

# Optimal control in a micro gas grid of prosumers using Model Predictive Control

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**Abstract**—This paper studies the optimal control of a micro grid of biogas prosumers equipped with local storage devices. Excess biogas can be upgraded and injected into the low-pressure gas grid or, alternatively, shipped per lorry to be used elsewhere in an effort to create revenue. The aim of the control process is to maximize the prosumers’ profit from the biogas they produce, with a size restriction on the local biogas storage. The problem is solved in centralized and distributed MPC schemes in order to compare their capabilities to control a micro grid. We perform simulations with a realistic average gas usage pattern over a year to study the economic feasibility of local biogas storage.

## I. INTRODUCTION

The drive towards a low-carbon economy coupled with the decreasing production of conventional gas in the Netherlands has motivated the Dutch government to subsidize the production and injection of biogas into the gas grid. The financial stimulation and incentives for renewable and sustainable energy production in The Netherlands are being provided by the Stimuleringsregeling Duurzame Energie (SDE) program. Additionally, the Dutch government has an ambition to meet 20% of the gas consumption with sustainable biomass supply in 2030 [1].

Biogas results from the anaerobic digestion process of biomass. Actors in various agricultural sectors can build digesters to produce biogas from organic waste. They can both produce and consume (bio)gas and are called prosumers. Assuming the (bio)gas is predominantly used for space heating, the consumption level fluctuates over a year due to climate and weather patterns. However, the production is kept at a nearly constant level due to the time and effort associated with establishing the right microbial populations. This leads to a mismatch between supply and demand that prosumers can mitigate by using (local) storage devices.

These developments could change the gas system to include (local) multi-producer multi-consumer markets. Prosumers can sell stored gas to other prosumers and even to consumers on the low pressure gas grid in an effort to create revenue from it. As a result, there will be a strong coupling among prosumers’ actions.

The produced biogas can be used in different ways. Prosumers can use special household appliances that can reliably and safely consume biogas. Alternatively, prosumers can convert the raw biogas into green gas before injecting it into the low pressure domestic grid in order to deliver it to

consumers. The conversion is done to meet the gas quality requirements for the low pressure gas grid. The third scenario is that prosumers convert biogas to green gas, compress, and transport it per lorry for injection in e.g. medium or high pressure gas grids. The combination of these three scenarios is studied in this paper and shown in Fig. 1. The notations in the figure are explained in Section 2.

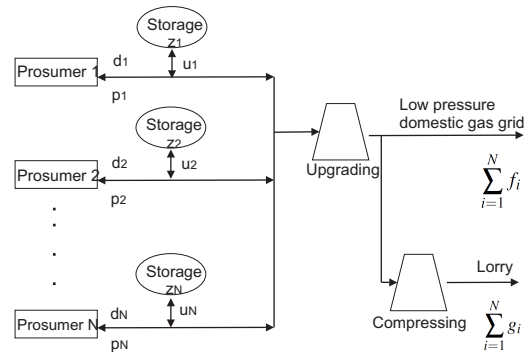


Fig. 1: Physical system illustration of a micro gas grid consisting of  $N$  prosumers and connected to the low and medium pressure gas grids.[2]

Relying on the fact that each prosumer likes to create revenue out of excess generation, but has to spend money on storage installation and connection to the grids, it is therefore of interest to study the value created by a particular size of storage device. In particular, given a size restriction on the storage device we study an optimal control gas flow in the micro gas grid consisting of a number of prosumers. If there is gas excess generation, should we store it locally, offer it to satisfy the consumers’ demand on the low pressure gas grid, or ship it per lorry?

In this paper, given a certain storage size we formulate a profit maximization by maximizing revenue as a result of satisfying the demand on the low pressure gas grid and shipping excess generation per lorry, and minimizing costs consisting of upgrading cost, compression cost, control cost, and storage cost. Regarding the suitable and common-used gas storage technology, i.e. an above-ground low-pressure vessel, we simulate all possible values of the storage size and calculate their corresponding profits in an effort to study the economic feasibility of the storage devices.

With respect to the presence of a storage device, the amount of gas stored/taken from the device, offered to satisfy the consumers’ demand on the low pressure gas grid, and shipped by a lorry per time unit are controllable even though they are subject to several physical constraints, such as a

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minimal and maximal pipeline pressure. It is then natural to solve the profit maximization in the model predictive control (MPC) framework, see [3].

