) \leq V \leq k_2(\tilde{x}^T\tilde{x} + \tilde{\eta}^T\tilde{\eta}).$$

Meanwhile, Assumption 1 guarantees that there exists a positive constant $\dot{w}^+$ such that $\|\dot{w}\| \leq \dot{w}^+$.

We compute

$$\begin{aligned}\dot{V} &= -\tilde{x}^T\tilde{x} - \frac{1}{\tau}\tilde{\eta}^T\tilde{\eta} + 2\tilde{x}^T P_1 A_{12}\tilde{\eta}\\ &\quad + 2\tilde{x}^T A_{21}^T P_2\tilde{\eta} + 2\tilde{\eta}^T P_2\tilde{A}_{22}\tilde{\eta} + 2\tilde{\eta}^T P_2 B_2 G^{-1}\dot{w}\\ &=: -\tilde{x}^T\tilde{x} - \frac{1}{\tau}\tilde{\eta}^T\tilde{\eta} + \tilde{x}^T\tilde{A}_1\tilde{\eta} + \tilde{\eta}^T\tilde{A}_2\tilde{\eta} + 2\tilde{\eta}^T P_2 B_2 G^{-1}\dot{w}\\ &\leq -\frac{1}{2}\tilde{x}^T\tilde{x} - \frac{1}{\tau}\tilde{\eta}^T\tilde{\eta} + \tilde{\eta}^T M(\mu^*)\tilde{\eta} + \mu^*(\dot{w}^+)^2\end{aligned}$$

where $\tilde{A}_2 = P_2\tilde{A}_{22} + \tilde{A}_{22}^T P_2$, μ^* is an arbitrary positive constant, and

$$M(\mu^*) := \frac{1}{2}\tilde{A}_1^T\tilde{A}_1 + \tilde{A}_2 + \frac{1}{\mu^*}P_2 B_2 G^{-1}G^{-1}B_2^T P_2.$$

Note that $M(\mu^*)$ depends on uncertain parameters. From the fact that all parameters belong to known compact sets, we can find $m(\mu^*)$ such that $M(\mu^*) \leq m(\mu^*)I$ for all uncertain parameters. We take $\tau^* > 0$ such that $-\frac{1}{\tau^*} + m(\mu^*) \leq -\frac{1}{2}$. Then, it follows for $0 < \tau < \tau^*$ that

$$\begin{aligned}\dot{V} &\leq -\frac{1}{2}\tilde{x}^T\tilde{x} - \frac{1}{2}\tilde{\eta}^T\tilde{\eta} + \mu^*(\dot{w}^+)^2\\ &\leq -\frac{1}{2k_2}V + \mu^*(\dot{w}^+)^2.\end{aligned}$$

Using the comparison lemma, we obtain

$$V(t) \leq e^{-\frac{1}{2k_2}t}V(0) + 2\mu^* k_2(\dot{w}^+)^2$$

which implies

$$\limsup_{t\to\infty} V(t) \leq 2\mu^* k_2(\dot{w}^+)^2.$$

Since μ^* is an arbitrary constant, it follows for a given ϵ that there exists τ^* such that for $0 < \tau < \tau^*$,

$$\limsup_{t\to\infty}\|x^i(t) - \bar{x}(t)\| \leq \epsilon$$

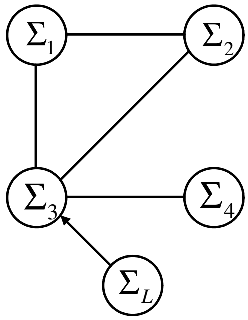

Fig. 2. Network topology among agents.

TABLE I
SYSTEM PARAMETERS AND DISTURBANCE INPUTS.

Agent #	ϕ_1^i	ϕ_2^i	g^i	d^i
1	-5.0	1.3	1.6	$0.3\sin 5t$
2	-1.3	-1.2	1.3	$-0.4\sin 10t$
3	0.8	-1.1	1.4	$-0.17\sin 15t$
4	-1.3	-1.8	2.0	$0.35\sin 20t$

which completes the proof. □

Remark 1: It is noted that the same problem has been studied in [1]. The main difference between the current work and [1] is that the controller of the current work is based on the reduced-order disturbance observer, while a disturbance observer with two Q-filters are used in [1]. The use of simpler controller results in not only a simpler stability analysis, but also a simpler computation to determine τ. These can be seen from the fact that A_{22} in (15) is the only component that is multiplied by $\frac{1}{\tau}$ while other components are constant matrices independent of τ. Note that the controller given in [1] has three components which are multiplied by $\frac{1}{\tau}$ and contains some components which are polynomials of τ, which will make the computation more complicated.

IV. SIMULATION

This section presents simulation results of the proposed controller. For that, interconnection topology depicted in Fig. 2 is used. We consider the case where only agent 3 gets information from the leader. The corresponding Laplacian matrix is given by

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 1 & -1 & 4 & -1 & -1 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Table I shows the parameters of the follower agents and the disturbances acting on them. Note that follower agents 1 and 3 have unstable open loop dynamics. The parameters for the leader (2) are

$$\bar{\phi}_1 = -1.5,\ \bar{\phi}_2 = -2.5,\ \bar{g} = 1,$$

and the input for the leader is $\bar{u}(t) = 2\sin t$. The parameters of the controller (6) are chosen as $\alpha_0 = 2,\ \alpha_1 = 1$ and $\tau = 0.001$. In order to show that the performance of the controller is independent of the initial conditions of the follower agents, the initial conditions are randomly selected, while that of the leader is set to zero. As shown in Figs 3 and 4, the practical

