21st International Symposium on
Mathematical Theory of Networks and Systems
July 7-11, 2014. Groningen, The Netherlands

# Synchronization and Transient Stability in Multi-machine Power System Through Edge Control Using TCSC

S. Kulkarni[1], S. Wagh[2] and N. Singh[3]

***Abstract*— Present paper proposes a method to provide necessary series compensation by formulating transient stability problem as a synchronization problem providing an edge control to keep the line flows within permissible limits. With the aim to satisfy relative rotor angle criteria in addition to line flow control having multiple constraints on a set of state variables as well as on control variables makes it necessary to call for an efficient optimization tool. Knowing the strengths and effectiveness of convex optimization we propose application of the same in finding necessary series compensation using TCSC to address this multidimensional problem in remarkably shortest possible time which facilitates on-line synchronization. In doing so, post fault conditions are mapped to pre-fault healthy conditions so as to guarantee stable and secure power flow even after credible line contingency without shedding any load (i.e. maintaining demand supply balance), which if not followed may lead to cascade tripping.**

***Index Terms*— Convex optimization, edge control, synchronization, transient stability**

## I. INTRODUCTION

The ever increasing power demand and limitations on additional infrastructure due to environmental and economical constraints, stress the existing power system, forcing it to operate at the edge of permissible limit, which directly affects the Transient Stability Margin (TSM). Being susceptible, survival of such a stressed power system becomes difficult under a large disturbance such as 3-phase short circuit fault, or a sudden outage of a critical transmission line which may result in catastrophic failure of the grid. In addition, for such a geographically wide-spread complex network the type, location, and time of occurrence of a fault is never known apriory and hence it is difficult to control the collapse if such a system is subjected to even a small disturbance in critical conditions.

A power system comprising of generation, transmission and distribution is topologically complex in itself. In addition, the control system required to maintain secure and uninterrupted service and the associated trading system contribute further increase in complexity. The most critical issue in power system is that, generation and consumption is achieved simultaneously to match the supply and instantaneous demand as electrical energy cannot be stored. Any unbalance in demand and supply causes instability in the system which may lead to power outages of different scales. Moreover, as energy markets are deregulated, energy management systems are more complicated.

[1]S. Kulkarni, Research scholar, Electrical Engineering, VJTI, Mumbai University, India samidhaskulkarni@yahoo.com

[2]S. Wagh and [3] N. Singh, Faculty in Electrical Engineering, VJTI, Mumbai University, India

Finding accurate control law for complex networks like a power grid is very difficult due to its large size and strong interconnections. Various methods for solving transient stability problems include time domain simulations (TDS) [1], Equal Area Criterion (EAC), direct stability methods [2], such as Transient Energy Function (TEF), Lyapunov methods [3]. As a matter of fact, synchronization is not the only issue for maintaining stability and integrity of the system as observed in the grid disturbance in the NEW grid.

On $30^{th} July, 2012$ there was a grid disturbance in the Northern Eastern Western (NEW) Indian grid that led to the separation of the Northern Region (NR) grid from the rest of the NEW grid and eventually NR system collapsed [4]. This was followed by another grid disturbance on $31^{st} July, 2012$ which resulted in collapse of Northern, Eastern and North-Eastern regional grids. The system was weakened by multiple outages of transmission lines in the WR-NR interface. The overdrawal by some of the NR utilities, utilizing Unscheduled Interchange (UI), contributed to high loading on Bina-Gwaliar-Agra tie line.

As it can be observed in the case above, despite the fact that demand-supply balance is maintained with sufficient generation to fulfil load demand, and no fault actually observed in the system, because of lack of available capacity in transmission line, the grid lost its synchrony leading to complete system collapse.

Critical analysis of this event has forced power system operators not only to focus on traditional criteria of rotor angle stability but also on line loadability. Motivated from this issue, present paper proposes an algorithm to address both these essential conditions to guarantee secure and stable post-fault operation.

In order to assess, control, and improve tie line capacity ensuring transient stability, the most appropriate solution is to implement series compensation using TCSC in critical transmission lines which helps in normal operating conditions as well as in post-fault scenario [5]. For line flow control, TCSC placed in transmission line provides necessary series compensation necessary for regaining post-transient stability.

For Single Machine Infinite Bus (SMIB) system with TCSC as a controller [6], [7], it is possible to design a linear as well as non linear controller. A constrained Model Predictive Control (MPC) with a TCSC controller has been successfully designed for an SMIB using a detailed Differential Algebraic Equations (DAE) model. However, for Multi-Machine Power System (MMPS), design of a controller using TCSC is difficult, because TCSC reactance becomes a non-seperable element of a higher order dense admittance matrix,

ISBN: 978-90-367-6321-9

and is not easily available as a control variable $u$.

In view of this, present paper proposes a method of edge-weight control by properly adjusting line flows, thereby avoiding overloading of transmission lines and regaining synchronization in post-fault conditions, which is confirmed by Condition for Synchronization (CfS) [8], [9] being satisfied. This is in line with adjustments of coupling weights in Kuramoto model of coupled oscillators so as to achieve phase and/or frequency synchronization among oscillators. For a power network, phase cohesiveness [10] among generators is achieved via edge-weight control, so that frequency synchronization in the network is assured.