Traditionally, such a grid is controlled by a centralized agent. However, the future gas grid becomes a highly complex system due to the fact that gas will come from multiple sources at multiple locations, flow in two directions, and perhaps have distinct and varying qualities. Additionally, large-scale prosumers embedding in the gas grid may be the case for the future gas value chain. Hence, we are interested in establishing whether the centralized MPC scheme is more practical for controlling parts of the future gas grid in comparison to the distributed MPC scheme. Recently some efforts have been made to solve optimization problems in a distributed fashion for an electricity grid using MPC, see e.g., [4]- [6].

The outline of the paper is as follows. In Section 2 we explain the components of the micro gas grid which is developed from the three use cases for biogas stated earlier. We also provide the performance criterion which is the running profit function in the presence of the storage cost and revenue that may occur as the prosumer satisfies a portion of demand on the low pressure grid and other prosumers' demand. The application of the MPC framework on the performance criteria is given in Section 3. A simulation of yearly operation, the economic feasibility of biogas storage devices, and several case studies to study the impact of system parameters on the performance of centralized and distributed MPC schemes are presented in Section 4. Some concluding remarks and suggestions for future studies are provided in Section 5.

## II. THE SYSTEM DESCRIPTION OF A MICRO GAS GRID

In this section, we briefly explain the model of a micro gas grid consisting of prosumers who generate biogas and operate a decentralized storage device. The micro gas grid has access to the low and medium pressure gas grids in an effort to create revenue from excess generation. The physical system illustration can be found in Fig. 1. The model specifies technical constraints on each prosumer and global restrictions on the gas grid consumption. We then introduce the information sharing within the micro gas grid to achieve a coordination to satisfy the consumers' demand on the low pressure gas grid and to ship excess generation per lorry for injection into the medium pressure gas grid in order to maximize the expected profit over a given time horizon.

### A. Gas imbalance dynamics

Each prosumer  $i = 1, \dots, N$  at time instant  $k \in \mathbb{Z}_+$  produces biogas, denoted by  $p_i(k)$ , and (partly) consumes it, given by  $d_i(k)$ . The production can also be offered to satisfy the consumers' demand on the low pressure gas grid, specified as  $f_i(k)$ , or excess generation can be shipped per lorry, defined

by  $g_i(k)$ . The prosumer may take some gas from the storage device represented by  $u_i(k) \in \mathbb{R}_+$ .

In addition, a prosumer can contribute to the local balancing of biogas by offering or asking biogas to neighboring prosumers. For this, the prosumer requires information about the imbalance in the micro gas grid. To provide the information, we take the communication structure proposed in [4]. We therefore introduce the gas imbalance information for prosumer  $i$  at time unit  $k$ ,  $x_i(k)$ , in the following discrete time state space equation

$$x_i(k+1) = \sum_{j=1}^N A_{ij}x_j(k) + p_i(k) + u_i(k) - d_i(k) - f_i(k) - g_i(k) \quad (1)$$

with  $u_i(k)$ ,  $f_i(k)$ , and  $g_i(k)$  are controllable flows, whereas  $d_i(k)$  and  $p_i(k)$  are simply measurable disturbances.

The imbalance information in (1) is determined by  $A \in \mathbb{R}^{N \times N}$ , specifying the topology and weighting the information in the micro gas grid. According to e.g., [4], there are four restrictions in designing the entries of the  $A$  matrix, given by

- $R_1$ :  $A_{ij} \neq 0$  if and only if there is imbalance information exchanged from prosumer  $j$  to  $i$ .
- $R_2$ :  $A_{ij} \geq 0, i, j = 1, \dots, N$ .
- $R_3$ :  $\sum_j A_{ij} = 1, j = 1, \dots, N$ .
- $R_4$ : The graph of the  $A$  matrix is strongly connected.

Furthermore, a prosumer may adjust the information weight  $A_{ii}$  higher than  $A_{ij}$  when he considers his own imbalance information is more important than the imbalance information which is received from neighbors  $j = 1, \dots, N$  with  $j \neq i$ .

