21st International Symposium on
Mathematical Theory of Networks and Systems
July 7-11, 2014. Groningen, The Netherlands

A Model-Free Approach to Wind Farm Control Using Simultaneous Perturbation Stochastic Approximation

Mohd Ashraf Ahmad, Shun-ichi Azuma and Toshiharu Sugie

Abstract— **This paper addresses an initial study of a model-free approach based on Simultaneous Perturbation Stochastic Approximation (SPSA) for wind farm control. The SPSA based method is used to optimize the control action of each turbine such that the overall power production of wind farm is maximized. In order to validate our model-free design, a wind farm model with dynamic characterization of wake interaction amongst turbines as presented in [4] is considered. For simplicity, the proposed method is tested on a single row wind farm. Simulation results illustrate that the SPSA based method achieves a maximum total power production with faster convergence as compared to the other existing model-free methods.**

I. INTRODUCTION

Nowadays, studies on improving the control algorithms to maximize energy production of an existing wind farms have attracted many researchers from the control community. However, it is a very challenging problem due to the aerodynamic interactions between turbines, which are complex and difficult to model. Therefore, a model-free approach has become a more reasonable solution recently.

In the class of induction control of wind farms, several model-free approaches have been investigated. In [1],[2] and [3], a game theory and cooperative control based approaches have been proposed for optimizing power production in wind farms. Their distributed learning algorithms have been applied for both local and global knowledge available to each turbine in a wind farm configuration. However, their approaches are validated to a static model of the wind farm. Therefore, the time efficiency of their learning algorithms is not guaranteed. As an alternative solution, a dynamic model of the wind farm has been proposed in [4]. Here, they adopt a delay structure when a wake travels from one turbine to the next. They have used a model-free approach based on Fixed-Step Maximum Power Point Tracking (FS-MPPT) method, which exploits the local knowledge available to each turbine. Their results show a much faster convergence with acceptable degradation in the maximum power production as compared to the game theoretic (GT) based approach in [1].

In most of the model-free approach, the on-line optimization or data-driven is more desirable than the off-line case since it reflects the current system immediately. Specifically for the wind farm control, the algorithm can capture and adapts the environment variations, e.g. change in wind direction and wind speed, wind turbine failure or communication disruption if the on-line data is used. Therefore, the wind farm model in [4] with the dynamic characterization of the wake interactions is the most favorable model framework for on-line model-free approach.

In this paper, we explore the applicability of the Simultaneous Perturbation Stochastic Approximation (SPSA) [5] as an on-line model-free approach and implement to the wind farm model in [4]. As an initial study, a single row wind farm is considered. Then, the performance of the proposed method is analyzed in terms of the convergence speed and the maximum total power production of wind farm. Finally, a comparative assessment between the SPSA based method and the FS-MPPT [4] and the GT based methods [1] is presented.

The rest of the paper is organized as follows. Section 2 provides a description of the wind farm model and the problem formulation. In Section 3, the benefit and methodology of the simultaneous perturbation stochastic approximation based algorithms are discussed. The SPSA based method is then tested to a single row wind farm in Section 4. The analysis and performance comparison between the SPSA based method and the other existing methods is also presented in this section. Finally, some concluding remarks are given in Section 5.

Notation: The symbols $\mathbb{R}$ and $\mathbb{R}_+$ represent the set of real numbers and the set of positive real numbers, respectively. The cardinality of a set $\mathcal{S}$ is denoted by $|\mathcal{S}|$. For the random variable V, the probability of event $V = a$ is represented by $\mathbb{P}(V = a)$.

II. WIND FARM MODEL

The wind farm to be considered consists of p wind turbines denoted by the set $\mathcal{X} = \{1, 2, ..., p\}$. Let $\mathcal{F} \subset \mathcal{X}$ be the set of turbines that are directly influencing the downstream turbines through wake interaction, and let $\mathcal{N}(i)$ be the index of the neighbor downstream turbine of turbine $i \in \mathcal{F}$ where it is directly influenced by turbine i. Next, let $\mathcal{L} = \{i \in \mathcal{X} \mid i \notin \mathcal{F}\}$ be the set of turbines that does not influence other turbines.