To address a two-fold transient stability issue of maintaining synchronization and assuring all line flows within acceptable limits poses a multidimensional problem calling for an optimization tool to handle transient stability problem as a special case of synchronization problem. The key contribution of the proposed idea is forming an optimization problem having objective function optimizing line flows with condition of rotor angle stability inherent in it. With the aim to satisfy relative rotor angle criteria in addition to line flow control having multiple constraints on a set of state variables as well as on control variables makes it necessary to call for an efficient optimization tool. Knowing the strengths and effectiveness of convex optimization [11], present paper proposes its application for finding necessary series compensation using TCSC to address this multidimensional problem in remarkably shortest possible time which facilitates on-line synchronization. In the process, post fault conditions are mapped to pre-fault healthy conditions so as to guarantee stable and secure power flow even after credible line contingency without shedding any load (i.e. maintaining demand supply balance), which if not followed may lead to cascade tripping.

The paper is organized as follows:
Mathematical preliminaries regarding necessity of Dini derivatives, convex functions, convex hull and elementary concepts in graph theory are explained in brief in Section II. Transient stability problem as a synchronization problem along with singular perturbation as a model reduction technique is discussed in section III. A systematic development of objective function and constraints for formulation of a convex optimization problem is described in Section IV. Starting with the need of convex optimization, an optimization problem is formulated for a representative 4-generator, 12-bus power network. Section VI supports the theory developed in the paper with MATLAB simulation results and analysis followed by conclusions in Section VII.

## II. Mathematical preliminaries

### A. Dini derivative

If a differentiable function $\psi : R^n \rightarrow R$ and a derivable trajectory $x(t)$ are given then it is possible to consider a composite function

$$\dot{\Psi}(x(t)) = \bigtriangledown \Psi(x(t))\dot{x}(t) \tag{1}$$

If a function is not continuously differentiable but only piecewise differentiable, then gradient is defined only in the interior. In such cases a gradient is replaced by sub-gradient and to account for discontinuities, instead of finding a regular derivative, a directional derivative is found referred to as Dini derivative of a function $\psi$ at $t$ as:

$$D^{+}\psi(x(t)) = \lim_{h \rightarrow 0^{+}} \sup \frac{\psi(x(t+h)) - \psi(x(t))}{h} \tag{2}$$

which is always defined if $\Psi$ is convex and $x(t)$ is a regular solution of differential equation and limit is replaced by limit supremum.

It is required to execute upper right Dini derivative [12] because main focus is on causal systems evolving in positive direction and also, future trend of evolution of the system is important. In addition, most of the analysis is carried out in worst case setting, which can guarantee reliable behaviour in any other scenario.

### B. Convex set

A set $S \in R^n$ is said to be convex if for all $x_1 \in S$ and $x_2 \in S$, $\alpha x_1 + (1-\alpha)x_2 \in S$ for all $0 < \alpha < 1$. The point, $x = \alpha x_1 + (1-\alpha)x_2$, with $0 < \alpha < 1$ is called convex combination of the pair $x_1$ and $x_2$. The set of all such points is the segment connecting $x_1$ and $x_2$. A set is convex if it includes all such segments connecting all the pairs of its points. Given a set $S$, a convex hull is the smallest [9] convex set containing $S$.

### C. Positive invariance

A set $S \subseteq O$ is said to be positively invariant w. r. t. $\dot{x} = f(x(t))$, if every solution of it with initial condition $x(0) \in S$ is globally defined and $x(t) \in S$ for $t > 0$. According to [13], [14], if every agent always moves toward the relative interior of the convex hull of the set of neighbour agents at each step, state agreement will be achieved.

### D. Graph theory

A graph G consists of n-dimensional vertex set V, and an e-dimensional edge set E, where an edge is an unordered pair of distinct vertices in G. A vertex $v_i$ is said to be a neighbour of another vertex $v_j$ if they are connected by an edge. A graph is said to be strongly connected if any two pairs of vertices can be connected by a walk [15], [16]. A weighted graph is a graph (V,E) together with a map $\varphi : E \rightarrow R$ that assigns a real number $w_{ij} = \varphi(e_{ij})$ is called a weight to an edge $e_{ij} = (v_i, v_j) \in E$. The set of all weights associated with E is denoted by W. A weighed graph can be represented as a triplet $G = (V, E, W)$.

### E. Phase cohesiveness

With reference to power system if every pair of connected generators has a phase distance smaller than some angle $\gamma \in [0, \pi/2[$, that is, $|\delta_i - \delta_j| \leq \gamma$ for every edge $(i, j) \in E$, then the generators are said to be phase cohesive.

## III. Transient stability as a synchronization problem

For a power network, during pre-fault, all the generators are in synchronism, giving perfect balance in demand and supply. However, power network being geographically wide spread, the type, location, and time of occurrence of a fault is never known apriory. If the fault is cleared after Critical Clearing Time (CCT), the system stability is lost pushing the system in unstable region. The region of attraction in such a situation can be increased by using various controllers such as Static Var Compensator (SVC), STATic COMpensator (STATCOM) etc to make the system stable. The major limitation of such controllers, is insufficient information about line loading in post-disturbed condition. To be specific, although, the generator demand-supply is balanced, the rescheduled line flows may reach their power transfer capability limits because of credible contingency. In such a stressed condition of power network, TCSC is the only best solution which provides necessary series compensation

- to regain synchronization and
- to reschedule power flows.