To make sure that gas flows from one station to another, it is necessary to ensure that the pressure difference occurs between two stations. Additionally, the gas pressure of the prosumer's pipeline needs to be maintained within certain acceptable limits in order to adhere to safety requirements. We determine the imbalance level here in a volume unit, i.e.  $Nm^3$  meaning that each  $m^3$  gas is considered at normal condition with the temperature of 273.15 K and the pressure of 1 atm. Boyle's law states that the relationship between gas pressure  $P$  and volume  $V$  is given by  $PV = k$  where  $k$  represents a constant value representative of the pressure and volume of a system. We then put physical constraints on the imbalance level, i.e.

$$x_i^{min} \leq x_i(k) \leq x_i^{max} \quad (2)$$

where  $x_i^{min}$  and  $x_i^{max}$  denote respectively the minimum and maximum imbalance levels, given in  $Nm^3$ .

### B. Storage dynamics

We consider here that each prosumer has a decentralized storage device, such as an above-ground low-pressure vessel. For prosumer  $i$ , let  $z_i(k)$  denote the amount of gas available in the storage device at time  $k \in \mathbb{Z}_+$ . Notice that the amount of gas taken in the storage device has been denoted earlier

by  $u_i(k) \in \mathbb{R}_+$ . One therefore can specify the evolution over time of the available gas in the storage device as follows

$$z_i(k+1) = z_i(k) - u_i(k) \quad (3)$$

with the initial condition  $z_i(0)$ .

Additionally, the storage level is bounded by a restriction on the storage size  $S_i$  which is given by

$$0 \leq z_i(k) \leq S_i. \quad (4)$$

Hence, the amount of gas taken ( $u_i(k) \geq 0$ ) and stored to the storage device is limited by

$$|u_i(k)| \leq \begin{cases} z_k^i & \text{for } u_k^i \geq 0 \\ S^i - z_k^i & \text{otherwise} \end{cases}$$

### C. Global restrictions on the gas grid consumption

Similar to the demand of a prosumer, the gas usage level of a consumer in the low pressure gas grid fluctuates over a year due to the climate and weather patterns. As the low pressure gas grid may consist of a number of gas consumers, the maximum amount of gas that can be fed into the low pressure gas grid is the total demand of consumers in that grid, denoted by  $F_k \in \mathbb{R}_+$ .

Suppose now that each prosumer has access to the low pressure gas grid to satisfy the demand on the low pressure gas grid. With respect to  $F_k$ , the total amount of gas fed by all prosumers to the low pressure gas grid needs to satisfy

$$\sum_{i=1}^N f_i(k) \leq F_k \quad (5)$$

As is mentioned before, another possibility is to ship excess generation per lorry to the medium pressure gas grid. It however needs to be upgraded to green gas and compressed. We assume that the prosumers in a micro gas grid have one central location to upgrade biogas and compress it before shipping it by a lorry. Hence, a physical constraint with respect to the amount of gas shipped per lorry is given by

$$\sum_{i=1}^N g_i(k) = G_r(k) \cdot L_r \quad (6)$$

where  $L_r$  is the capacity of the lorry and

$$G_r(k) = \begin{cases} \mathbb{Z}_+ & \text{for filling lorry} \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

### D. Objective function

The objective of each prosumer  $i$  is to maximize profit by maximizing expected revenue as a result of satisfying the consumers' demand on the low pressure gas grid and shipping excess generation per lorry, and minimizing costs involved. Hence, our goal is to find appropriate control inputs  $u_i(k)$ ,  $f_i(k)$ , and  $g_i(k)$  resulting in maximum revenue and minimum costs involved, given the influence from neighboring prosumers and physical grid constraints.

Given a particular storage size  $S_i$ , the amount of gas taken from the storage device  $u_i(k)$ , the amount of gas injected into the low pressure gas grid  $f_i(k)$ , the amount of gas shipped by lorry  $g_i(k)$ , and the storage level  $z_i(k)$ , we associate total cost  $C_i(u_i(k), f_i(k), g_i(k), z_i(k), S_i)$  for a prosumer  $i$  at time  $k$  given by

$$C_i(u_i(k), f_i(k), g_i(k), z_i(k), S_i) = c_{upi} \cdot (f_i(k) + g_i(k)) + c_{compi} \cdot g_i(k) + c_{ci} \cdot u_i(k) + c_{oppi} \cdot z_i(k) + C_{si} \cdot S_i \quad (8)$$

where  $c_{upi}$ ,  $c_{compi}$ ,  $c_{ci}$ ,  $c_{oppi}$ , and  $C_{si}$  correspond respectively to the cost of upgrading biogas to green gas, the cost of compressing the green gas, the cost of control effort, opportunity cost of stored gas, and the storage cost of prosumer  $i$ .

