

FRF parameter identification with arbitrary input sequence from noisy input–output measurements

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Abstract—This paper deals with the identification of errors–in–variables (EIV) models corrupted by additive and uncorrelated white noises when the noise–free input is an arbitrary signal, not necessarily periodic. In particular, a frequency domain method is proposed, under the assumption that the ratio of the noise variances is known.

I. INTRODUCTION

System representations where both inputs and outputs are affected by additive errors are called errors–in–variables (EIV) models and play an important role in several engineering applications. The identification of EIV models has been deeply investigated in the literature and many solutions have been proposed, in the time and frequency domain, see [17], [18] and the references therein.

The full equivalence, in the likelihood framework, of time- and frequency- domain methods has been formalized in a recent paper [1]. However, from a practical point of view, there can be several reasons that lead the user to prefer a solution to the other. In this respect, an interesting comparison of the pros and cons of these choices is given in [16].

Frequency domain system identification [14] is a huge area of research, characterized by parametric and non parametric approaches and by many and different techniques, see for example [11], [12], [13].

In this work, the EIV identification problem is addressed by applying the Koopmans–Levin approach [9], [7] in the frequency domain. The system is described by a parametric model that links the Discrete Fourier Transforms (DFTs) of the input and output signals. According to the theoretic results described in [15], a polynomial term is added in the system equation in order to take into account the leakage and transient effects. Compared with other well–known Frequency Response Function (FRF) estimators [4], the usage of this extended model allows to deal with shorter data sequences and with arbitrary noise-free input signals, not necessarily periodic [5].

In this paper, the additive noises are assumed as mutually uncorrelated white processes with unknown variances, but with known variance ratio. The proposed identification method can be considered as the frequential counterpart of the time domain Koopmans–Levin approach. However, the performances of this new method are potentially superior to those of the corresponding time domain version, thanks to its

intrinsic possibility of filtering the data, by fitting the model only within a desired frequency range.

The organization of the paper is as follows. Section II defines the EIV identification problem in the frequency domain. Section III describes the Koopmans–Levin solution for frequency data. In Section IV the effectiveness of the proposed approach is verified by means of Monte Carlo simulations. Finally some concluding remarks are reported in Section V.

II. STATEMENT OF THE PROBLEM

Consider the linear time–invariant SISO system described in Figure 1. The noise–free input and output $\hat{u}(t)$, $\hat{y}(t)$ are linked by the linear difference equation

$$A(z^{-1})\hat{y}(t) = B(z^{-1})\hat{u}(t), \quad (1)$$

where $A(z^{-1})$ and $B(z^{-1})$ are polynomials in the backward shift operator z^{-1}

$$A(z^{-1}) = 1 + \alpha_1 z^{-1} + \dots + \alpha_n z^{-n} \quad (2)$$

$$B(z^{-1}) = \beta_0 + \beta_1 z^{-1} + \dots + \beta_n z^{-n}. \quad (3)$$

In the EIV environment the input and output measurements are assumed as corrupted by additive noise so that the available observations are

$$u(t) = \hat{u}(t) + \tilde{u}(t) \quad (4)$$

$$y(t) = \hat{y}(t) + \tilde{y}(t). \quad (5)$$

The following assumptions are made.

- A1. The system (1) is asymptotically stable.
- A2. $A(z^{-1})$ and $B(z^{-1})$ do not share any common factor.
- A3. The order n of the system is assumed as *a priori* known.
- A4. The noise–free input $\hat{u}(t)$ is a quasi–stationary bounded deterministic signal [10] and is persistently exciting of sufficiently high order.
- A5. $\tilde{u}(t)$ and $\tilde{y}(t)$ are zero–mean ergodic white processes with unknown variances λ_u and λ_y , respectively. These processes are mutually uncorrelated and uncorrelated with the noise–free input $\hat{u}(t)$.
- A6. The noise variances λ_u and λ_y are unknown but their ratio $\rho = \lambda_y/\lambda_u$ is assumed as known, with $0 < \rho < \infty$.