Florian Dorfler and Francesco Bullo presented coupled oscillator approach to the problem of synchronization and transient stability in the power network [9] and procedure to obtain a purely algebraic conditions sufficient for ensuring the same. The idea is briefly explained here for completeness of the theory. For detailed proofs and analysis the interested reader may refer [9].

To solve a multi-dimensional problem discussed above, an efficient optimization tool is required. Hence, a constrained optimization problem is formed where the synchronization criteria is formulated in terms of line flows to assure both the issues- rotor angle stability and proper line loading. The proposed algorithm ensures maintaining power balance without load-shedding in spite of a contingency. This fact is validated by checking demand-supply balance in pre-fault and post-fault, which is found to remain constant after re-distribution of the line flows. TCSC provides an edge-weight control wherein, change in impedance in the transmission line in which TCSC is placed causes re-distribution of line flows in the entire network due to strong interconnections.

First step towards the solution of present problem is, proper modelling of the system. If model of the system is accurate but too complex, it becomes unsuitable for analysis, optimization, or control design. In such a situation model reduction is performed so as to capture important dynamics of the system and ignoring comparatively less important dynamics.

Singular perturbation [9], [17] is a model reduction technique in which system equations are separated in two parts, fast and slow modes. Then order of the model is reduced first by ignoring the fast modes and then quality of approximation is improved by reintroducing their effects as boundary layer corrections. Singular perturbation proves to be the best model reduction technique for power system because, power system dynamic analysis encompasses a wide time span ranging from lightening phenomenon in microseconds to Automatic Generation Control (AGC) taking a few minutes.

In a power network comprising of $n$ generator nodes, and $m$ load nodes, for transient stability assessment, rotor dynamics of generators need to be monitored, which can be represented by swing equations as:

$$\dot{\delta}_i = \omega_i - \omega_{ref} \tag{3}$$

and

$$M_i \dot{\omega}_i = P_{mi} - P_{ei} - D_i(\omega_i - \omega_{ref}) \tag{4}$$

Where, $P_{mi}$ is mechanical power input and $P_{ei}$ is electrical power output, which is computed as:

$$P_{ei} = \sum_{j=1}^{n} |E_i||E_j|[\Re(Y_{ij})\cos(\delta_i - \delta_j) + \Im(Y_{ij})\sin(\delta_i - \delta_j)] \tag{5}$$

For $j = i$, $P_{ei}$ becomes $E_i^2 G_{ii}$ which modifies (4) as:

$$M_i \ddot{\delta}_i = P_{mi} - E_i^2 G_{ii} - D_i \dot{\delta}_i - P_{ei} \tag{6}$$

for $i \in \{1, \cdots, n\},\ i \neq j$.

Defining effective power input to generator $i$ as $(P_{mi} - E_i^2 G_{ii})$ and coupling weights represented by power transferred between generator $i$ and $j$ as $P_{ij} = |E_i||E_j||Y_{ij}|$ with $P_{ii} = 0$, (6) is rewritten as:

$$M_i \ddot{\delta}_i = -D_i \dot{\delta}_i + \omega_i - \sum_{j=1}^{n} P_{ij} \sin(\delta_i - \delta_j + \varphi_{ij}) \tag{7}$$

where, phase shift $\varphi_{ij} = \arctan(\Re(Y_{ij})/\Im(Y_{ij})) \in [0, \pi/2[$ depicting energy loss due to transfer conductances, appearing in case of lossy network. For lossless network $\varphi_{ij} = 0$. For small perturbation parameter $\epsilon = M_{min}/D_{max}$, singular perturbation can be applied to seperate slow and fast dynamics of the system. Fast eigen value corresponding to frequency damping can be neglected in the long-term phase dynamics, reducing (7) to (8):

$$D_i \dot{\delta}_i = \omega_i - \sum_{j=1}^{n} P_{ij} \sin(\delta_i - \delta_j + \varphi_{ij}) \tag{8}$$

which captures the power system dynamics sufficiently well during first swing. Equation (8) is equivalent to Kuramoto Mean Field Model (KMFM) of N phase-coupled oscillators modified to accommodate non-uniformities due to, damping constants, and non-trivial transfer conductances [19], to the original uniform model represented by (9) as:

$$\dot{\theta}_i = \omega_i - \sum_{j=1}^{N} K_{ij} \sin(\theta_j - \theta_i), \tag{9}$$

for $i = 1, \cdots, N$, where $\theta_i$ represents phase of oscillator which is analogous to rotor angle $\delta_i$ in a power network.

In KMFM, each oscillator oscillates independently at its natural frequency while coupling $K_{ij}$ tends to synchronize it to all others. When the coupling is weak, oscillators run incoherently whereas for coupling strength beyond a certain threshold, oscillators are synchronized with each other.

KMFM proves to be the most appropriate model for MMPS as it neither requires SMIB structure, nor uniform damping for the generators. On the contrary, it works fine even in the presence of transfer conductances. For a power network, all-to-all connectivity requirement of KMFM is achieved using Kron reduction [8], [9], [18], [20] by modelling loads as passive admittances and merging them with generator nodes. The original electrical network is replaced with a simpler network (in the sense of number of nodes) that still provides same relationships between voltages and currents at the terminals of the synchronous generators in the process of Kron reduction. Line flows among the generators play the role of coupling by adjusting which frequency synchronization can be achieved in the post-disturbed condition using necessary series compensation in shortest possible time without requiring any load-shedding, which is the key contribution of present work.