Related to the amount of gas injected into the low pressure gas grid  $f_i(k)$  and the amount of gas shipped by lorry  $g_i(k)$ , we associate the revenue  $R_i(f_i(k), g_i(k))$  denoted by

$$R_i(f_i(k), g_i(k)) = r_{fi} \cdot f_i(k) + r_{gi} \cdot g_i(k) \quad (9)$$

where  $r_{fi}$  and  $r_{gi}$  represent respectively the revenue per  $Nm^3$  of gas injected to satisfy the consumers' demand on the low pressure gas grid and the revenue of gas shipped per lorry. Thus, a profit maximization model of prosumer  $i$  can be expressed by

$$\max_{u_i(k), f_i(k), g_i(k)} \sum_{k \in \mathbb{Z}_+} R_i(f_i(k), g_i(k)) - C_i(f_i(k), g_i(k), u_i(k), z_i(k), S_i) \quad (10)$$

subject to (1)-(7).

*Remark 1* As we have a binary condition to indicate the number of lorries defined in (7), the problem (10) turns out to be a mixed integer program. It leads then to a non-convex problem formulation. A framework for controlling systems which are described by binary logic rules has been treated in [8].

## III. MPC FOR THE MICRO GAS GRID

To maximize the profit over a prediction horizon while taking into account technical constraints of each prosumer and global restrictions on the gas grid consumption, we formulate the problem in MPC framework. We denote by  $x_i(k+j|k)$  and  $z_i(k+j|k)$ , with  $j \geq 0$  the internal time, the imbalance and storage levels at time step  $k+j$  predicted at time  $k$ , respectively.

Given a prediction horizon  $L$  and initial conditions of the imbalance level  $x_i(k)$  and inventory level  $z_i(k)$  at every time instant  $k$ , the MPC algorithm calculates the optimal control sequences. Next, only the first control input sequences are implemented in state equations (1) and (3). Subsequently, the horizon is shifted and the new states are estimated at the next sampling time to solve a new optimization problem using the new information.

In the following subsections, we derive the centralized MPC optimization problem to be a base problem and then

provide the distributed MPC problem by decomposing the centralized MPC optimization problem into separate criteria for each prosumer.

### A. Centralized MPC

As has been discussed in [9], the centralized MPC controller is based on iterative solutions of the finite horizon prediction of (10) given by

$$\begin{aligned} & \max_{u,f,g} \sum_{l=0}^L \sum_{k \in \mathbb{Z}_+} R_i(f_i(k+l|k), g_i(k+l|k)) \\ & - C_i(f_i(k+l|k), g_i(k+l|k), u_i(k+l|k), z_i(k+l|k)S) \end{aligned} \quad (11)$$

subject to

$$\begin{aligned} x_i(k+l+1|k) &= \sum_{j=1}^N A_{ij}x_j(k+l|k) + p_i(k+l|k) \\ &+ u_i(k+l|k) - d_i(k+l|k) \\ &- f_i(k+l|k) - g_i(k+l|k) \end{aligned} \quad (12)$$

$$z_i(k+l+1|k) = z_i(k+l|k) - u_i(k+l|k) \quad (13)$$

$$\sum_i f_i(k+l|k) \leq F(k+l|k) \quad (14)$$

$$\sum_i g_i(k+l|k) = G_r(k+l|k) \cdot L_r \quad (15)$$

$$G_r(k+l|k) = \begin{cases} \mathbb{Z}_+ & \text{for filling lorry} \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