Assumptions A1–A4 are commonly imposed when dealing with general identification algorithms, not necessarily in an EIV context [19], [10]. Assumption A5 is common in the Frisch scheme context [3], [8] and as a basic assumption for analysing the EIV identification methods [17], [20]. For

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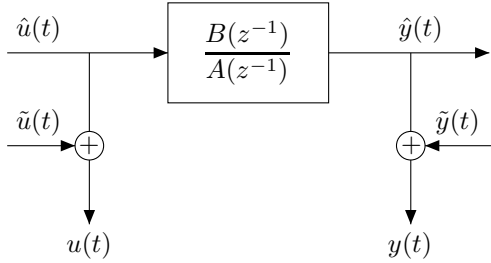


Fig. 1. Errors-in-variables model

several EIV identification approaches, this assumption can be relaxed and correlated noises or coloured (output) noises can be considered [18]. More general conditions on the identifiability of EIV systems can be found in [2]. Assumption A6 is a standard assumption for all TLS-based estimators in EIV problems. Even if mathematically restrictive, this condition is often satisfied in practical situations [23], with the benefit of a simple and fast solution to the EIV identification problem.

Let $\{u(t)\}_{t=0}^{(N-1)\Delta t}$ and $\{y(t)\}_{t=0}^{(N-1)\Delta t}$ be a set of input and output observations at N equidistant time instants $t, t + \Delta t$. In the following we will assume $\Delta t = 1$, without loss of generality. The corresponding DFTs are defined as

$$U(\omega_k) = \frac{1}{\sqrt{N}} \sum_{t=0}^{N-1} u(t) e^{-j\omega_k t} \quad (6)$$

$$Y(\omega_k) = \frac{1}{\sqrt{N}} \sum_{t=0}^{N-1} y(t) e^{-j\omega_k t}, \quad (7)$$

where $\omega_k = 2\pi k/N$ and $k = 0, \dots, N-1$. The system transfer function is represented as

$$G(e^{-j\omega_k}) = \frac{B(e^{-j\omega_k})}{A(e^{-j\omega_k})}. \quad (8)$$

The DFTs defined in (6) and (7) can be expressed in matrix form by introducing the $N \times N$ Fourier matrix F_N [1] whose entries are defined as follows

$$F_N = [f_{ik}] \quad (9)$$

$$f_{ik} = \frac{1}{\sqrt{N}} e^{-j\frac{2\pi}{N}(i-1)(k-1)} \quad i, k = 1, \dots, N. \quad (10)$$

It can be proved that matrix F_N is unitary, i.e. $F_N F_N^H = I$, where $(\cdot)^H$ denotes the transpose and conjugate operation.

Defining the following vectors in time and frequency domain

$$v_u = [u(0), \dots, u(N-1)]^T \quad (11)$$

$$v_y = [y(0), \dots, y(N-1)]^T \quad (12)$$

$$V_U = [U(\omega_0), \dots, U(\omega_{N-1})]^T \quad (13)$$

$$V_Y = [Y(\omega_0), \dots, Y(\omega_{N-1})]^T \quad (14)$$

the relations (6) and (7) can be represented by the linear transformations

$$V_U = F_N v_u \quad (15)$$

$$V_Y = F_N v_y. \quad (16)$$

In the frequency domain, the problem under investigation can be stated as follows.

Problem 1. Let $U(\omega_k), Y(\omega_k)$ be a set of noisy measurements generated by an EIV system of type (1)–(5), under assumptions A1–A6, where $\omega_k = 2\pi k/N$ and $k = 0, \dots, N-1$. Estimate the system parameters α_i ($i = 1, \dots, n$), β_i ($i = 0, \dots, n$) and the noise variance λ_u .

Recently in [22] a frequency domain solution to Problem 1 has been proposed in more general conditions, when only Assumptions A1–A5 hold. The solution, however, is much more involved. For the sake of simplicity this case will not be treated here.

III. THE KOOPMANS–LEVIN SOLUTION

In the following a possible solution of Problem 1 is proposed, analogue to the time domain Koopmans–Levin solution [9], [7].

