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Geometric-PBC based Control of 4-DOF Underactuated Overhead Crane System

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***Abstract*— The control of 4-DOF underactuated overhead crane system poses a challenging control problem as it has extra degree of freedom (DOF) compared to its popular 3-DOF variant. The extra DOF represents strong state coupling and hence more complicated system dynamics. We propose Geometric - Passivity Based Control (PBC) methodology for synthesis of nonlinear stabilising feedback control law. The structure of the split tangent space is modified along the actuated direction in such a way that the power flow is established between the controller and the un-actuated subsystems. The passivating outputs of the modified system are identified which are utilized for energy shaping of the system. The nonlinear control law thus obtained achieves the control objective of precise payload positioning with elimination of payload swings. The main advantage of proposed nonlinear design methodology is that obviates the need of solving PDE's or obtaining nonlinear transformations to synthesize the control law. The simulation results are presented to validate the nonlinear control law. The system parameters and constraints on input forces are considered to represent the experimental setup of 4-DOF overhead crane system.**

***Index Terms*— Nonlinear systems, Lyapunov function, stability, Control of mechanical systems, Overhead crane**

I. INTRODUCTION

Overhead cranes are widely used for industrial applications involving heavy material handling and transportation. The practical controller requirements, precise payload positioning with minimum swinging motion, are simple but difficult to achieve. The overhead crane models are evolving to include many design changes and adding more degree of freedoms which makes the system dynamics more complex and control objectives more challenging. Most of the present controller design methodologies are implemented on two degree-of freedom (2-DOF) and 3-DOF overhead crane system.

In last two decades various control methodologies have been applied on different overhead crane models. In 2-DOF crane the trolley motion and payload swing are in 2-D plane. For the 2-DOF crane system, nonlinear feedback were developed in [1], [2]. The overhead cranes models with additional DOF, corresponding to 3-D space, were developed initially by [3], [4]. One of the main limiting factors associated with control design and stability analysis is that crane system nonlinearities are not sufficiently accounted in design. To overcome the drawback, several control approaches have been explored that considers for the nonlinear dynamics of overhead cranes. The techniques include sliding mode methods [5], [6], input shaping [7], and optimal control [8]. The passivity and energy based control laws are implemented in [14], [15]. The energy shaping based well known control techniques for underactuated mechanical system are Controlled Lagrangian [9] and Interconnection and Damping Assignment Passivity Based Control (IDA-PBC) [10]. The control laws were generated by solving the matching conditions, which transform the original Lagrangian (or Hamiltonian) system into chosen Lagrangian (or Hamiltonian) form with desired energy function. The methodologies were applied to overhead crane system in [11], [12] and [13]. The control method, applied on 2D spidercrane, achieves objective through potential shaping. The kinetic shaping is implemented to further improve the transient behaviour.

The 4-DOF overhead crane model was introduced in [16]. The 4-DOF crane modelling represents more complicated dynamics and much stronger state coupling, thus bringing more technical challenges for controller design. In this paper we present a nonlinear design methodology 'Geometric-Passivty Based Control' for control of 4-DOF underactuated overhead crane. The control design is based on the intrinsic geometry of the mechanical system being controlled and passivity based control (PBC) [20] . The tangent space of mechanical system can be decomposed into a vertical component along the external variables and a horizontal component which projects onto the base space. The external variables are actuated variables and shape variables on base space are unactuated variables. The proposed control methodology is based on manipulating the symmetric structure of the system which modifies the structure of split tangent space along the actuated direction. This results in establishing flow of power between the controller and the unactuated part of system. The passive outputs of the subsystems are identified which facilitate the development of passivity based control problem formulation. The energy shaping idea can be followed from the passive outputs and the desired controller form is obtained by using passivity based techniques. The potential energy shaping based controller is proposed which gives the satisfactory transient control performance. The control law achieves both the control objective of precise payload positioning and minimum swing payload motion. The control law is derived by fully considering the constraints in input forces. The main advantage of the proposed method is that it eliminates the need of solving PDE's for synthesis of control law.

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The rest of paper is organised as follows: the preliminaries required for geometric theory are presented in Section-2. The concept of Geometric-PBC design methodology for 4-DOF crane is illustrated in Section-3. Energy shaping and control law derivation is conducted in Section -4. In Section-5, numerical simulations results are presented.

II. PRELIMINARIES

The geometric theory which forms a background to Geometric–PBC approach for controlling a class of underactuated mechanical systems is briefly described in this section. The notation and presentation style are in standard form as followed in [17] and are repeated here for completeness.