$$x_i^{\min} \leq x_i(k+l|k) \leq x_i^{\max} \quad (16)$$

$$0 \leq z_i(k+l|k) \leq S_i \quad (17)$$

$$x_i(k|k) = x_i(k) \quad (18)$$

$$z_i(k|k) = z_i(k) \quad (19)$$

$$x_i(k+l|k) \in \mathcal{X}_i, z_i(k+l|k) \in \mathcal{Z}_i \quad (20)$$

$$u_i(k+l|k) \in \mathcal{U}_i, d_i(k+l|k) \in \mathcal{D}_i \quad (21)$$

$$f_i(k+l|k) \in \mathcal{F}_i, g_i(k+l|k) \in \mathcal{G}_i \quad (22)$$

where  $\mathcal{X}_i, \mathcal{Z}_i, \mathcal{U}_i, \mathcal{D}_i, \mathcal{F}_i, \mathcal{G}_i$  are convex sets, see [7].

As is noted in [4], it is widely agreed that the centralized scheme leads to the computational complexity when the grid consists of a large number of prosumers. We therefore need a control strategy that scales well with such a large grid. Hence, in the following subsection we set an MPC optimization model in a distributed fashion.

### B. Distributed MPC

In order to build a distributed MPC scheme, the centralized MPC optimization model is decomposed using dual decomposition and sub-gradient iterations. We follow the notation introduced in [9] to provide the decoupled state equations

$$\begin{aligned} x_i(k+l+1|k) &= A_{ii}x_i(k+l|k) + v_i(k+l|k) \\ &+ p_i(k+l|k) + u_i(k+l|k) - d_i(k+l|k) \\ &- f_i(k+l|k) - g_i(k+l|k) \end{aligned} \quad (23)$$

with the additional constraints

$$v_i(k+l|k) = \sum_{j \neq i} A_{ij}x_j(k+l|k) \quad (24)$$

having the interpretation over the expected influence of other prosumers on the evolution of states  $x_i(k+l|k)$ .

To simplify the notation of (10), we further rewrite it into

$$\max_{u,f,g} \sum_{l=0}^L \sum_k R_i(k+l|k) - C_i(k+l|k) \quad (25)$$

In order to solve the optimization problem along with the additional constraints stated in (24), one can incorporate the constraints into (25) using standard Lagrangian relaxation. To do so, we introduce the adjoint variables  $\lambda_i(k+l|k)$  corresponding to the constraints (24). One can then obtain the following Lagrangian function

$$\begin{aligned} & \min_{\lambda} \max_{u,f,g} \sum_{l=0}^L \sum_k R_i(k+l|k) - C_i(k+l|k) \\ & + \lambda_i(k+l|k)^T \left( v_i(k+l|k) - \sum_{j \neq i} A_{ij}x_j(k+l|k) \right) \end{aligned} \quad (26)$$

or

$$\begin{aligned} & = \min_{\lambda} \max_{u,f,g} \sum_{l=0}^L \sum_k \sum_i R_i(k+l|k) - C_i(k+l|k) \\ & \quad + \lambda_i(k+l|k)^T v_i(k+l|k) \\ & \quad - x_i^T(k+l|k) \sum_{j \neq i} A_{ji}^T \lambda_j(k+l|k) \end{aligned} \quad (27)$$

The minimum in (27) is attained iff the gradient of adjoint variables  $\lambda$  is zero [9]. However, finding the optimal adjoint variables requires coordination. In [9], optimal adjoint variables are found as the limits of a gradient iteration in order to compute the optimization problem locally. For this, the adjoint variables are updated distributively by

$$\begin{aligned} \lambda_i^{r+1}(k+l|k) &= \lambda_i^r(k+l|k) \\ &+ \gamma_i(k)^r v_i^r(k+l|k) \\ &- \gamma_i(k)^r \sum_{j \neq i} A_{ij} x_j^r(k+l|k) \end{aligned} \quad (28)$$

where  $r$  refers to the iteration number and the gradient-steps  $\gamma_i(k)^r$  are chosen to converge into a saddle point, see [9]. At each iteration the centralized computer collects the coupling constraints  $v_i^r(k+l|k)$  and imbalance from prosumers  $x_j^r(k+l|k)$ . Next, the updated adjoint variables are calculated and sent back to the prosumers. The algorithm is terminated when the update of adjoint variables stays within a bound  $\xi$ .