A. The noise-free case

With reference to the noise-free signals $\hat{u}(t)$ and $\hat{y}(t)$, definitions similar to (11)–(14) and (15)–(16) hold, i.e.

$$\hat{v}_u = [\hat{u}(0), \dots, \hat{u}(N-1)]^T \quad (17)$$

$$\hat{v}_y = [\hat{y}(0), \dots, \hat{y}(N-1)]^T \quad (18)$$

$$\hat{V}_U = [\hat{U}(\omega_0), \dots, \hat{U}(\omega_{N-1})]^T \quad (19)$$

$$\hat{V}_Y = [\hat{Y}(\omega_0), \dots, \hat{Y}(\omega_{N-1})]^T, \quad (20)$$

where

$$\hat{V}_U = F_N \hat{v}_u \quad (21)$$

$$\hat{V}_Y = F_N \hat{v}_y. \quad (22)$$

It is a well-known fact [15] that for finite N , even in absence of noise, the ratio of the DFTs $\hat{Y}(\omega_k)$ and $\hat{U}(\omega_k)$ ($\omega_k = 2\pi k/N$) is not equal to the true transfer function

$$G(e^{-j\omega_k}) \neq \frac{\hat{Y}(\omega_k)}{\hat{U}(\omega_k)}. \quad (23)$$

As a matter of fact, it can be proved that the DFTs $\hat{Y}(\omega_k)$ and $\hat{U}(\omega_k)$ exactly satisfy an extended model that includes also a transient term, i.e.

$$A(e^{-j\omega_k}) \hat{Y}(\omega_k) = B(e^{-j\omega_k}) \hat{U}(\omega_k) + T(e^{-j\omega_k}), \quad (24)$$

where $T(z^{-1})$ is a polynomial of order $n-1$

$$T(z^{-1}) = \tau_0 + \tau_1 z^{-1} + \dots + \tau_{n-1} z^{-n+1} \quad (25)$$

that takes into account the effects of the initial and final conditions of the experiment.

By considering the whole number of frequencies, eq. (24) can be rewritten in a matrix form. For this purpose, introduce the parameter vectors

$$\theta_\alpha = [\alpha_n, \dots, \alpha_1, 1]^T \quad (26)$$

$$\theta_\beta = [\beta_n, \dots, \beta_1, \beta_0]^T \quad (27)$$

$$\theta_\tau = [\tau_{n-1}, \dots, \tau_0]^T. \quad (28)$$

and define the following vector Θ , with dimension $p = 3n + 2$, containing the whole number of parameters

$$\Theta = [\theta_\alpha^T, -\theta_\beta^T, -\theta_\tau^T]^T. \quad (29)$$

In absence of noise, the system parameters can be recovered by means of the following procedure.

Define the row vectors

$$Z_{n+1}(\omega_k) = [e^{-jn\omega_k}, e^{-j(n-1)\omega_k}, \dots, e^{-j\omega_k}, 1] \quad (30)$$

$$Z_n(\omega_k) = [e^{-j(n-1)\omega_k}, \dots, e^{-j\omega_k}, 1], \quad (31)$$

whose entries are constructed with multiple frequencies of ω_k , and construct the following matrices

$$\Pi = \begin{bmatrix} Z_{n+1}(\omega_0) \\ \vdots \\ Z_{n+1}(\omega_{N-1}) \end{bmatrix} \quad \Psi = \begin{bmatrix} Z_n(\omega_0) \\ \vdots \\ Z_n(\omega_{N-1}) \end{bmatrix}. \quad (32)$$

of dimension $N \times (n+1)$ and $N \times n$, respectively.

With the noise-free input-output DFTs (19) and (20) construct the following $N \times N$ diagonal matrices

$$\hat{V}_U^{diag} = \text{diag}[\hat{U}(\omega_0), \hat{U}(\omega_1), \dots, \hat{U}(\omega_{N-1})] \quad (33)$$

$$\hat{V}_Y^{diag} = \text{diag}[\hat{Y}(\omega_0), \hat{Y}(\omega_1), \dots, \hat{Y}(\omega_{N-1})]. \quad (34)$$

Compute the $N \times (n+1)$ matrices

$$\hat{\Phi}_B = \hat{V}_U^{diag} \Pi; \quad \hat{\Phi}_A = \hat{V}_Y^{diag} \Pi \quad (35)$$

and set

$$\hat{\Phi}_T = \Psi. \quad (36)$$

Construct the $N \times p$ matrix

$$\hat{\Phi} = [\hat{\Phi}_A | \hat{\Phi}_B | \hat{\Phi}_T]. \quad (37)$$

Thus, eq. (24) can be rewritten as

$$\hat{\Phi} \Theta = 0. \quad (38)$$

It then holds

$$\hat{\Sigma} \Theta = 0, \quad (39)$$

where $\hat{\Sigma}$ is the $p \times p$ matrix

$$\hat{\Sigma} = \frac{1}{N} (\hat{\Phi}^H \hat{\Phi}). \quad (40)$$

Remark 1. Because of assumption A2, relation (24) cannot be satisfied by polynomials $A(z^{-1})$ and $B(z^{-1})$ with order lower than n . Therefore, matrix $\hat{\Sigma}$ in (40) is positive semidefinite, with only one null eigenvalue, i.e.

$$\hat{\Sigma} \geq 0 \quad \dim \ker \hat{\Sigma} = 1. \quad (41)$$

The previous considerations can be summed up as follows.