The system has configuration space Q and Lie group G that acts freely and properly on Q. In our case $Q = S \times G$ with the Lie group G acting on left by group multiplication. The objective of Geometric-PBC theory is to control the variables lying in the shape space Q/G using actuation which acts directly on G only. For a large class of underactuated mechanical system of interest the Lagrangian $L : TQ \to R$ is invariant under the action of G on Q. This implies that the Lagrangian L is cyclic in G variable. A special case of the above is when only the kinetic energy term of the Lagrangian has the property of being invariant under the action of G.

A principal connection on the principal bundle $\pi : Q \to Q/G$ is a map $A : TQ \to \mathfrak{g}$ (where A is a $\mathfrak{g}$-valued one-form) that is linear on each tangent space and at each point $q \in Q$ we have the decomposition of the tangent space T_qQ = Hor$_q$ $\oplus$ Ver$_q$. The horizontal space of the connection at $q \in Q$ is the linear space, $\text{Hor}_q = (v_q \in T_qQ \mid A(v_q) = 0)$.

A connection is uniquely defined by the specification of its horizontal space. Given a connection A the vector $v_q \in T_qQ$ is decomposed as v_q = Hor v_q+ Ver v_q where $\text{Ver}_q v = [A(q,v)]_Q(q)$ and $\text{Hor}_q v = v - \text{Ver}_q v$. Here, $[A(q,v)]_Q$ denotes the infinitesimal generator corresponding to the Lie algebra element $[A(q,v)]$. The principal connection is a special case of the vertical valued Ehresmann connection defined on fiber bundle.

The symmetry of the mechanical system under the action of the Lie group G is the invariance of the Lagrangian under G. This invariance gives rise to mechanical connection on the principal bundle $\pi : Q \to Q/G$. The action of the mechanical connection can be described in terms of τ_m a Lie-algebra valued horizontal one form on Q.

τ_m is a horizontal one form on Q with values in the Lie algebra $\mathfrak{g}$ of G that annihilates the vertical vectors. If v is a vertical vector, i.e. a vector along the group direction then $[\tau_m(v)]_Q$ is the zero vector field on Q. The τ_m horizontal space at $q \in Q$ consists of tangent vectors at $q \in Q$ of the form, $\text{Hor}_{\tau_m} v_q = \text{Hor}(v_q) - [\tau_m(v)]_Q(q)$ and $\text{Ver}_{\tau_m} v_q = \text{Ver}(v_q) + [\tau_m(v)]_Q(q)$. It is obvious that $v_q = \text{Hor}_{\tau_m}(v_q) + \text{Ver}_{\tau_m}(v_q)$.

III. GEOMETRIC-PBC METHODOLOGY

A. Dynamics of 4-DOF crane

The 4-DOF underactuated overhead crane system is introduced in [16] and the model is derived from the 3 DOF crane by introducing one extra degree of freedom.

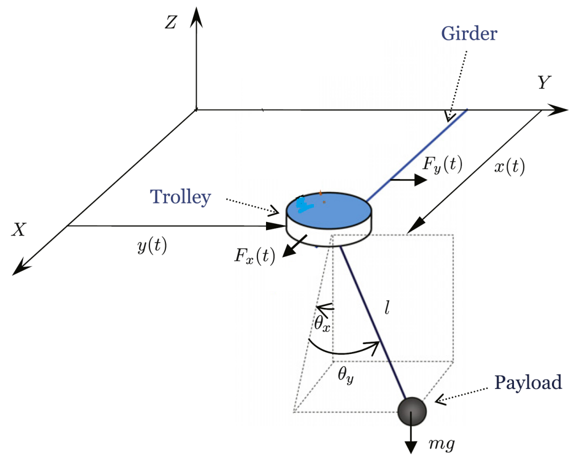


Fig. 1: 4-DOF overhead crane representation

The system has following dynamics:

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = u \tag{1}$$

where $q \in \mathbf{R}^4$ denotes the state vector which are defined as $\begin{pmatrix} x & y & \theta_x & \theta_y \end{pmatrix}^T$, $M(q) \in \mathbf{R}^{4\times4}$ denotes the inertia matrix, $C(q,\dot{q}) \in \mathbf{R}^{4\times4}$ denotes the Centripetal–Coriolis matrix, $G(q) \in \mathbf{R}^4$ is the gravity vector, and $u \in \mathbf{R}^4$ denotes the control vector.