*Remark 2* In the cost function formulated in (27),  $\lambda_i(k+l|k)^T v_i(k+l|k)$  can be interpreted as some kind of expected revenue that prosumer  $i$  receives from others, whereas  $x_i^T(k+l|k)$

$l|k) \sum_{j \neq i} A_{ji}^T p_j(k+l|k)$  can be seen as costs that prosumer  $i$  needs to make in satisfying his unfulfilled demand  $x_i(k+l|k)$ . Hence, one can further consider the adjoint variables  $\lambda_i(k+l|k)$  as shadow prices. [10]

Notice that in (14) there is a maximum amount of gas that can be fed in the low pressure gas grid. The prosumers only know such a global restriction of total gas demand in the low pressure gas grid, but they do not know to which consumers they need to transfer their excess generation. To decompose the global constraint, we formulate for each prosumer

$$f_i(k+l|k) \leq \frac{1}{N} F(k+l|k) \quad (29)$$

here  $N$  refers to the number of prosumers in the micro gas grid. Notice that this decomposition is fair if the prosumers are identical in the biogas production and consumption levels. When we consider heterogeneous sizes of prosumers, diverse weighting factors can be applied to decompose the global constraint.

Another challenge to make the original optimization problem fully distributed is to decompose the global constraint stated in (15). The lorry capacity is much larger than the production level of prosumer  $i$  at time instant  $k$ . Hence, all prosumers compress the upgraded biogas at a central location and ship it per lorry to the medium pressure gas grid. It is therefore not possible to create a local constraint from this particular global constraint. We therefore assume that there will be always a lorry that can be continuously filled and transported to the medium or high pressure gas grid.

#### IV. SIMULATION RESULTS

A micro gas grid consisting of 15 biogas prosumers and 100 consumers on the low pressure gas grid is here constructed. Each consumer and prosumer operate a household. The average gas usage of a household over a year is provided by DNV KEMA, The Netherlands, and shown in Fig. 2. It is a base pattern to generate the consumption of distinct prosumers and consumers. We multiply the base pattern with a random variable between 0.75 and 1.25 and shift it randomly between -2 and +2 hours to provide different gas usage patterns. The production level of each prosumer is set at a constant level which is between  $22.5 \text{ Nm}^3$  and  $27.5 \text{ Nm}^3$  per quarter of an hour. The imbalance information topology is depicted by

$$A = \begin{pmatrix} 0.6 & 0.2 & 0 & \cdots & 0 & 0.2 \\ 0.2 & 0.6 & 0.2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0.2 & 0.6 & 0.2 \\ 0.2 & 0 & \cdots & 0 & 0.2 & 0.6 \end{pmatrix}$$

In the simulation, we use MATLAB 2012b to implement the model, YALMIP to formulate the MPC optimization problem, and Gurobi 5.5 to solve the problem. To provide the results in this section, the prediction and control horizons are set to be 40 time steps and the time step is 15 minutes.

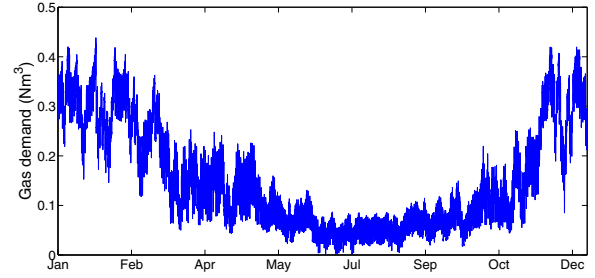


Fig. 2: The average gas usage of a household for the whole year.[11]

The parameters which are related to the cost and revenue mentioned in (8) and (9) are given in Table 1. See [2] for the comprehensive overview of the parameters.

TABLE I: The parameter values involved in the objective function [2]

Parameter	Value (in euro/ $\text{Nm}^3$ )
$r_{fi}$	0.209
$r_{gi}$	0.161
$c_{upi}$	0.083
$c_{compi}$	0.033
$c_{oppi}$	$2.763e-5$
$C_{si}$	0.001

#### A. Demonstration of yearly operation

Now we study the prosumers' control strategy when their overall gas demands are both high and low. Hence, the prosumers' and consumers' gas usages over a year are here simulated. Each prosumer has a storage device of  $1500 \text{ Nm}^3$  with zero initial condition.