Result 1. Let $\hat{U}(\omega_k)$, $\hat{Y}(\omega_k)$ be a set of noise-free measurements generated by a dynamic system of type (1)–(3), under assumptions A1–A4, where $\omega_k = 2\pi k/N$ and $k = 0, \dots, N-1$.

Thus, for every frequency ω_k , the DFTs $\hat{U}(\omega_k)$, $\hat{Y}(\omega_k)$ exactly satisfy relation (24), where $A(z^{-1})$, $B(z^{-1})$ are the system polynomials. The parameter vector Θ , see (29), can be recovered by computing the kernel of the matrix $\hat{\Sigma}$ defined in (40), after normalizing the $(n+1)$ -th entry to 1.

B. The noisy case

In presence of noise, the previous procedure can be modified as follows.

With the noisy input-output DFTs (13) and (14) construct the $N \times N$ diagonal matrices

$$V_U^{diag} = \text{diag}[U(\omega_0), U(\omega_1), \dots, U(\omega_{N-1})] \quad (42)$$

$$V_Y^{diag} = \text{diag}[Y(\omega_0), Y(\omega_1), \dots, Y(\omega_{N-1})] \quad (43)$$

and compute the matrices

$$\Phi_B = V_U^{diag} \Pi; \quad \Phi_A = V_Y^{diag} \Pi; \quad \Phi_T = \Psi. \quad (44)$$

Then construct the $N \times p$ matrix

$$\Phi = [\Phi_A | \Phi_B | \Phi_T] \quad (45)$$

and compute the $p \times p$ positive definite matrix

$$\Sigma = \frac{1}{N} (\Phi^H \Phi). \quad (46)$$

Remark 2. Because of assumption A5, when $N \rightarrow \infty$, matrix Σ in (46) is constituted by the sum of two matrices

$$\Sigma = \hat{\Sigma} + \tilde{\Sigma}, \quad (47)$$

where $\hat{\Sigma}$ satisfies the condition (41) and

$$\tilde{\Sigma} = \begin{bmatrix} \lambda_y I_{n+1} & 0 & 0 \\ 0 & \lambda_u I_{n+1} & 0 \\ 0 & 0 & 0_n \end{bmatrix}. \quad (48)$$

Moreover, because of assumption A6, matrix $\tilde{\Sigma}$ results as

$$\tilde{\Sigma} = \lambda_u \begin{bmatrix} \rho I_{n+1} & 0 & 0 \\ 0 & I_{n+1} & 0 \\ 0 & 0 & 0_n \end{bmatrix} = \lambda_u \Lambda, \quad (49)$$

where λ_u is unknown.

Remark 3. The noise variance λ_u is obtained by computing the value of λ that satisfies condition (41), i.e.

$$(\Sigma - \lambda \Lambda) \geq 0 \quad \dim \ker (\Sigma - \lambda \Lambda) = 1. \quad (50)$$

It can be easily verified that the solution is

$$\frac{1}{\lambda_u} = \max \left(\text{eig} (\Sigma^{-1} \Lambda) \right). \quad (51)$$

The previous considerations can be summed up as follows.

Result 2. Let $U(\omega_k)$, $Y(\omega_k)$ be a set of noisy measurements generated by an EIV system of type (1)–(5), under assumptions A1–A6, where $\omega_k = 2\pi k/N$ and $k = 0, \dots, N-1$.

Thus, matrix Σ in (46) is the sum of two matrices, see (47), where matrix $\tilde{\Sigma}$ is known up to a scalar factor λ_u . The solution for λ_u is given by relation (51). The parameter vector Θ , defined in (29), can be obtained as the kernel of

$$(\Sigma - \lambda_u \Lambda) \Theta = 0, \quad (52)$$

after normalizing the $(n + 1)$ -th entry to 1.