As discussed in [9], the set $S^1$ denotes the unit circle and state of generator (i.e. rotor angle) is a point $\delta \in S^1$. An arc is a connected subset of $S^1$. The geodesic distance $|\delta_1 - \delta_2|$ between two angles $\delta_1, \delta_2 \in S^1$ is the minimum of counter-clockwise and clockwise arc length connecting $\delta_1$ and $\delta_2$. The $n$-dimensional torus is the product set $T^n = S^1 \times \cdots \times S^1$ of $n$ unit circles. Fig. 1(a) shows Kuramoto model representation of a 4-generator power network. For $\gamma \in [0, \pi]$, let $\Delta(\gamma) \subset T^n$ be the set of angle arrays $(\delta_1, \cdots, \delta_n) \in T^n$ such that there exists an arc of length $\gamma$ containing all $\delta_1, \cdots, \delta_n$ in its interior. Thus an array of angles $\delta \in \Delta(\gamma)$ satisfies $\max_{i,j \in (1,\cdots,n)} |\delta_i - \delta_j| < \gamma$.

According to Tikhonov's theorem, since the product of compact spaces is compact, $n$-dimensional torus formed by $S^1 \times S^1 \cdots \times S^1$ is compact. However, Brouwer's theorem [14] states that, if the convex and compact set $S$ is positively invariant, then it necessarily includes a stationary point for the system, which is solution of (8) in case of a power network or of (9) in case of KMFM. More precisely, there exists $\bar{x} \in S$ such that $f(\bar{x}) = \bar{x}$. The angles $\delta_1, \delta_2, \cdots, \delta_n$ are contained in an open half circle which ensures that all machines are in the generating mode and synchronized. For fixed point to exist on a torus, it should be compact as well as convex. Although $n$-dimensional torus is compact, it is not convex hence it is required for the system to evolve on an Euclidean space which results into an $R^{n-1}$ dimensional grounded base configuration as depicted in Fig 1(b). To ensure that all angles remain in $\Delta(\gamma)$ for any time $t$ requires that there exists a non-smooth function $V(\delta)$ such that,

$$V(\delta) = \max_{i,j \in 1,\cdots,n} |\delta_i - \delta_j| \tag{10}$$

It becomes necessary that $V(\delta(t)) \leq \gamma$ for any time $t$ to ensure positive invariance. In other words, function $V(\delta(t))$ should be non-increasing at any time $t$. The function being non-smooth (not continuously differentiable), it is necessary to compute $Dini$ derivative. For testing worst case scenario the upper right Dini derivative of corresponding non-smooth and bounded function (Nagumo's theorem) $V(\delta)$, is computed as:

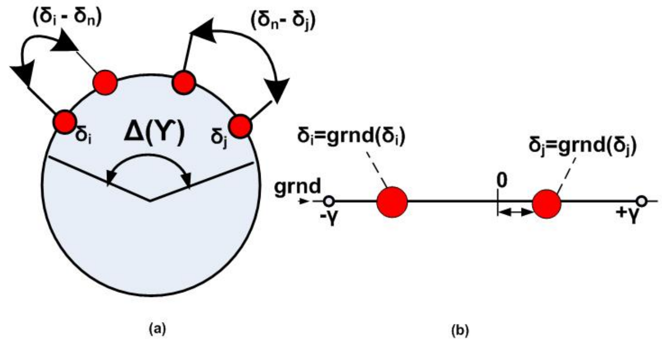


Fig. 1. (a) Kuramoto model representation of 4-gen, 12-bus system (b) Grounded base configuration of Kuramoto model

$$D^+V(\delta(t)) = \lim_{h \to 0^+} (\sup \frac{V(\delta(t+h)) - V(\delta(h))}{h}) \tag{11}$$

along the dynamical system, such that

$$D^+V(\delta(t)) \leq 0 \tag{12}$$

Thus $V(\delta(t))$ is non increasing for all $\delta(t) \in \Delta(\gamma)$ if the upper right Dini derivative is non-increasing. Mathematically, the Dini derivative is computed as:

$$D^+V(\delta(t)) = \dot{\delta}_m(t) - \dot{\delta}_l(t) \tag{13}$$

where $\delta_m$ and $\delta_l$ are the extreme ends of the arc $\Delta(\gamma)$ and the corresponding $\dot{\delta_m}$ and $\dot{\delta_l}$ are obtained by expanding (8) as:

$$\dot{\delta}_m(t) = \frac{\omega_m}{D_m} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{P_{mj}}{D_m} (\cos(\varphi_{mj}) \sin(\delta_m - \delta_j) + \sin(\varphi_{mj}) \cos(\delta_m - \delta_j)) \tag{14}$$

Similarly, $\dot{\delta}_l(t)$ can be computed. Equation (14) leads to CfS, an algebraic condition (15) which is sufficient to obtain synchronization of generators [9]:

$$\Gamma_{min} \geq \Gamma_{critical} \tag{15}$$

Where

$$\Gamma_{min} = n \min_{i \neq j} (\frac{P_{ij}}{D_i} \cos(\varphi_{ij})) \tag{16}$$

and $\Gamma_{critical}$ comprising of 2 components, namely,

$$\Gamma_{crit1} = \max_{i \neq j} |\frac{\omega_i}{D_i} - \frac{\omega_j}{D_j}| \tag{17}$$

and

$$\Gamma_{crit2} = 2 \max_{i \in 1,\cdots,n} \sum_{j=1}^{n} (\frac{P_{ij}}{D_i} \sin(\varphi_{ij})) \tag{18}$$

$$\Gamma_{critical} = \frac{1}{\cos(\varphi_{max})} (\Gamma_{crit1} + \Gamma_{crit2}) \tag{19}$$

For complete synchronization, CfS (15) must be satisfied.