Fig. 3 displays the storage level of a prosumer during the whole year. It gives more or less a similar storage level for other prosumers in the micro gas grid as they are only slightly different in the production and consumption levels over a year. As can be seen in the figure, the storage device is totally empty for 64% of the year. The highest level of storage device which is at 67% of the storage size is between May and September, i.e. summer time.

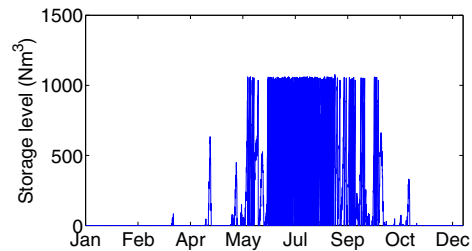


Fig. 3: The storage level of a prosumer,  $z_i$ , over a year. The resolution is 15 minutes. When making use of the storage device of  $1500 \text{ Nm}^3$ , the highest storage level is at 67% of the storage capacity.

The total amount of gas which is injected into the low pressure gas grid in comparison with the total demand on that grid is portrayed in Fig. 4. Note that the injected gas is green gas which is upgraded from raw biogas. There is clear demonstration in the figure that the prosumers are only able to completely satisfy the total demand on the low pressure gas grid during the low demand period, which is during May - September. In that period, it is financially more rewarding to ship it per lorry than to store excess generation to the storage device. Hence, one can find in Fig. 3 that the storage device are not full during those five months. In addition, one can see in Fig. 5 that the lorry of  $10.000 Nm^3$  is always full within May - September.

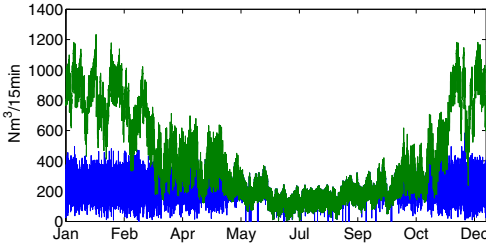


Fig. 4: The total amount of gas injected into the low pressure gas grid (blue line) versus the total demand on that grid (green line). The prosumers can only completely fulfill the consumers' demand on the low pressure gas grid during May - September.

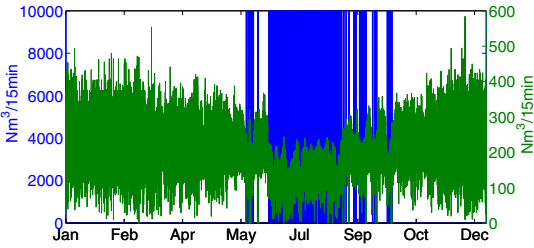


Fig. 5: The total amount of gas injected into the low pressure gas grid (green line) versus the total amount of gas shipped per a lorry (blue line). The lorry size is  $10.000 Nm^3$ .

### B. Storage sizing

Today, the suitable and common used storage technology in the low pressure gas grid is above-ground low-pressure vessel. It costs approximately at  $\text{€}306\text{-}\text{€}374/\text{Nm}^3$  to install. The storage size is at a range of  $100\text{-}2500 Nm^3$ . We here simulate all possible values of the storage size to study their impact on the prosumers' total profit over a year.

Fig. 6a shows how the total profit of 15 prosumers for the whole year changes with respect to the storage size, when the prosumers call a lorry with the capacity of  $10.000 Nm^3$ . The figure explicitly tells that the storage cost dominates the total cost. As a result, the profit decreases linearly with the storage size. Notice that the minimum storage size in Fig. 6a is at  $1111 Nm^3$ . It is due to the fact that 15 prosumers

need to fill the lorry at each time step with green gas. Hence, the minimum storage size is given by  $\frac{10.000Nm^3/15}{0.6}$ , with the factor of 0.6 is due to biogas consists of 60% green gas and 40%  $CO_2$ . The solution gets infeasible when the storage size is smaller than the minimum size.

Moreover, we are interested in analyzing the case when the storage cost becomes much cheaper than today. To do so, we take the storage cost 100 times lower than today-storage cost. The simulation result is presented in Fig. 6b. It clearly shows that the profit will not linearly decrease along with the increase of storage size, as the storage cost will be no longer dominating the total cost. Effectively, the profit seems to be independent to the storage investment in this case.