Remark 4. The proposed procedure can be applied also when only a subset W of the frequency range is used, i.e. $\omega_k \in W = [\omega_i, \omega_f]$, with $i \geq 0$ and $f \leq N - 1$. The subset $W = [\omega_i, \omega_f]$, with $L = f - i + 1$ frequencies, must be chosen by the user on the basis of *a priori* knowledge of the frequency properties of the transfer function $G(e^{-j\omega_k})$ and of the noise-free input $\hat{U}(\omega_k)$.

IV. NUMERICAL RESULTS

Example 1. The proposed algorithm has been tested on sequences generated by a second-order model of type (1), already proposed in [6]

$$A(z^{-1}) = 1 - 0.5z^{-1} + 0.3z^{-2} \quad (53)$$

$$B(z^{-1}) = 2 - 1.2z^{-1} - 0.6z^{-2}. \quad (54)$$

The input is a pseudo random binary sequence with unit variance and length N . Monte Carlo simulations of 100 independent runs have been performed by adding to the noise-free sequences $\hat{u}(\cdot)$, $\hat{y}(\cdot)$ different white noise realizations with variances $\lambda_u^* = 0.1$, $\lambda_y^* = 0.6$ ($\rho = 6$), corresponding to a signal to noise ratio (SNR) of about 10 dB on both input and output.

In order to verify the improvement of the accuracy in the estimates for increasing values of data, the Monte Carlo simulations have been performed with $N = 250$, $N = 500$ and $N = 1000$.

Tables 1 and 2 report the empirical means of the parameter estimates together with the corresponding standard deviations, obtained with the proposed Frequency Domain Koopmans–Levin (FD–KL) algorithm.

The last column of Table 1 reports the empirical mean of the estimates of λ_u , and the corresponding standard deviation, for the different values of N .

These tables show the effectiveness of the proposed method, comparable with that of the time domain KL approach, see also [21].

Example 2. In order to verify the selective properties described in Remark 1, the proposed FD–KL algorithm has been tested on sequences generated by a fourth-order model of type (1), also proposed in [24]

$$A(z^{-1}) = 1 - 3.57z^{-1} + 5.13z^{-2} - 3.5z^{-3} + 0.96z^{-4} \quad (55)$$

$$B(z^{-1}) = 10^{-2} \times p(z^{-1}) \quad (56)$$

$$p(z^{-1}) = 1 + 1.15z^{-1} + 1.02z^{-2} + 0.27z^{-3} + 0.05z^{-4}.$$

The noise-free input $\hat{u}(t)$ is a white noise process with unit variance and length N . A Monte Carlo simulation of 100 independent runs have been performed by adding to

TABLE I

TRUE AND ESTIMATED VALUES OF α_i ($i = 1, 2$) AND λ_u , OBTAINED BY MEANS OF THE FD–KL ALGORITHM FOR DIFFERENT DATA LENGTH N

	α_1	α_2	λ_u
true	-0.5	0.3	0.1
$N = 250$	-0.5055 ± 0.0688	0.3109 ± 0.0706	0.0967 ± 0.0060
$N = 500$	-0.5005 ± 0.0507	0.3063 ± 0.0485	0.0986 ± 0.0039
$N = 1000$	-0.4974 ± 0.0391	0.2974 ± 0.0370	0.0993 ± 0.0029

TABLE II

TRUE AND ESTIMATED VALUES OF β_i ($i = 1, 2, 3$) OBTAINED BY MEANS OF THE FD–KL ALGORITHM FOR DIFFERENT DATA LENGTH N

	β_0	β_1	β_2
true	2	-1.2	-0.6
$N = 250$	2.0064 ± 0.0413	-1.2179 ± 0.1632	-0.5711 ± 0.1763
$N = 500$	2.0032 ± 0.0283	-1.2035 ± 0.1195	-0.5800 ± 0.1265
$N = 1000$	2.0001 ± 0.0170	-1.1955 ± 0.0903	-0.6092 ± 0.1003

the noise-free sequences $\hat{u}(\cdot)$, $\hat{y}(\cdot)$ different white noise realizations with variances $\lambda_u^* = 0.1$, $\lambda_y^* = 0.6$ ($\rho = 6$), corresponding to a SNR of about 10 dB on both input and output.

The Monte Carlo simulation has been performed with $N = 2000$ frequencies. However, the transfer function $G(e^{-j\omega_k})$, defined in (8), has been identified by using the $L = 200$ frequencies in the window $W = [\omega_0, \omega_{199}] = [0, 0.625]$ radians.

