21st International Symposium on
Mathematical Theory of Networks and Systems
July 7-11, 2014. Groningen, The Netherlands

A COMPOSITIONAL APPROACH TO TRANSIENT STABILITY OF MULTI-MACHINE POWER SYSTEMS

SINA Y. CALISKAN AND PAULO TABUADA

1. Extended Abstract

Under normal operating conditions, the electrical signals in a power system are sinusoidal waveforms with the same constant frequency, also known as synchronous frequency. Since a deviation from the synchronous frequency may cause serious damage in the components of a power system, one of the most important problems in power systems, also known as the transient stability problem, is the synchronization of all the signals on the synchronous frequency.

Traditional models used for the study of transient stability problem are based on several assumptions: 1) all waveforms in the model are sinusoidal; 2) the frequencies of these waveforms are very close to the synchronous frequency; 3) the reactive power is neglected in the power balance equation of each generator. These assumptions are not compatible with transients during which waveforms are known not to be sinusoidal. Moreover, the models based on these restrictive assumptions prevent us from having a clear understanding of how energy moves between different components of a power system. Using the framework of port-Hamiltonian systems it is possible to derive a more general model from first principles. Such model is still tractable and it does not require *any* of the stated assumptions. Furthermore, we show, using the aforementioned model, how to infer transient stability of the interconnected system by checking a simple condition for each individual generator. Since we reason about the interconnected system by applying a condition to the individual generators, the presented results are also compositional. This energy-based framework is already used in the recent works [3, 4], where a single machine connected to a load is analyzed in the restrictive scenario that there are no losses. The transient stability conditions presented in this extended abstract are applicable for a larger class of power systems which are composed of lossy transmission lines, constant impedance and constant current loads.

1.1. **Single generator.** Every synchronous generator consists of a mechanical part and an electrical part. We assume that the rotor of the generator has two poles, however the results can be easily extended to the machines with more than two poles. The rotor has a kinetic energy $H_{\text{kinetic}} = \frac{1}{2}M\omega^2$, where $\omega = \dot{\theta}$ is the angular velocity of the rotor. The electrical part is composed of electrical circuits called windings. Three of these windings are connected to the stator. These circuits, also known as *stator windings*, are identical and are labeled by the letters a, b, and c. The remaining windings, which are called *field windings*, are connected to the rotor. In this work, we assume that the rotors of the generators have cylindrical rotor structure. Such generators are mostly used for modeling nuclear and thermal

---

Key words and phrases. Electrical power networks, nonlinear systems, stability.

generation units [5]. In a cylindrical rotor structure, we can assume that there is a single field winding because the aggregated effect of the field windings can be modeled by a single circuit. This single field winding is commonly labeled by the letter f. The model we use in this work can also be used to model wound rotor synchronous generators, if the saliency effects are neglected. Using Kirchoff's Voltage Law, we can obtain the dynamical equation for the flux generated at the phase-a winding, denoted by λ_a, as:

$$\dot{\lambda}_a = -r_a I_a + V_a. \tag{1.1}$$

where r_a is the winding resistance, I_a is the current entering through the positive pole of the winding terminal, and V_a is the voltage at the terminal windings. The equations for the other stator windings and the field winding can be obtained by changing the subscript a in (1.1) to b, c and f, respectively. Since the circuits for the stator windings are identical, we have $r_a = r_b = r_c = r$. In a cylindrical rotor synchronous machine or a wound rotor synchronous machine with negligible saliency, the fluxes $\lambda_{abc} = (\lambda_a, \lambda_b, \lambda_c, \lambda_f)$ and the currents $I_{abc} = (I_a, I_b, I_c, I_f)$ are related by $\lambda_{abc} = \mathbb{L}_{abc} I_{abc}$, where

$$\mathbb{L}_{abc} = \begin{bmatrix} L_s + L_{s0} & -L_{s0} & -L_{s0} & L_{sf}\cos(\theta) \\ -L_{s0} & L_s + L_{s0} & -L_{s0} & L_{sf}\cos\left(\theta - \frac{2\pi}{3}\right) \\ -L_{s0} & -L_{s0} & L_s + L_{s0} & L_{sf}\cos\left(\theta + \frac{2\pi}{3}\right) \\ L_{sf}\cos(\theta) & L_{sf}\cos\left(\theta - \frac{2\pi}{3}\right) & L_{sf}\cos\left(\theta + \frac{2\pi}{3}\right) & L_f \end{bmatrix},$$

a matrix that can be obtained by neglecting the saliency terms of the inductance matrix in [5, page 273]. At steady state, the rotor of the generator has a constant angular speed. Since the rotor has two-poles, this results in sinusoidal currents and voltages in the stator winding when the field winding current I_f is constant. Studying the stability of an equilibrium point is easier that studying the stability of a trajectory. Therefore, we perform a change of coordinates and map phase-abc currents, voltages and fluxes to the corresponding phase-xyz variables using the transformations $\lambda_{xyz} = T_\theta \lambda_{abc}$, $I_{xyz} = T_\theta I_{abc}$, and $V_{xyz} = T_\theta V_{abc}$, where

$$T_\theta = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) & 0 \\ \sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & \sqrt{\frac{3}{2}} \end{bmatrix}.$$