$$M = \begin{pmatrix} m+m_x & 0 & mlC_xC_y & -mlS_xS_y \\ 0 & m+m_y & 0 & mlC_y \\ mlC_xC_y & 0 & ml^2C_y^2 & 0 \\ -mlS_xS_y & mlC_y & 0 & ml^2 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 0 & -mlS_xC_y\dot{\theta}_x \; -mlC_xS_y\dot{\theta}_y & -mlC_xS_y\dot{\theta}_x \; -mlS_xC_y\dot{\theta}_y \\ 0 & 0 & 0 & -mlS_x\dot{\theta}_y \\ 0 & 0 & -ml^2S_yC_y\dot{\theta}_y & -ml^2S_yC_y\dot{\theta}_x \\ 0 & 0 & ml^2S_yC_y\dot{\theta}_x & 0 \end{pmatrix}$$

$$G = \begin{pmatrix} 0 & 0 & mglS_xC_y & mglC_xS_y \end{pmatrix}^T$$
$$u = \begin{pmatrix} F_x & F_y & 0 & 0 \end{pmatrix}^T$$

where m represents the payload mass, m_x consists of the trolley mass and some additional equivalent components in direction X, and m_y consists of the trolley and girder masses and some additional equivalent components in direction Y, such as the motor mass, which corresponds to Lagrange's modelling method; l is the rope length; $x(t)$ and $y(t)$ are the trolley displacements along the X and Y axes respectively. $\theta_x(t)$ and $\theta_y(t)$ denote the projected swing signals; S_x, S_y, C_x, C_y are abbreviations for $\sin\theta_x$, $\sin\theta_y$, $\cos\theta_x$, and $\cos\theta_y$ respectively; $F_x(t), F_y(t)$ denote the actuating forces supplied by the motors in directions X and Y respectively.

$$(m+m_x)\ddot{x}+mlC_xC_y\ddot{\theta}_x-mlS_xS_y\ddot{\theta}_y-mlS_xC_y\dot{\theta}_x^2$$
$$-2mlC_xS_y\dot{\theta}_x\dot{\theta}_y-mlS_xC_y\dot{\theta}_y^2=F_x \quad (2)$$
$$(m+m_y)\ddot{y}+mlC_y\ddot{\theta}_y-mlS_y\dot{\theta}_y^2=F_y \quad (3)$$
$$mlC_xC_y\ddot{x}+ml^2C_y^2\ddot{\theta}_x-2ml^2S_xC_y\dot{\theta}_x\dot{\theta}_y+mglS_xC_y=0 \quad (4)$$
$$mlS_xS_y\ddot{x}-mlC_y\ddot{y}-ml^2\ddot{\theta}_y-ml^2S_yC_y\dot{\theta}_x^2-mglC_xS_y=0 \quad (5)$$

The payload position can be calculated as: $x_p(t)=x+lS_xS_y$, $y_p(t)=x+lS_y$ and $z_p(t)=l(1-C_xC_y)$.

The common assumption in crane model is that the payload always remains lower than the trolley position i.e. $-\pi/2<\theta_x(t),\theta_y(t)<\forall t\geq 0$.

The desired equilibrium point is

$$(q_d^T,\dot{q}_d^T)^T=(x_d,y_d,0,0,0,0,0,0)^T \quad (6)$$

where $q_d(t),\dot{q}_d(t)\in\mathbf{R}^4$ and x_d,y_d being the desired trolley positions in directions X and Y respectively. Unlike the inverted pendulum, the crane system has stable behaviour around its desired equilibrium point. The crane system has natural equilibrium point at $(q_d,\dot{q}_d)$.

B. Geometrical setting

The role of geometry in formulating a control law for the 4-DOF crane can be explained in terms of decomposition of T_qQ given a mechanical connection. For the configuration space Q a vector $q\in Q$ defined as (q_x,q_s) and corresponding tangent vector $v_q\in T_qQ$ is $(\dot{q}_x,\dot{q}_s)$. The q_x forms the external variable along the direction of fiber bundle i.e. actuated variable and q_s is the shape variable along the base space Q/G which is unactuated variable. For 4-DOF crane system, the external forces are applied along x and y direction hence the external (actuated) variables are $q_x=(x,y)$ and shape (unactuated) variables to be controlled are payload angles $q_s=(\theta_x,\theta_y)$.