Figure 2 reports the true values of the magnitude and phase of $G(e^{-j\omega_k})$ in red (solid) line, together with the mean of the estimated transfer functions (TF), in blue (dashed) line. The advantageous effects of filtering are evident in the frequency region around the two peaks of $|G(e^{-j\omega_k})|$.

Example 3. Finally, the FD–KL method has been tested by means of the 3 Degrees Of Freedom (DOF) mechanical,

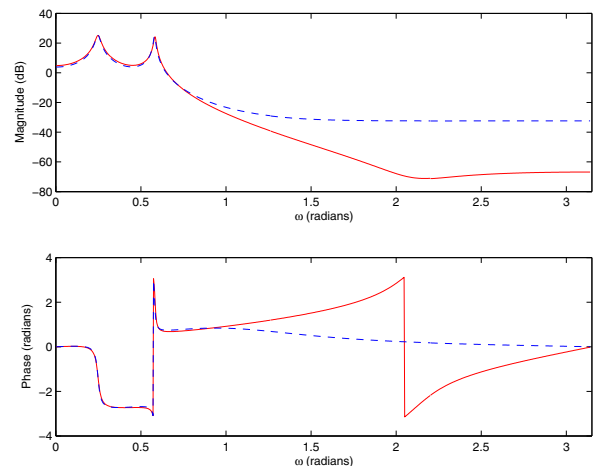


Fig. 2. True TF: red (solid); Estimated TF (mean) with $N=2000$ and window W : blue (dashed).

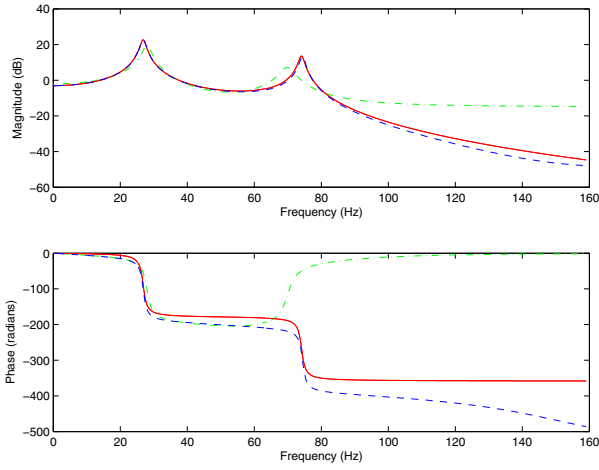


Fig. 3. True TF: red (solid); Estimated TF (mean) with FD-KL method: blue (dashed); Estimated TF (mean) with TLS approach: green (dash-dotted).

lumped-parameter system governed by the equation

$$M \frac{d^2 \hat{\mathbf{y}}(t)}{dt^2} + C \frac{d\hat{\mathbf{y}}(t)}{dt} + K \hat{\mathbf{y}}(t) = \mathbf{F}(t), \quad (57)$$

where $\hat{\mathbf{y}}(t)$ is the vector of the nodal displacements and $\mathbf{F}(t)$ is the vector of the external forces, not accessible for measurements. The mass matrix M is

$$M = \text{diag}[m_1 \ m_2 \ m_3], \quad (58)$$

where $m_1 = 11$, $m_2 = 5$ and $m_3 = 10$ (Kg).

The stiffness matrix K is

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 + k_4 \end{bmatrix}, \quad (59)$$

where $k_1 = 10^5$, $k_2 = 4 \times 10^5$, $k_3 = 6 \times 10^5$ and $k_4 = 2 \times 10^5$ (N/m).

The damping matrix C is

$$C = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 + c_5 & -c_3 \\ 0 & -c_3 & c_3 + c_4 \end{bmatrix}, \quad (60)$$

where $c_1 = 110.15$, $c_2 = 0.6$, $c_3 = 0.9$, $c_4 = 100.3$ and $c_5 = 50$ (Kg/s).

The external force is applied to the 3-rd DOF. The FRF to be identified is the transmissibility function $G_{13}(j\omega) = \hat{Y}_1(j\omega) / \hat{Y}_3(j\omega)$ between the 1-st and 3-rd DOFs. It results a rational function of order $2n = 4$, where n denotes the number of pairs of complex conjugate poles. The magnitude and the phase of $G_{13}(j\omega)$ are reported in Figure 3 (solid, red line).