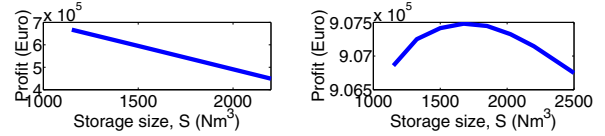


Fig. 6: The left figure represents total profit of 15 prosumers for the whole year changes with respect to the storage size, with today-storage cost. The minimum storage size of a prosumer is  $1111 Nm^3$ . The right figure shows the total profit of 15 prosumers for the whole year, with the storage cost is 100 times lower than today-storage cost.

### C. Comparison between the centralized and distributed MPC

We here compare the result of calculating the performance criteria which are obtained from the centralized and distributed MPC schemes. To provide the comparison, we simulate a day in the middle of May which represents a high demand consumption relative to the production level followed by low level and further end the day up with the high demand. It turns out that these two schemes give similar results in calculating the total profit of a prosumer for the whole day, i.e.  $\text{€}301.33$  and  $\text{€}301.27$  for centralized MPC and distributed MPC schemes, respectively.

We aim as well in this subsection at comparing the centralized and distributed MPC schemes from a computational point of view. We set the gradient step in (28) to be  $\gamma_i(k)^r = \frac{0.8}{\sqrt{r}}$ , where  $r$  is the iteration number. The convergence rate of the distributed MPC algorithm is fairly slow. Following [9], we counter the undesirable property by providing a stopping criterion. The algorithm is terminated when distance between previous and current adjoint variables stays within  $\xi = 3$  for each prosumer  $i = 1, \dots, N$  and  $l = 0, \dots, L$ .

It is confirmed in Table 1 that the distributed MPC scheme scales better than the centralized MPC scheme when the micro gas grid consists of larger numbers of prosumers. A similar computation time occurs when the distributed and centralized MPC schemes are employed for a micro gas grid consisting of 5 prosumers. However, the computation time of the centralized MPC scheme becomes approximately twice longer than the distributed MPC scheme when the micro gas grid consists of 15 and 30 prosumers. We simulate as

well the micro gas grid consisting of 250 prosumers with the production level in the range of 0-55 Nm<sup>3</sup> per 15 minutes. As we make use of the university computer with the memory of 32 GB RAM for simulation, more than 250 prosumers is not computable anymore for the centralized case, where it is for the distributed case.

TABLE II: The scalability of centralized and distributed implementations

Implementation	5 prosumers	15 prosumers	30 prosumers
Centralized	81.2 sec	249.7 sec	611.6 sec
Distributed	88.3 sec	109.6 sec	331.1 sec

## V. CONCLUSION

By connecting the micro gas grid which consists of biogas prosumers to the low pressure gas grid and inclusion of a lorry, the prosumers can get rid of their excess generation and create revenue from it. However, there is a strong coupling among prosumers' actions due to their gas imbalance information and global restrictions of the consumption on the low pressure gas grid and transportation of the excess generation per lorry.

To determine the profit, the storage cost plays the most important role among other parameter values involved. With today-storage cost, the profit linearly decreases along with the increase of storage size. Based on the simulation result, the economically optimal storage size is at 1111 Nm<sup>3</sup>. When the storage cost becomes much cheaper in the future, the profit appears to be almost independent from the local storage size.

The centralized MPC scheme works well to deal with the physical constraints on the imbalance and storage levels, influence from neighboring prosumers, and the binary condition (7) leading to a non-convex problem formulation. The distributed MPC scheme can be applied as well with some adjustment on the global restrictions which are stated in (14) and (15). Based on the simulation results in Section 4, the distributed MPC scheme scales better than the centralized MPC scheme. Nevertheless, the centralized one is still capable to control the micro gas grid consisting of 250 prosumers and more realistic with respect to the global constraints.

In this study, each prosumer exchanges his imbalance information dynamically and synchronously to connected prosumers. In other words, the connected prosumers are assumed to have a common clock to update the information. Future work may focus on providing an algorithm for the asynchronous updates of the information. Another further study may be conducted to include a possibility to make use of produced gas for generating electricity and heat in an effort to satisfy local electricity demand and heat demand. The coupling among these three energies can be promisingly done through a micro combined heat and power ( $\mu$ -CHP) [12].

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