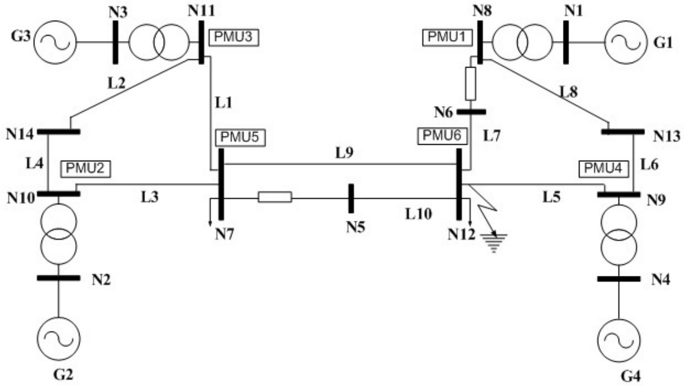


Fig. 2. Representative 4-gen, 12-bus system

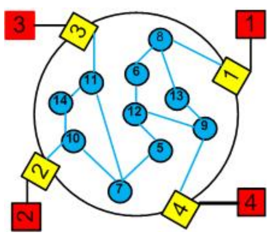

Fig. 3. Undirected graph of representative system

## IV. Controlling power network using TCSC-Formulation of objective function and constraints

It is assumed that system parameters and some of the network topological details are available from off-line data base while some of the inputs need to be collected on-line from PMUs. After forming the complete non-linear complex dynamical model using these inputs, Kron reduction is performed for application of KMFM, and then convex optimization algorithm is run to obtain necessary compensation. The major contribution of present work lies in real-time control in regaining synchronization in a disturbed power network along with proper distribution of line flows without shedding any load, in spite of credible contingency which is otherwise impossible in the case of a Wide Area Network (WAN). The working of proposed controller to provide necessary compensation in an interconnected network is summarized in Fig.5 along with a detailed formulation of objective function and constraints in Fig. 6 via flow-charts.

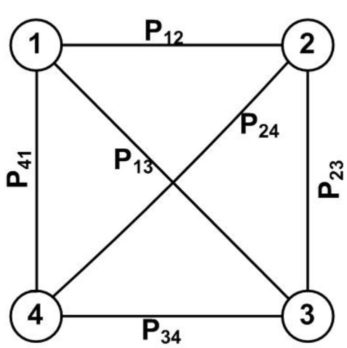


Fig. 4. All-to-all connectivity using Kron reduction

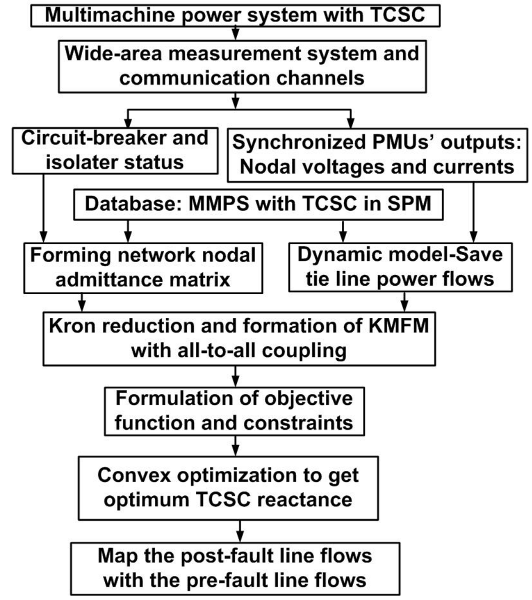


Fig. 5. Main flow chart with complete algorithm

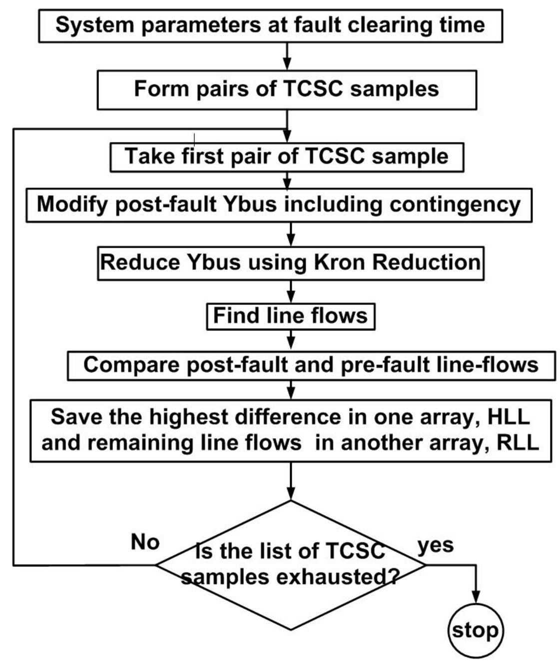


Fig. 6. Flow chart for formulation of objective function and constraints

The optimization problem formulation is explained in detail with the help of a 4-generator, 12-bus power system with two TCSCs placed between node-pairs $N_5 - N_7$ and $N_6 - N_8$ respectively. To formulate an objective function using future control strategies, various line flows are observed. The lines which are overloaded or loaded closest to their limit, are collected to obtain a function which derives a mathematical relation of line flow, directly in terms of control variables. Although, different controllers can be used, for the representative system, series compensation is implemented using TCSCs, the constraints on which are formed based on the operating characteristics as given in CIGRE standards [21].