The tangent vector v_q can be expressed as sum of its vertical and horizontal component.

$$v_q=(q_x,q_s)=\text{Ver}(v_q)+\text{Hor}(v_q) \quad (7)$$

In general the horizontal one form is given by $[\tau_m(\dot{q}_s)]_Q(q)=m_{xx}^{-1}(q_s)m_{xs}(q_s)\dot{q}_s$. For 4-DOF crane system (2)–(5):

1) One form corresponding to X-axis is given by $[\tau_{mx}(\dot{\theta}_x)]_Q(q)=m_{11}^{-1}m_{13}\dot{\theta}_x=(m+m_x)^{-1}(mlC_xC_y)\dot{\theta}_x$
2) One form corresponding to Y-axis is given by $[\tau_{my}(\dot{\theta}_y)]_Q(q)=m_{22}^{-1}m_{14}\dot{\theta}_y=(m+m_y)^{-1}(mlC_y)\dot{\theta}_y$

The tangent vector v_q is projected onto the vertical component, $\text{Ver}_{\tau_m}(v_q)=(\dot{q}_x+m_{xx}^{-1}(q_s)m_{xs}(q_s)\dot{q}_s,0)$ and the horizontal component, $\text{Hor}_{\tau_m}(v_q)=(-m_{xx}^{-1}(q_s)m_{xs}(q_s)\dot{q}_s,\dot{q}_s)$. For 4-DOF crane system, the vertical component is given by $\text{Ver}_{\tau_m}(v_q)=\left(\left[\dot{x}+(m+m_x)^{-1}mlC_xC_y\dot{\theta}_x\right],\left[\dot{y}+(m+m_y)^{-1}mlC_y\dot{\theta}_y\right],0,0\right)$ and horizontal component is $\text{Hor}_{\tau_m}(v_q)=\left(\left[-(m+m_x)^{-1}mlC_xC_y\dot{\theta}_x\right],\left[-(m+m_y)^{-1}mlC_y\dot{\theta}_y\right],\dot{\theta}_x,\dot{\theta}_y\right)$

Since the actuation is along the group direction G the power flow between the controller and system is

$$P=\begin{bmatrix}\dot{q}_x & \dot{q}_s\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\tau=\dot{q}_xf \quad (8)$$

where f denotes the external forces. It is straight forward to show that, f has no direct control on shape variables $q_s=(\theta_x,\theta_y)$ i.e. on the payload angles. To control the payload angles new control law needs to be designed so that the power flow is established between controller and unactuated part i.e. the payload subsystem. This can be obtained if $\text{Ver}_{\tau_m}(v_q)$ is changed by suitably modifying the Euler Lagrange equation along the actuated direction using control input u. The subsequent modification of $\text{Ver}_{\tau_m}(v_q)$ to $\tilde{\text{V}}\text{er}_{\tilde{\tau}_m}(v_q)$ is explained in the following subsection.

C. Modified system equations

The Euler Lagrange equation (2) can be modified as follows,

$$\ddot{x}=u_x \quad (9)$$
$$\ddot{y}=u_y \quad (10)$$
$$mlC_xC_y\ddot{x}+ml^2C_y^2\ddot{\theta}_x-2ml^2S_xC_y\dot{\theta}_x\dot{\theta}_y+mglS_xC_y=0 \quad (11)$$
$$mlS_xS_y\ddot{x}-mlC_y\ddot{y}-ml^2\ddot{\theta}_y-ml^2S_yC_y\dot{\theta}_x^2-mglC_xS_y=0 \quad (12)$$

under the influence of nonlinear feedback control law,

$$F_x=mlC_xC_y\ddot{\theta}_x-mlS_xS_y\ddot{\theta}_y-mlS_xC_y\dot{\theta}_x^2-2mlC_xS_y\dot{\theta}_x\dot{\theta}_y$$
$$-mlS_xC_y\dot{\theta}_y^2+(m+m_x)u_x \quad (13)$$
$$F_y=mlC_y\ddot{\theta}_y-mlS_y\dot{\theta}_y^2+(m+m_y)u_y \quad (14)$$

The fundamental vector field of horizontal one form for the modified system is given by $[\tilde{\tau}_m(\dot{q}_s)]_Q(q)=m_{xs}(q_s)\dot{q}_s$ and $\text{Ver}_{\tau_m}(v_q)\rightarrow\tilde{\text{V}}\text{er}_{\tilde{\tau}_m}(v_q)=(\dot{q}_x,0)$ and $\text{Hor}_{\tilde{\tau}_m}(v_q)$ remains unchanged since the unactuated subsystem part is not altered. For 4-DOF crane system, the modified vertical component is given by $\tilde{\text{V}}\text{er}_{\tilde{\tau}_m}(v_q)=(\dot{x},\dot{y},0,0)$ however the horizontal component is unaltered, $\text{Hor}_{\tilde{\tau}_m}(v_q)=\left(\left[-(m+m_x)^{-1}mlC_xC_y\dot{\theta}_x\right],\left[-(m+m_y)^{-1}mlC_y\dot{\theta}_y\right],\dot{\theta}_x,\dot{\theta}_y\right)$