The wind farm model used in this paper is based on the well-known Park model studied in several literatures [6],[7]. This model presents a characterization of wake by estimating the velocity profile of a single turbine. In order to evaluate the time efficiency of our model-free approach, our wind

Mohd Ashraf Ahmad is a PhD student of the Department of Systems Science, Kyoto University, Uji, Kyoto 611-0011, Japan, ahmad@robot.kuass.kyoto-u.ac.jp
Shun-ichi Azuma is an Associate Professor in the Department of Systems Science, Kyoto University, Uji, Kyoto 611-0011, Japan, sazuma@i.kyoto-u.ac.jp
Toshiharu Sugie is a Professor in the Department of Systems Science, Kyoto University, Uji, Kyoto 611-0011, Japan, sugie@i.kyoto-u.ac.jp

ISBN: 978-90-367-6321-9

farm model also considers a delay model, which is firstly introduced in [4]. This delay structure is a dynamic version of the Park model and it is practical to explore the model-free approach in a real-time environment.

Let a_i $(i = 1, 2, ..., p)$, D_i $(i = 1, 2, ..., p)$ and $V_i(x, r, a_i)$ be the axial induction factor, the rotor diameter, velocity profile of each turbine, respectively, where x is the distance to the rotor disk plane of the turbine, and r is the distance to the centerline of the rotor axis. Then, the velocity profile of the Park model can be represented as

$$V_i(x, r, a_i) = V_\omega(1 - \delta V_i(x, r, a_i)) \tag{1}$$

where the fractional velocity deficit $\delta V_i(x, r, a_i)$ is given by

$$\delta V_i(x, r, a_i) = \begin{cases} 2a_i \left(\frac{D_i}{D_i + 2\phi x}\right)^2 & \text{if } r \leq \frac{D_i + 2\phi x}{2} \\ 0 & \text{if } r > \frac{D_i + 2\phi x}{2} \end{cases} \tag{2}$$

where ϕ is a roughness coefficient, which represents the slope of wake expansion. Next, the wake model is extended to include multiples turbine. In order to illustrate the interaction of wake model for multiple turbines, the aggregate wind velocity $\overline{V}_j$ for $i \in \mathcal{X}$ is introduced, where it is evaluated based on the aggregation of the wind velocity deficit created by each upstream turbine. The aggregate wind velocity is given by

$$\overline{V}_j = V_\omega(1 - \delta\overline{V}_j) \tag{3}$$

for

$$\delta\overline{V}_j = 2\sqrt{\sum_{i \in \mathcal{X}: x_i < x_j} \left(a_i \left(\frac{D_i}{D_i + 2\phi(x_j - x_i)}\right)^2 \frac{A^{\text{ov}}_{i\to j}}{A_j}\right)^2} \tag{4}$$

where A_j is the rotor swept area of turbine j, $A^{\text{ov}}_{i\to j}$ is the overlap area between the wake generated by an upstream turbine i and rotor swept area of turbine j. The phenomena of wake interaction between the two turbines is illustrated in Fig. 1. Further, the power of each turbine can be represented as

$$P_j = \frac{1}{2}\rho A_j C_P(a_j)\overline{V}_j^3 \tag{5}$$

where ρ is the air density and $C_P(a_j)$ is the power efficiency coefficient which is given by

$$C_P(a_j) = 4a_j(1 - a_j)^2. \tag{6}$$

Next, the dynamic of the wake interaction is illustrated based on the estimation of the wake travel time from one turbine to another turbine as studied in [4]. Therefore, the time interval for the wake to travel to the whole wind farm can be approximated as

$$T_w \approx \max_{i \in \mathcal{X}} \left(\sum_{j \in \mathcal{R}} \frac{x_{\mathcal{N}(j)} - x_j}{\frac{1}{2}(\overline{V}_j(1 - 2a_j) + \overline{V}_{\mathcal{N}(j)})}\right) \tag{7}$$

where $\mathcal{R} = \{i, \mathcal{N}(i), \mathcal{N}(\mathcal{N}(i)), ...\}$ is a set that includes turbine i and the other downstream turbines in a row with maximum number of turbines that are affected by turbine i.

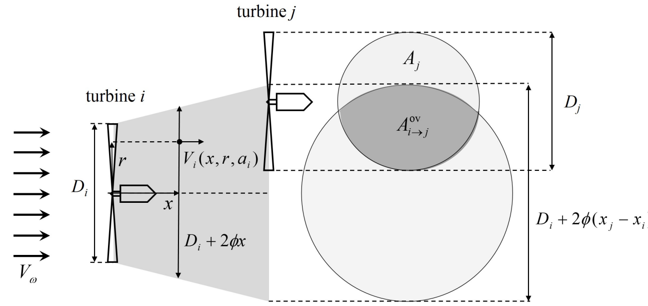

Fig. 1. The parameters of the wake expansion in the Park model

From the above wind farm model, it is clearly shown that the wake effect of the upstream turbines always reduces the incoming wind speed of the downstream turbines. Accordingly, the power production of the downstream turbines is decreased. In contrast, the incoming wind speed of the downstream turbines is expected to increase (which consequently increases the power production of the downstream turbine) by simply regulate the control variable of the upstream turbines such that its power production is reduced. On the other hand, in this study, the axial induction factor a_i $(\forall i \in \mathcal{L})$ is set to an optimal axial induction factor of individual wind turbine, which is $1/3$ since there are no further downstream turbines [8]. Therefore, our task is only to tune the axial induction factor a_i $(\forall i \in \mathcal{F})$ using the proposed model-free approach.