The system has been discretized with zero-order-hold reconstruction, at the sampling frequency $F_s = 400$ Hz, which is compatible with the frequency band of the system, as illustrated in Figure 3. The system has been excited on the 3-rd mass by a discrete-time white noise process with

TABLE III

TRUE AND ESTIMATED VALUES OF THE COMPLEX CONJUGATE POLES

	s_1, s_2	s_3, s_4
true	$-0.0502 \pm j 1.6872$	$-0.0516 \pm j 4.6576$
FD - KL	$-0.0494 \pm j 1.6873$	$-0.0492 \pm j 4.6758$
TLS	$-0.0687 \pm j 1.7582$	$0.1294 \pm j 4.3910$

length $N = 4000$. Thus, the discrete-time system of type (1) to be identified is

$$G_{13}(e^{-j\omega_k}) = \frac{\hat{Y}_1(\omega_k)}{\hat{Y}_3(\omega_k)} = \frac{H_{13}(e^{-j\omega_k}) F_3(\omega_k)}{H_{33}(e^{-j\omega_k}) F_3(\omega_k)} = \frac{H_{13}(e^{-j\omega_k})}{H_{33}(e^{-j\omega_k})}, \quad (61)$$

where $\omega_k = 2\pi k/N$ and $F_3(\omega_k)$ is the DFT of the external, not accessible, force.

A Monte Carlo simulation of 100 independent runs has been performed by adding to the noise-free sequences $\hat{y}_3(\cdot)$, $\hat{y}_1(\cdot)$ different white noise realizations with variances $\lambda_{y_3}^* = 0.02$, $\lambda_{y_1}^* = 0.02$ ($\rho = 1$), corresponding to a SNR of about 30 dB on both the signals (equivalent to an external force with variance $\sigma_f = 10^{12}$).

In order to evaluate the effects in the identification algorithm of the leakage and transient phenomena, the FD-KL method has been compared with a classical TLS approach in which the polynomial term $T(e^{-j\omega_k})$ in (24) is not present. Also in this case, a data sequence with length $N = 4000$ has been considered and no averaging over different data blocks has been performed.

Figure 3 reports in red (solid) line the true values of the magnitude and phase of $G_{13}(j\omega)$, discretized at the sampling time $\Delta t = 1/F_s = 0.0025$ sec. The figure reports also the means of the estimated discrete-time transfer function $G_{13}(e^{-j\omega_k})$. The blue (dashed) line refers to the FD-KL solution, while the green (dash-dotted) line refers to the TLS solution. This figure well illustrates the theoretic correctness of equation (24) and the effectiveness of the proposed FD-KL method.

Once the system parameters have been determined, the poles of the transmissibility function $G_{13}(j\omega)$ can be easily estimated. Firstly, the poles p_i ($i = 1, \dots, 2n$) of the identified discrete-time function $G_{13}(e^{-j\omega_k})$ are estimated, by computing the roots of the polynomial $A(z^{-1})$. Then, they are transformed into the continuous-time poles s_i by means of the relation

$$s_i = \log_e(p_i) \quad i = 1, \dots, 2n. \quad (62)$$

Table 4 reports the true values of the $n = 2$ pairs of complex poles, together with their estimates obtained with the FD-KL method and the TLS approach. It must be observed that the TLS method fails to give a feasible solution for s_3, s_4 since this pair exhibits a positive real part. Also in this case, the results verify the good features of the proposed method.

V. CONCLUSIONS

In this paper a new frequency domain method has been proposed for the identification of EIV models with additive white noises. The method applies for general inputs, but requires the *a priori* knowledge of the noise variance ratio. Monte Carlo simulations have confirmed the good performances of the proposed algorithm, in different working conditions.

