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Density flow over networks: a mean-field game theoretic approach

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A distributed routing control algorithm for dynamical networks has recently been presented in the literature [2], [3]. The model describes the time evolution of the density at the edges of a network and describes a distributed routing control that allows the density to converge to a Wardrop equilibrium. This is characterized by an equal traffic density on all used paths. We borrow the idea and rearrange the density model recasting this within the framework of mean-field games [4], [5]. The problem we analyze in this paper has striking similarities with the optimal planning problem [1] which in turn can be linked back to mean-field games. Essentially, in optimal planning problems the idea is to drive the density of players from a given initial configuration to a target one at time T by an appropriate design of the optimal decisions of the agents.

The problem setup involves a population of individuals or players traversing the edges of a network in the attempt to reach a destination node starting from a source node. From a microscopic standpoint, each player jumps from one edge to an adjacent one according to a continuous-time Markov model. Players select the transition rates, which represent the control. From a macroscopic perspective, each edge is then characterized by dynamics describing the time evolution of the density of individuals on that edge. These dynamics take the form of a classical forward Kolmogorov ordinary differential equation (ODE). In the second part of the paper, we extend our analysis to the case where the Kolmogorov equation turns into a stochastic differential equation (SDE) driven by a Brownian motion.

Main results. For the problem at hand we highlight three main results. First, we provide a mean-field game formulation of the problem at hand. Second, we illustrate an extended state space solution approach in the same spirit as in [6]. Third, we study the stochastic case where the density evolution is driven by a Brownian motion.

Keywords: mean-field games; consensus; multiagent systems; network flow.

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