For the initial exploration of our approach to the problem of maximizing the power production of wind farm, several assumptions have been adopted as follows:

- The incoming wind speed V_ω is uniform and constant with a fixed direction.
- The diameter of the wake has a circular cross-section and expand proportional to the distance x.
- The turbine axes are parallel to the wind direction for all $i \in \mathcal{X}$.
- The sets $\mathcal{X}$, $\mathcal{F}$, $\mathcal{L}$ can be updated based on the given wind farm configuration and the wind direction.
- The proposed model-free approach requires that each of the individual turbines to only have knowledge of the total power produced in the wind farm after each time interval T_w.

Then, the model-free optimization problem can be described as follows.

Problem 2.1 Let a total power production of a given wind farm be defined as $\overline{P}(V_\omega, a_1, ..., a_p, T_w) = \sum_i^n P_i(V_\omega, a_1, ..., a_i, T_w)$. Then, find axial induction factors a_i $(i \in \mathcal{F})$ of each turbine such that the $\overline{P}(V_\omega, a_1, ..., a_p, T_w)$ is maximized. □

III. MODEL-FREE DESIGN USING SIMULTANEOUS PERTURBATION STOCHASTIC APPROXIMATION

This section presents the main idea to solve Problem 2.1. First, the SPSA algorithm proposed by [5] is briefly described. Then, it is presented how to effectively implement the model-free design based on the SPSA algorithm.

A. Simultaneous Perturbation Stochastic Approximation

The essential feature of the SPSA is the gradient approximation with not the objective function, but two measurements of the objective function. Therefore, this optimization method is an efficient model-free approach for wind farm control.

A general optimization problem is given by

$$\max_{\theta \in \mathbb{R}^n} \; f(\theta) \tag{8}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is the objective function and $\theta \in \mathbb{R}^n$ is the design variable.

The SPSA algorithm [5] iteratively updates the design parameter to search an optimal solution $\theta^* \in \mathbb{R}^n$ of (8). The update law is

$$\theta(k+1) = \theta(k) + a(k)g(\theta(k)), \tag{9}$$

for $k = 0, 1, ...,$ where $a(k)$ is the gain, $g(\theta(k))$ is the estimation of the gradient at the iterate $\theta(k)$, which is given by

$$g(\theta(k)) = \begin{bmatrix} \frac{f(\theta(k)+c(k)\triangle(k)) - f(\theta(k)-c(k)\triangle(k))}{2c(k)\triangle_1(k)} \\ \vdots \\ \frac{f(\theta(k)+c(k)\triangle(k)) - f(\theta(k)-c(k)\triangle(k))}{2c(k)\triangle_n(k)} \end{bmatrix}. \tag{10}$$

In (10), $c(k)$ is another gain, and $\triangle(k)$ is the n-dimensional random perturbation vector. For example, the gains $a(k)$ and $c(k)$ are given by $a(k) = a/(A+k+1)^\alpha$ and $c(k) = c/(k+1)^\gamma$, respectively, for non-negative numbers a, c, A, α and γ. Meanwhile, $\triangle(k)$ is, for example, drawn from the Bernoulli distribution

$$\begin{cases} \mathbb{P}(\triangle_i(k) = 1) = 0.5 \\ \mathbb{P}(\triangle_i(k) = -1) = 0.5 \end{cases} \tag{11}$$

and $\triangle_i(k)$ is its i-th component. Note that, the selection of non-negative coefficients a, c, A, α and γ will be performed by some guidance reported in [5].

Then, the standard SPSA algorithm consists of the following steps:

Step I: Select the non-negative coefficients a, c, A, α and γ for the SPSA gain sequences $a(k) = a/(A+k+1)^\alpha$ and $c(k) = c/(k+1)^\gamma$. Set the initial conditions of the design parameters $\theta(0)$ and set $k = 0$.

Step II: If a pre-specified termination criterion is satisfied, the algorithm terminates with the solution $\theta^* := \arg \max_{\theta \in \{\theta(0), \theta(1), ..., \theta(k)\}} f(\theta)$. Otherwise, go to Step III.