VI. ACKNOWLEDGEMENTS

The present study was supported by the Regione Emilia-Romagna, Progetto Tecnopolo POR FESR 2007-2013

REFERENCES

- [1] J.C. Agüero, J.I. Yuz, G.C. Goodwin and R.A. Delgado, On the equivalence of time and frequency domain maximum likelihood estimation, *Automatica*, vol. 46, n. 2, pp. 260–270, 2010.
- [2] J.C. Agüero and G.C. Goodwin, Identifiability of errors in variables dynamic systems, *Automatica*, vol. 44, n. 2, pp. 371–382, 2008.
- [3] S. BegHELLI, R. Guidorzi and U. Soverini, The Frisch scheme in dynamic system identification, *Automatica*, vol. 26, n. 1, pp. 171–176, 1990.
- [4] J.S. Bendat and A.G. Piersol, *Random Data: Analysis and Measurement Procedures*, John Wiley, NY, 2010.
- [5] B. Cauberghe, P. Guillaume, P. Verboven, S. Vanlanduit and E. Parloo, Frequency response function-based parameter identification from short data sequences, *Mechanical Systems and Signal Processing*, vol. 18, pp. 1097–1116, 2004.
- [6] R. Diversi, R. Guidorzi and U. Soverini, Maximum likelihood identification of noisy input–output models, *Automatica*, vol. 43, n. 3, pp. 464–472, 2007.
- [7] K.V. Fernando and H. Nicholson, Identification of linear systems with input and output noise: the Koopmans–Levin method, *IEE Proceedings*, Part D, 132, pp. 30–36, 1985.
- [8] R. Guidorzi, R. Diversi and U. Soverini, The Frisch scheme in algebraic and dynamic identification problems, *Kybernetika*, vol. 44, no. 5, pp. 585–616, 2008.
- [9] M.J. Levin, Estimation of a system pulse transfer function in the presence of noise, *IEEE Transactions on Automatic Control*, vol. 9, n. 3, pp. 229–235, 1964.
- [10] L. Ljung, *System identification—Theory for the user* (2nd ed.), Upper Saddle River, NJ, USA: Prentice–Hall, 1999.
- [11] T. McKelvey, Frequency domain identification methods, *Circuits Systems Signal Processing*, vol. 21, n.1, pp. 39–55, 2002.
- [12] T. McKelvey, H. Akcay and L. Ljung, Subspace-based multivariable system identification from frequency response data, *IEEE Trans. on Automatic Control*, vol. 41, n. 7, pp. 960–979, 1996.
- [13] R. Pintelon, P. Guillaume, G. Vandersteen and Y. Rolain, Analyses, development and applications of TLS algorithms in frequency domain system identification, *SIAM J. Matrix Anal. Appl.*, vol. 19, n. 4, pp. 983–1004, 1998.
- [14] R. Pintelon and J. Schoukens, *System identification: a frequency domain approach* (2nd ed.). NY: IEEE Press, 2012.
- [15] R. Pintelon, J. Schoukens and G. Vandersteen, Frequency domain system identification using arbitrary signals, *IEEE Transactions on Automatic Control*, vol. 42, n. 12, pp. 1717–1720, 1997.
- [16] J. Schoukens, Y. Rolain and R. Pintelon, On the Use of Parametric and Non-Parametric Noise-models in time- and frequency- domain System Identification, *Proc. 49-th IEEE Conference on Decision and Control*, Atlanta, USA, pp. 316–321, December 2010.
- [17] T. Söderström, Errors-in-Variables methods in system identification, *Automatica*, vol. 43, n. 6, pp. 939–958, 2007.
- [18] T. Söderström, System identification for the errors-in-variables problem, *Transaction of the Institute of Measurement and Control*, vol. 34, n. 7, pp. 780–792, 2012.
- [19] T. Söderström and P. Stoica, *System identification*, Hemel Hempstead: Prentice–Hall, 1989.
- [20] T. Söderström, L. Wang, R. Pintelon and J. Schoukens, Can errors-in-variables systems be identified from closed-loop experiments?, *Automatica*, vol. 49, n. 2, pp. 681–684, 2013.
- [21] U. Soverini and T. Söderström, Frequency domain maximum likelihood identification of noisy input–output models, *Proc. XX IFAC World Conference*, Cape Town, South Africa, August 2014.
- [22] U. Soverini and T. Söderström, Frequency domain EIV identification: a Frisch Scheme approach, *Proc. XX IFAC World Conference*, Cape Town, South Africa, August 2014.
- [23] S. Van Huffel and P. Lemmerling (Eds.) *Total Least Squares Techniques and Errors-in-Variables Modelling: Analysis, Algorithms and Applications*, Dordrecht, The Netherlands: Kluwer Academic Publishers, 2002.
- [24] E. Zhang, R. Pintelon and J. Schoukens, Errors-in-variables identification of dynamic systems excited by arbitrary non-white input, *Automatica*, vol. 49, n. 10, pp. 3032–3041, 2013.