The power flow for the modified system is,

$$P_m=\begin{bmatrix}\dot{q}_x & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}u+\begin{bmatrix}-m_{xs}(q_s)\dot{q}_s & \dot{q}_s\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}u$$
$$=(\dot{q}_x+\{-m_{xs}(q_s)\dot{q}_s\})u \quad (15)$$

For the modified system dynamics (9)-(12) under control law u, Ver_{τ_m} is changed to $\tilde{\text{V}}\text{er}_{\tilde{\tau}_m}$ and the power flow is established between the trolley (actuated) and the payload (unactuated) subsystems. Thus the basic setup for control law design for the stabilization of the 4-DOF crane system is constituted. Further, the passivity theory and energy shaping concept are used in the design of control law for which $y_x=\dot{q}_x$ and $y_s=-m_{xs}(q_s)\dot{q}_s$ in (15) are defined as passivating outputs of actuated and unactuated subsystems respectively.

IV. Energy Shaping and Controller Design

The energy shaping is one of the most effective approach for control of overhead crane system. The existing methods involves solving the PDE's which can be very difficult to solve. The proposed Geometric-PBC control methodology obviates the need of solving PDEs to obtain the control law. The control objective of precise trolley positioning and elimination of swinging motion is achieved by potential energy shaping.

The total energy of crane system is

$$E(t) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + mgl(1 - C_x C_y) \tag{16}$$

which is locally positive definite w.r.t $\dot{q}(t)$, $\theta_x(t)$ and $\theta_y(t)$, $M(q)$ is positive definite, and $mgl(1-C_xC_y) \geq 0$.

A. Passivating outputs

With the modified Euler Lagrange equations (9) and (12), the energy of actuating subsystem is

$$E_a = E_{ax} + E_{ay} = \frac{1}{2}\dot{x}^2 + \frac{1}{2}\dot{y}^2 \tag{17}$$

$$\dot{E}_a = \dot{E}_{ax} + \dot{E}_{ay} = \dot{x}\ddot{x} + \dot{y}\ddot{y} = y_1 u_x + y_2 u_y \tag{18}$$

where $y_1 = \dot{x}$ and $y_2 = \dot{y}$. The y_1 and y_2 defines the passive outputs of the actuated subsystem, $y_x = y_1 + y_2$.

The energy of the unactuated subsystem can be obtained by considering the actuated subsystem (9)-(10) at rest ($\dot{q}_x = 0$). Putting the values $\dot{x} = 0$ and $\dot{y} = 0$ in (16) and solving, the energy of the unactuated subsystem (11)-(12) is,

$$E_s = E_{sx} + E_{sy} = \frac{1}{2}ml^2 C_y^2 \dot{\theta}_x^2 + \frac{1}{2}ml^2\dot{\theta}_y^2 - mglC_xC_y \tag{19}$$

Taking the time derivative to obtain,

$$\begin{aligned}\dot{E}_s = \dot{E}_{sx} + \dot{E}_{sy} &= ml^2C_y^2\dot{\theta}_x\ddot{\theta}_x - ml^2\dot{\theta}_x^2 C_yS_y\dot{\theta}_y \\ &+ ml^2\dot{\theta}_y\ddot{\theta}_y + mglS_x\dot{\theta}_xC_y + mglC_xS_y\dot{\theta}_y\end{aligned} \tag{20}$$

Obtaining the values of $\ddot{\theta}_x$ and $\ddot{\theta}_y$ from (11) and (12) we get,

$$\ddot{\theta}_x = \frac{1}{lC_y^2}[-C_xC_yu_x + 2lS_xC_y\dot{\theta}_x\dot{\theta}_y - gS_xC_y] \tag{21}$$

$$\ddot{\theta}_y = \frac{1}{l}[S_xS_yu_x - C_yu_y - lS_yC_y\dot{\theta}_x^2 - gC_xS_y] \tag{22}$$

Substituting the above values into the expression of $\dot{E}_s$ and performing some calculations yield,

$$\begin{aligned}\dot{E}_s &= \left[-ml\dot{\theta}_xC_xC_y + ml\dot{\theta}_yS_xS_y\right]u_x - ml\dot{\theta}_yC_yu_y \\ \dot{E}_s &= \dot{E}_{sx} + \dot{E}_{sy} = y_3u_x + y_4u_y\end{aligned} \tag{23}$$

where $y_3 = -ml\dot{\theta}_xC_xC_y + ml\dot{\theta}_yS_xS_y$ and $y_4 = -ml\dot{\theta}_yC_y$. The y_3 and y_4 defines the passive outputs of the unactuated subsystem, $y_s = y_3 + y_4$.