Step III: Generate n-dimensional random perturbation vector $\triangle(k)$.

Step IV: Obtain two measurements of the objective functions $f(\theta(k) + c(k) \triangle (k))$ and $f(\theta(k) - c(k) \triangle (k))$ with $c(k)$ and $\triangle(k)$ from Steps I and III, respectively.

Step V: Calculate the vector $g(\theta(k))$ in (10).

Step VI: Apply (9). Set $k = k + 1$ and then go to Step II.

In the algorithm, an example of termination criterion in Step II is based on the maximum number of iterations; i.e., the algorithm terminates after a user-determined number of iterations k_{max}.

B. Model-Free Design

Based on the SPSA algorithm in the Section A, our model-free design procedure for wind farm control is summarized as follows:

Step 1: Determine the number k_{max} of the maximum iterations and the time interval T_w. Then, n design parameters a_i $(i = 1, 2, ..., n)$ are selected, where $n = |\mathcal{F}|$.

Step 2: Perform the SPSA algorithm in Section A to solve Problem 2.1.

Step 3: After k_{max} iterations, the optimal axial induction factors $a_i(k_{max})$ $(i = 1, 2, ..., n)$ and the total power production $\overline{P}(V_\omega, a_1, ..., a_p, T_w)$ are recorded, and the convergence speed of the on-line optimization process is analyzed.

IV. SIMULATION RESULTS

In this section, the performance of the SPSA based method is demonstrated to the simple four-turbine row farm example, as shown in Fig. 2. Each turbine has a rotor diameter of 80 meters and the distance between each turbine is 400 meters. In the simulations, a constant incoming wind $V_\omega = 16$ m/s in the direction of the positive x-axis is assumed. The roughness coefficient is $\phi = 0.075$ and the air density is $\rho = 1.225$ kg/m^3. From the given wind farm configuration, the time interval for the wake to travel to the whole wind farm is approximated as $T_w = 150$ s.

Next, we set the parameters of the SPSA based algorithm $a(k) = 2 \times 10^{-8}/(k+19)^{0.6}$, $c(k) = 0.001/(k+1)^{1/3}$, and $k_{max} = 72$. The initial conditions of the design parameters are set to the value $a_i(0) = 1/3, \forall i \in \mathcal{F}$. In order to compare our proposed method to the current existing model-free approaches, which are the FS-MPPT [4] and GT based methods [1], the parameters for both methods are provided in this paper. Here, the FS-MPPT based approach with $K = 0.003$ and the GT based approach with $K = 0.05$ and $E = 0.5$ are used. See [1] and [4] for the detail of both algorithms. In order to observe the randomization effect, we perform 50 designs for the SPSA and GT based approaches.

Fig. 3 shows the response of the best total power production $\overline{P}(V_\omega, a_1, ..., a_4, T_w)$ for each method from 50 designs. In general, each of the methods successfully improves the

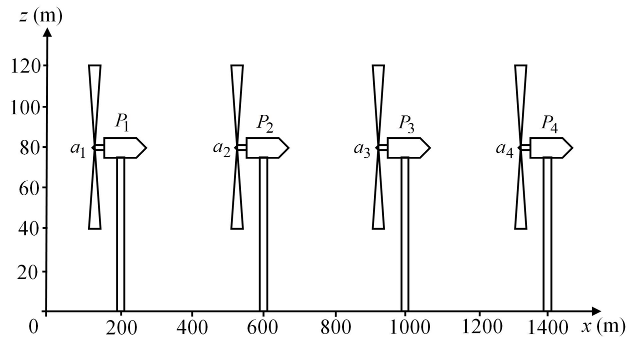

Fig. 2. Four-turbine wind farm

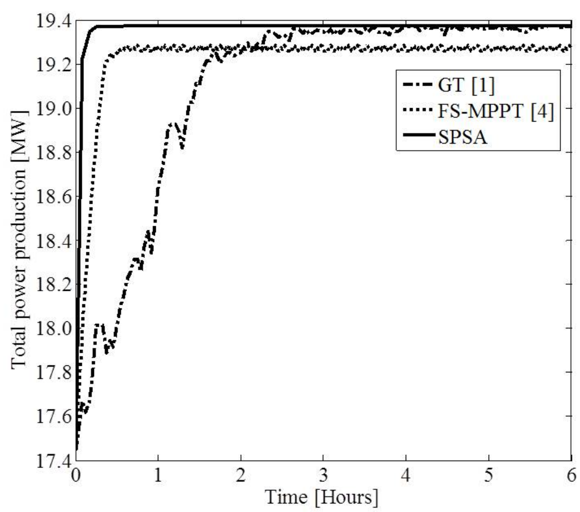

Fig. 3. Convergence of the best total power production $\overline{P}(V_\omega, a_1, ..., a_4, T_w)$ from 50 designs