With 2 TCSCs placed as shown in Fig. 2, let $s_u$ be the number of input control strategies i.e TCSC reactances. In the present case, $u_1^i$ and $u_2^i$, $i = 1, 2, \cdots, s_u$ are the two TCSC reactances forming input control strategies. The systematic procedure summarized in Flow chart Fig. 6 leads to formation two arrays :

TABLE I
PREDICTED LINE FLOWS FOR DIFFERENT CONTROL ACTIONS

| Control input(s) | $P_{12}$ | $P_{23}$ | $P_{34}$ | $P_{41}$ | $P_{24}$ | $P_{13}$ |
|---|---|---|---|---|---|---|
| $u_1^1, u_2^1$ | $\star$ | zz | zz | zz | zz | zz |
| $u_1^2, u_2^2$ | zz | zz | zz | zz | zz | $\star$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $u_1^{s_u}, u_2^{s_u}$ | zz | $\star$ | zz | zz | zz | zz |

$\star$ - Highest line loading (pu)
zz- Line loading excluding HLL(pu)

1) $HLL_{(s_u \times 1)}$: Array of Highest Line Loading and
2) $LL_{(s_u \times (\frac{n(n-1)}{2}-1)}$: Array of Line Loadings excluding HLL, as shown in (20), extracted from Table. 1.

$$\underbrace{\begin{bmatrix} \star \\ \star \\ \cdots \\ \star \end{bmatrix}}_{HLL(s_u \times 1)} \; and \; \underbrace{\begin{bmatrix} zz & zz & \cdots & zz \\ zz & zz & \cdots & zz \\ \cdots & \cdots & \cdots & \cdots \\ zz & zz & \cdots & zz \end{bmatrix}}_{LL(s_u \times (\frac{n(n-1)}{2}-1)} \tag{20}$$

Order of an objective function expressing the algebraic relation between line flows and control variables, is determined based on the desired accuracy and number of control variables used. For example, for a second order objective function relating line flows and two control variables corresponding to reactances of two TCSCs for the system shown in Fig.2, number of coefficients in the objective function are six, i.e. $n_c = 6$. For a second order objective function, using $s_u$ pairs of control variables, a $(s_u \times n_c)$ dimensional control matrix $U$ matrix is formed as:

$$U = \underbrace{\begin{bmatrix} (u_1^1)^2 & (u_1^1)(u_2^1) & (u_2^1)^2 & (u_1^1) & (u_2^1) & 1 \\ (u_1^2)^2 & (u_1^2)(u_2^2) & (u_2^2)^2 & (u_1^2) & (u_2^2) & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ (u_1^{s_u})^2 & (u_1^{s_u})(u_2^{s_u}) & (u_2^{s_u})^2 & (u_1)^{s_u} & (u_2)^{s_u} & 1 \end{bmatrix}}_{s_u \times n_c} \tag{21}$$

The coefficients of objective function giving Objective function Coefficient Matrix (OCM), are computed as:

$$[OCM]_{(n_c \times 1)} = [U]^{-1}_{(n_c \times s_u)} \times [HLL]_{(s_u \times 1)} \tag{22}$$

An objective function expressing relation between line flows and control variables is given by :

$$[Objfun]_{(1 \times 1)} = [OCM]'_{(1 \times n_c)} * [MCV]_{(n_c \times 1)} \tag{23}$$

where, $MCV$ is the Matrix of Control Variables that are obtained by running convex optimization algorithm to give necessary control action. Similarly coefficients of constraint equations denoted by Constraint Coefficient Matrix (CCM) are computed as:

$$[CCM]_{(n_c \times (\frac{n(n-1)}{2}-1))} = [U]^{-1}_{(n_c \times s_u)} \times [LL]_{(s_u \times (\frac{n(n-1)}{2}-1))} \tag{24}$$

where

$$CCM = \overbrace{\begin{bmatrix} b_5^1 & b_5^2 & b_5^3 & b_5^4 & b_5^5 \\ b_4^1 & b_4^2 & b_4^3 & b_4^4 & b_4^5 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b_0^1 & b_0^2 & b_0^3 & b_0^4 & b_0^5 \end{bmatrix}}^{Number\,of\,lines}_{(n_c \times (\frac{n(n-1)}{2}-1))} \tag{25}$$

After constructing objective function and constraints, it is required to formulate the same in a form acceptable by Convex Optimization Toolbox (CVX) which is used in MATLAB environment.