The map T_θ has the inverse $T_\theta^{-1} = T_\theta^T$. The total magnetic energy stored in the stator and the field windings is $H_{\text{magnetic}} = \frac{1}{2}\lambda_{abc}^T \mathbb{L}_{abc}^{-1} \lambda_{abc}$. In the new coordinates, the Hamiltonian becomes independent of θ and we can express the equations of motion for a generator as:

$$\dot{\theta} = \omega, \tag{1.2}$$

$$\dot{\xi} = (\mathcal{J}(\xi) - \mathcal{R}) \frac{\partial H}{\partial \xi} + g \begin{bmatrix} \tau_m \\ V_{xyz} \end{bmatrix}, \tag{1.3}$$

where $\xi = (M\omega, \lambda_{xyz})$, $\mathcal{J}(\xi) = \mathcal{J}^T(\xi)$, $\mathcal{R} = \mathbf{diag}(r, r, r, r_f)$ is a diagonal matrix, and $g = \begin{bmatrix} 1 & 0_{1\times 4} \\ 0_{4\times 1} & T_\theta^{-1} \end{bmatrix}$. In [1], it was shown that for any given steady state voltage V_{xyz}^*, one can choose a torque value τ_m^* such that $\xi^* = \left(M\omega_s, \lambda_{xyz}^*\right)$ is an equilibrium point. Our stability condition will be expressed in terms of steady state currents

I_x^* and I_y^*. These currents can be obtained from ξ^* using $I_{xyz}^* = \mathbb{L}_{xyz}^{-1}\lambda_{xyz}^*$, where $\mathbb{L}_{xyz}^{-1} = T_\theta \mathbb{L}_{abc}^{-1} T_\theta^T$ is a matrix that *does not* depend on θ. The following stability result is proved in [1], [2]:

Theorem 1.1. [1], [2] *Let* ξ^* *be an equilibrium point of a single generator, described by equation* (1.3), *with a constant field winding current* I_f. *When we have* $\tau_m = \tau_m^*$ *and* $V_{xyz} = V_{xyz}^*$, *the equilibrium point* ξ^* *is globally asymptotically stable if*

$$(I_x^*)^2 + (I_y^*)^2 < \frac{4Dr}{(L_s + 2L_{s0})^2}. \tag{1.4}$$

1.2. **Multi-machine power systems.** We consider a power system which is composed of N generators, a transmission grid and loads, where the transmission grid and loads are modeled as port-Hamiltonian systems. The parameters of the i^{th} generator are obtained by adding the subscript i to the parameters defined in the previous section. The following result can be obtained by connecting generators, transmission grid and loads using energy-preserving interconnections.

Theorem 1.2. [1], [2] *Let* ξ_i^* *be an equilibrium point of the* i^{th} *generator, described by equation* (1.3), *with a constant field winding current* $I_{f,i}$. *When we have* $\tau_{m,i} = \tau_{m,i}^*$ *and* $V_{xyz,i} = V_{xyz,i}^*$ *for each* i^{th} *generator, the equilibrium point* $\xi^* = (\xi_1^*, \ldots, \xi_N^*)$, *which is consistent with all the equations that describe the power system, is globally asymptotically stable if*

$$(I_{x,i}^*)^2 + (I_{y,i}^*)^2 < \frac{4D_i r_i}{(L_{s,i} + 2L_{s0,i})^2} \qquad \forall i \in \{1, \ldots, N\}. \tag{1.5}$$

In general, the constants D_i and r_i are small which prevents the applicably of (1.5) for large steady state currents. However, (1.5) suggests that transient stability can be achieved by simply providing mechanical damping to enlarge D_i or electrical damping to enlarge r_i. Existing techniques to increase the mechanical damping include fast valving and to increase electrical damping include breaking resistors and diverse FACTS devices such as unified power flow controllers. Further research is required to assess if we can neglect the dynamics of these mechanisms and simply model their operation by an increase in D_i and r_i.

References

[1] S. Y. Caliskan, P. Tabuada, *Towards a compositional analysis of multi-machine power systems transient stability*, Proceedings of the 52nd IEEE Conference of Decision and Control, pp. 3969–3974, December 10–13 2013, Florence, Italy.

[2] S. Y. Caliskan, P. Tabuada, *Compositional transient stability analysis of multimachine power networks*, IEEE Transactions on Control of Network Systems, Vol. 1, Issue 1, pp. 4-14, March 2014.

[3] F. Shaik, D. Zonetti, R. Ortega, J. M. A. Scherpen, and A. J. van der Schaft, *On port-Hamiltonian modeling of the synchronous generator and ultimate boundedness of its solutions*, 4th IFAC Workshop on Lagrangian and Hamiltonian Methods for Non Linear Control, August 29-31 2012, Bertinoro, Italy.

[4] S. Fiaz, D. Zonetti, R. Ortega, J. M. A. Scherpen, and A. J. van der Schaft, *A port-Hamiltonian approach to power network modeling and analysis*, European Journal of Control, Volume 19, Issue 6, pp. 477–485, December 2013.

[5] A. E. Fitzgerald, C. Kingsley, S. D. Umans, *Electric machinery*, 5th Ed., McGraw-Hill, New York, 1990.