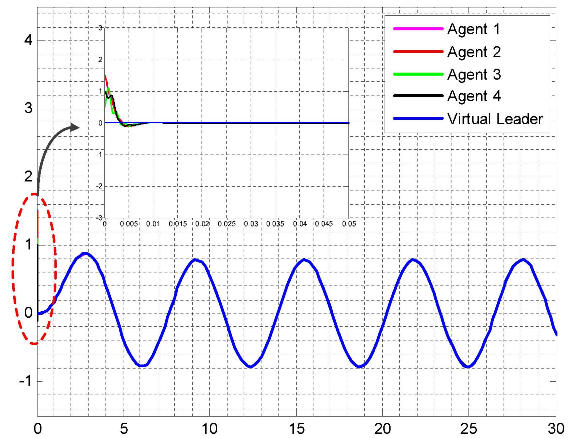

Fig. 3. Response of x_1^i using the controller (6).

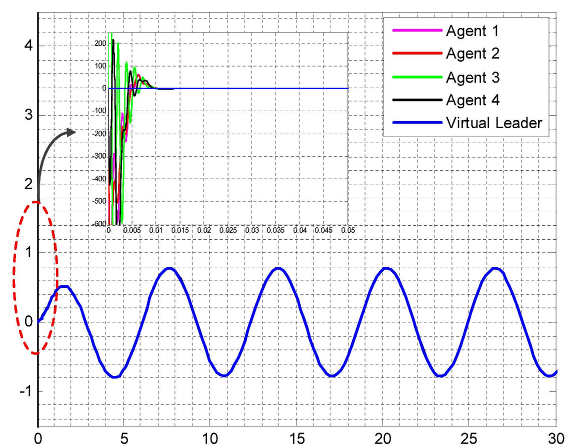

Fig. 4. Response of x_2^i using the controller (6).

tracking is achieved by the proposed controller as time goes by. However, as shown in the sub-panels in Figs 3 and 4, big overshoots occur in the transient response since the controller state q^i exhibits a peak as τ gets smaller, which is known as a peaking phenomenon [9]. Since such peaking phenomena can deteriorate system performance and even can destabilize the system, it is necessary to cope with the problem properly. As suggested in [9], the malicious effect of the peaking can be suppressed using the saturation function in the controller output. That is to say, we use $u^i = \text{sat}_\sigma\left(q_1^i + \frac{\alpha_0}{\tau^2}(\frac{1}{\bar{g}}z^i)\right)$ where the function sat_σ, with $\sigma > 0$, is defined by $\text{sat}_\sigma(x) = x$ if $|x| \le \sigma$ and $\text{sat}_\sigma(x) = \frac{x}{|x|}\sigma$ if if $|x| > \sigma$. The saturation level σ is selected as 15 in this simulation. Figs. 5 and 6 demonstrate that not only the proposed controller solves the practical coordinated tracking problem but also the saturation function in the controller output successfully restrains the peaking.

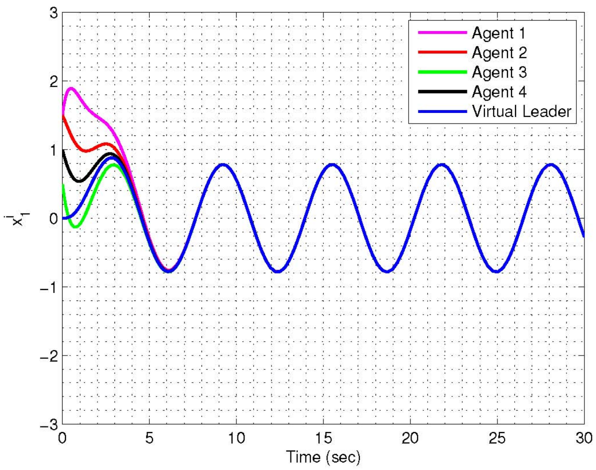

Fig. 5. Response of x_1^i using the controller (6) with saturation.

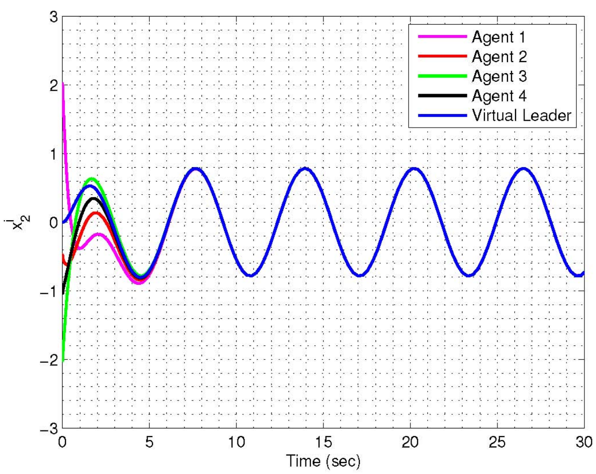

Fig. 6. Response of x_2^i using the controller (6) with saturation.

V. CONCLUSIONS

This paper presented a dynamic controller for practical coordinated tracking of multi-agent systems comprised of uncertain heterogeneous second-order agents on the basis of reduced-order disturbance observer theory. Since the controller is based on the relative measurements, the amount of information that agents will exchange will not increase even if the order of agent dynamics increases. Future research topics include extensions to the cases where the network is switching and nonlinear agents are involved.

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