## V. Need for convex optimization

There are great advantages in recognizing or formulating a problem as a convex optimization problem after which the problem can then be solved, reliably and efficiently, using interior-point methods or other special methods. These solution methods are reliable enough to be embedded in a computer-aided design or analysis tool, or even a real-time automatic control system. Convex optimization can be considered to be a generalization of linear programming as convexity is more general than linearity wherein inequality replaces the more restrictive equality [11].Since differentiability of the objective and constraint functions is the only requirement for most local optimization methods, formulating a practical problem as a non-linear optimization problem is relatively straightforward. Despite the fact that formulation of convex optimization problem is rather tedious, once formulated it converges fast to give a global solution.

As mentioned in Section IV, on occurrence of a large disturbance, if the fault gets cleared after CCT, the system becomes desynchronized if corrective actions are delayed.To regain rotor angle stability accompanied by proper distribution of power flows, keeping demand-supply balance, without any load-shedding in spite of a topological change due to credible contingency is a multidimensional problem. Moreover, the level of instability and the rate of damage caused in such a critical scenario needs an efficient tool capable of giving a real-time control for fast recovery of the system.

Applying the procedure explained in Section IV, objective function to be minimized and constraint equations are obtained which are required to be transformed into a quadratic form as given in (26) which is acceptable in CVX

$$u'Au + C'u + D \tag{26}$$

A convex optimization problem can be defined as:
*minimize* $Objfun$
*subject to* constraints equations for $((\frac{n(n-1)}{2}) - 1)$ lines,
*so as to satisfy* $\Gamma_{min} > \Gamma_{critical}$.

For a 4-generator,12-bus system, $n(n-1)/2 = 6$, out of which highest overloaded line contributes in forming objective function and remaining 5 lines form constraint equations. Sine and cosine terms in (16) and (18) make the constraint non-convex, and need modification to convexify the same. If the transfer conductances are assumed to be sufficiently small, then power network can be represented by most popular uniform Kuramoto model (9).
For lossless network $\varphi_{ij}$ being zero,
(16) and (18) become,

$$\Gamma_{min} = n \min_{i \neq j}(\frac{P_{ij}}{D_i})$$

and

$$\Gamma_{crit2} = 2 \max_{i \in 1,\cdots,n} \sum_{j=1}^{n}(\frac{P_{ij}}{D_i} sin(\varphi_{ij})) = 0$$

respectively.
Since there is no significant deviation in $\omega_i$ and $\omega_j$, $\Gamma_{crit1}$ is zero. Thus, CFS is rewritten as:

$$\Gamma_{min} > 0.$$

For lossy power network, due to non-trivial transfer conductances, $\varphi_{ij}$ is finite. In the worst case, $\varphi_{ij}$ is $\cong 90^0$. Considering $\varphi_{ij}$ slightly less than $90^0$, $\Gamma_{min}$ and $\Gamma_{critical}$ are computed and CfS is checked. The CVX will return a value of TCSC reactance that adjusts the power flows so that CfS is satisfied.

## VI. Simulation Results and Analysis

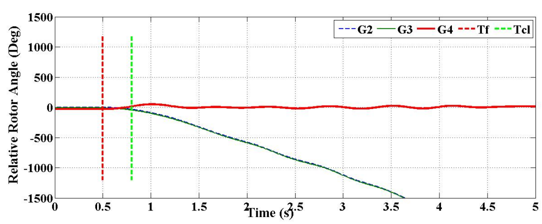


Fig. 7. Variation of rotor angle delta without controller

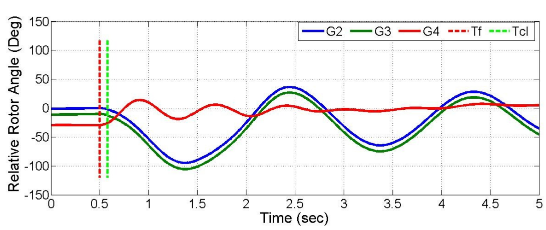


Fig. 8. Variation of rotor angle delta with TCSC

The convex optimization algorithm developed in the previous sections is implemented on a 4-generator, 12-bus system shown in Fig. 2 as a case study. During pre-fault all the generators are in frequency-synchronism, indicated by the phase-cohesive behaviour of the relative rotor angles in Fig. 7, and Fig. 8 for a time span of 50ms (pre-fault region). The line flows match the demand and supply, thus maintaining stable operation of the grid satisfying CfS, ensuring synchronization among generators. For a given system CCT is found to be 170 ms and any fault cleared after CCT gives rise to instability resulting in loss of synchronism among the generators as can be observed in Fig. 7 after 50 msec. Instead of complete synchronization, different clusters of generators are formed giving rise to islands. Generators $G_2, G_3$ form a group and $G_4$ is seperated. Although, generators within the group are in synchronism with each other, the two groups behave incoherently.

Assuming a severe disturbance in the system such as a 3-phase dead short circuit in one of the most critical transmission lines, for example, line $L_5$, near node $N_{12}$. The scenario is worsened if the fault gets cleared after CCT (by tripping the circuit breakers), in which case the system is bound to become unstable. The $(n-1)$ contingency as a result of line outage causes a change in network topology which in turn results into a totally different power flow distribution that may overload some of the lines. In such a disturbed condition it becomes essential to provide a compensation so as to regain synchronization.