total power production during 6 hours of simulation time. The statistical analysis of the final value of the total power production and the convergence time is recorded in Table I. The optimal axial induction factors of the SPSA based method in comparison with the FS-MPPT and the GT based methods are tabulated in Table II. Notice that, the SPSA and GT based methods achieve almost the same maximum total power production with 11.04 % of the total power improvement. This is followed by the FS-MPPT based methods with 10.42 % of the total power improvement.

However, in terms of the convergence speed, the SPSA and FS-MPPT based methods converge much faster than the GT based method. This also can be clearly seen from Fig. 3 for the first 1 hour of the simulation time. Hence, from the comparative assessment, we can confirm the superiority of the SPSA based algorithm in producing a maximum total power production with reasonable convergence speed.

V. CONCLUSIONS

In this paper, an initial study of a model-free approach based on Simultaneous Perturbation Stochastic Approximation (SPSA) for wind farm control has been performed. The proposed method is simulated on a single row wind farm based on the dynamic wind farm model in [4]. In

TABLE I
PERFORMANCE COMPARISON OF THE SPSA AND EXISTING MODEL-FREE APPROACHES FROM 50 DESIGNS

Model-free approaches		SPSA	FS-MPPT [4]	GT [1]
Total power production [MW]	Mean	19.37	-	19.36
	Best	19.37	19.27	19.37
	Worst	19.37	-	19.34
	Std.	0.00	-	0.01
Computation time [Hour]	Mean	0.61	-	2.59
	Best	0.13	0.36	1.83
	Worst	1.88	-	3.88
	Std.	0.43	-	0.46

TABLE II
THE BEST OPTIMAL AXIAL INDUCTION FACTORS OF THE SPSA METHOD AND EXISTING MODEL-FREE APPROACHES FROM 50 DESIGNS

Optimal axial induction factors	SPSA	FS-MPPT [4]	GT [1]
$a_1(k_{max})$	0.2289	0.2528	0.2297
$a_2(k_{max})$	0.1967	0.2178	0.1962
$a_3(k_{max})$	0.2104	0.2388	0.2099
$a_4(k_{max})$	0.3333	0.3333	0.3333

the simulation study, the SPSA based method yields a maximum power production with reasonable convergence speed as compared to the existing model free approaches. In this sense, the proposed method may be practical than the existing methods, especially for on-line model-free approach.

In the future, the applicability of the SPSA based method will be tested to a large scale wind farm with a chaotic environment, e.g. incoming wind with time-varying speed and direction.

ACKNOWLEDGMENT

This work was partly supported by JSPS KAKENHI Grant Number 25249058 and 24686051.

REFERENCES

[1] J.R. Marden, S.D. Ruben, and L.Y. Pao, Surveying game theoretic approaches for wind farm optimization, in Proc. of the AIAA Aerospace Sciences Meeting, 2012.
[2] J.R. Marden, S.D. Ruben, and L.Y. Pao, A model-free approach to wind farm control using game theoretic methods, IEEE Transactions on Control Systems Technology, Vol. 21(4), pp. 1207–1214, 2013.
[3] J. Park, S. Kwon and K.H. Law, Wind farm power maximization based on a cooperative static game approach, in Proc. SPIE 8688, Active and Passive Smart Structures and Integrated Systems 2013.
[4] P.M.O. Gebraad, F.C. van Dam, and J.-W. van Wingerden, A model-free distributed approach for wind plant control, in 2013 American Control Conference, Washington, DC, 2013, pp. 628-633.
[5] J.C. Spall, Multivariate stochastic approximation using a simultaneous perturbation gradient approximation, IEEE Transactions on Automatic Control, Vol. 37(3), pp. 332–341, 1992.
[6] A. Scholbrock, Optimizing wind farm control strategies to minimize wake loss effects, Master's Thesis, University of Colorado Boulder, United States, 2011.
[7] T. Sorensen, M.L. Thogersen, P. Nielsen, A. Grotzner, and S. Chun, Adapting and calibration of existing wake models to meet the conditions inside offshore wind farms, EMS International A/S Report, 2008.
[8] F. Bianchi, H.D. Battista, and R. Mantz, Wind turbine control systems; principles, modelling, and gain scheduling design, Springer-Verlag London, 2007.