As discussed in Section IV, input data is available partly from the off-line data base, and partly acquired from PMUs located at different places in the WAN. Optimal placement of PMUs based on complete observability requires 4 PMUs at the generator buses. However, due to placement of TCSC at nodes, $N_5$, and $N_6$, for acquiring real-time data required for deriving necessary control law, needs two additional PMUs at two nodes of active TCSCs. Even though the number of PMUs used are more than the optimal placement calculation it will be useful in giving better observability under $(n-1)$ contingency.

Using the Convex Optimization Algorithm (COA), in which the objective function minimizes the power flow in the highest overloaded line, subjected to constraints which take care of power flows in all the lines excluding the highest overloaded line so as to satisfy CFS. The COA generates control law that computes minimum reactance compensation to achieve both- rotor angle stability and proper power distribution. The algorithm developed generates a control law which gives a global minimum value that can be used as a set point to fire TCSC.

## VII. Conclusions

The most obvious solutions suggested in literature for regaining transient stability after severe disturbance followed by contingency, are limited in assuring first swing rotor angle stability with least or no concern about post-disturbance line loading condition which may result in cascade tripping. In view of this, the paper proposes a guaranteed solution not only for satisfying first swing rotor angle criteria but also maintaining rescheduled line flows within required stability margin. The major contribution of the paper is in deriving a control law by which necessary compensation applying edge-weight control to satisfy synchronization condition using real time data acquired from PMUs. The method gives an edge over traditional TDS methods by minimizing computation burden and time which is the necessity of real time controllers. The idea can be further explored for distributed control strategy for adaptive controller coordination schemes. A global minimum value of TCSC reactance obtained using convex optimization assures rotor angle stability as well as synchronization in the post-fault condition to maintain perfect demand supply balance without load-shedding, even after a credible contingency.

## References


[1] R. Zarate-Minano, Thierry V C, Federico M, and Antonio J C, ”Securing transient stability using time-domain simulations within an optimal power flow”, *IEEE Transactions on Power Systems*, 25(1):243253, 2010.

[2] Chiang Hsiao Dong and Chu Chia Chi, ”Direct Stability Analysis of Electric Power Systems Using Energy Functions: Theory, Applications, and Perspective”, *Proceedings of the IEEE*, Vol. 83(11), pp 1497-1529, 1995.

[3] Willems J L and Willems J C, ”The application of Lyapunov methods to the computation of transient stability regions for multimachine power systems”, *IEEE Transactions on Power Apparatus and Systems*, N0. 5, pp. 795-801, 1970.

[4] ”Report on Grid Disturbances on $30^{th}$ July, and $31^{st}$ July, 2012”, submitted in compliance to CERC orderin Petition no 167/SuoMotu/2012, $1^{st}$ Aug, 2012.

[5] Noroozian M and Angquist L and Ghandhari M and Andersson, ”Improving power system dynamics by series-connected FACTS devices”, *IEEE Transactions on Power Delivery*, Vol. 12(4), pp. 1635-1641, 1997.

[6] Abido M A, ”Power system stability enhancement using FACTS controllers : A review”, The Arabian Journal for Science and Engineering, April 2009, Vol. 34(1), pp.153-172.

[7] Wagh S, Kamath A and Singh N, ”Non-linear Model Predictive Control for improving transient stability of power system using TCSC controller”, 7th IEEE Asian Control Conference, 2009.

[8] Dorfler F and Bullo F, ”Synchronization of power networks: Network reduction and effective resistance”, IFAC Workshop on Distributed Estimation and Control in Networked Systems, 2010.

[9] Dorfler F and Bullo F, ”Synchronization and transient stability in power networks and nonuniform Kuramoto oscillators”, SIAM Journal on Control and Optimization, Vol. 50(3), pp.1616-1642,2012

[10] Lin Z and Francis B and Maggiore M, ”State agreement for continuous-time coupled nonlinear systems”, SIAM Journal on Control and Optimization, Vol. 46(1), pp.288-307, 2007.

[11] Boyd S and Vandenberghe L, ”Convex optimization”, Cambridge university press, 2004.

[12] Sarlette Alain, ”Geometry and symmetries in coordination control”, 2009.

[13] Moreau L, ”Stability of multiagent systems with time-dependent communication links”, *IEEE Transactions on Automatic Control*, Vol.50(2), pp.169-182, 2005.

[14] Blanchini F and Miani S, ”Set-theoretic methods in control”, Springer, 2008.

[15] Bondy J A and Murty U S R, ”Graph theory with applications”, Macmillan London, 1976.

[16] Bollobas Bela, ”Graph Theory: An Introductory Course”, Springer, 1994.

[17] Khalil H K, ”Nonlinear systems”, Prentice hall Upper Saddle River, 2002.

[18] Dorfler F and B F, ”Kron reduction of graphs with applications to electrical networks”, IEEE, 2012.

[19] R Ortega, M Galaz, A Astoln, Y Sun and T Shen, ”Transient stabilization of multi machine power systems with nontrivial transfer conductances”, *IEEE Transactions on Automatic Control*, Vol. 50(1), January 2005.

[20] Grainger J J, ”Power system analysis”, Tata McGraw-Hill Education, 2003.

[21] CIGRE, ”Modelling of power electronics equipments (FACTS) in load flow and stability programs: A representative guide for power system planning and analysis”, 1999 August.