B. Potential energy shaping

Let V denote the total energy of the modified system (9)–(12) including the energy shaping terms. Taking V as the storage function, which also forms the Lyapunov function, the passivity property of the system can be achieved. In general the candidate Lyapunov function is

$$\begin{aligned}V &= k_e(k_1E_a + k_2E_s) + \frac{1}{2}k_v(k_1y_1 + k_2y_2)^2 \\ &+ \frac{1}{2}k_p(k_1\int y_1dt + k_2\int y_2dt)^2\end{aligned} \tag{24}$$

The second term in the R.H.S. of above equation represents the shaped kinetic energy and the third term represents the shaped potential energy. For 4-DOF crane example, we propose only the potential energy shaping. The kinetic energy shaping is not required as the equilibrium point of system is stable unlike the inverted pendulum like systems. The improved transient performance is achieved with the potential energy shaping only. Putting $k_v = 0$,

$$V = k_e[k_1E_a + k_2E_s] + \frac{1}{2}k_p[k_1\int y_xdt + k_2\int y_sdt]^2$$

For 4-DOF crane system, the passive outputs y_1 and y_3 corresponds to X axis and y_2 and y_4 corresponds to Y axis.

$$\begin{aligned}V &= k_e[k_1E_{ax} + k_3E_{sx}] + \frac{1}{2}k_p[k_1\int_{x_0}^{x_d} y_1dt \\ &+ k_3\int_{x_0}^{x_d} y_3dt + k_2\int_{y_0}^{y_d} y_2dt + k_4\int_{y_0}^{y_d} y_4dt]^2\end{aligned} \tag{25}$$

where x_0 and y_0 corresponds to initial position, x_d and y_d corresponds to desired position of trolley.

Taking the time derivative, substituting the values of $\dot{E}_ax$ and $\dot{E}_sx$ and making some mathematical arrangements produce,

$$\begin{aligned}\dot{V} &= (k_1y_1 + k_3y_3)[k_pk_1\int y_1dx + k_pk_3\int y_3dx + k_eu_x] \\ &+ (k_2y_2 + k_4y_4)[k_pk_2\int y_2dy + k_pk_4\int y_4dy + k_eu_y]\end{aligned} \tag{26}$$

The integration of passivating outputs y_x and y_s with respect to time is as follows,

$$y_1 = \dot{x}, \quad \int_{x_0}^{x_d} y_1dt = \int_{x_0}^{x_d}\dot{x}dx = (x_0 - x_d)$$

$$y_2 = \dot{y}, \quad \int_{y_0}^{y_d} y_2dt = \int_{y_0}^{y_d}\dot{y}dy = (y_0 - y_d)$$

$$y_3 = -ml\dot{\theta}_xC_xC_y + ml\dot{\theta}_yS_xC_y, \quad \int y_3dt = mlC_yS_x$$

$$y_4 = ml\dot{\theta}_yC_y, \quad \int y_4dt = mlS_y$$

Substituting the above values and performing some calculations yield,

$$\begin{aligned}\dot{V} &= (k_1y_1 + k_3y_3)[k_pk_1mlC_yS_x - k_pk_1(x_0 - x_d) + k_eu_x] \\ &+ (k_2y_2 + k_4y_4)[k_pk_2mlS_y - k_pk_2(y_0 - y_d) + k_eu_y]\end{aligned} \tag{27}$$

The system can be made passive by selecting the control law,

$$u_x = \frac{-1}{k_e}\left[k_p k_1 ml C_y S_x - k_p k_1 (x_0 - x_d) - k_{d1}(y_1 + y_3)\right] \quad (28)$$

$$u_y = \frac{-1}{k_e}\left[k_p k_2 ml S_y - k_p k_2 (y_0 - y_d) - k_{d2}(y_2 + y_4)\right] \quad (29)$$

which will lead to $\dot{V} = -k_{d1}(y_1 + ky_3)^2 - k_{d2}(y_2 + ky_4)^2$

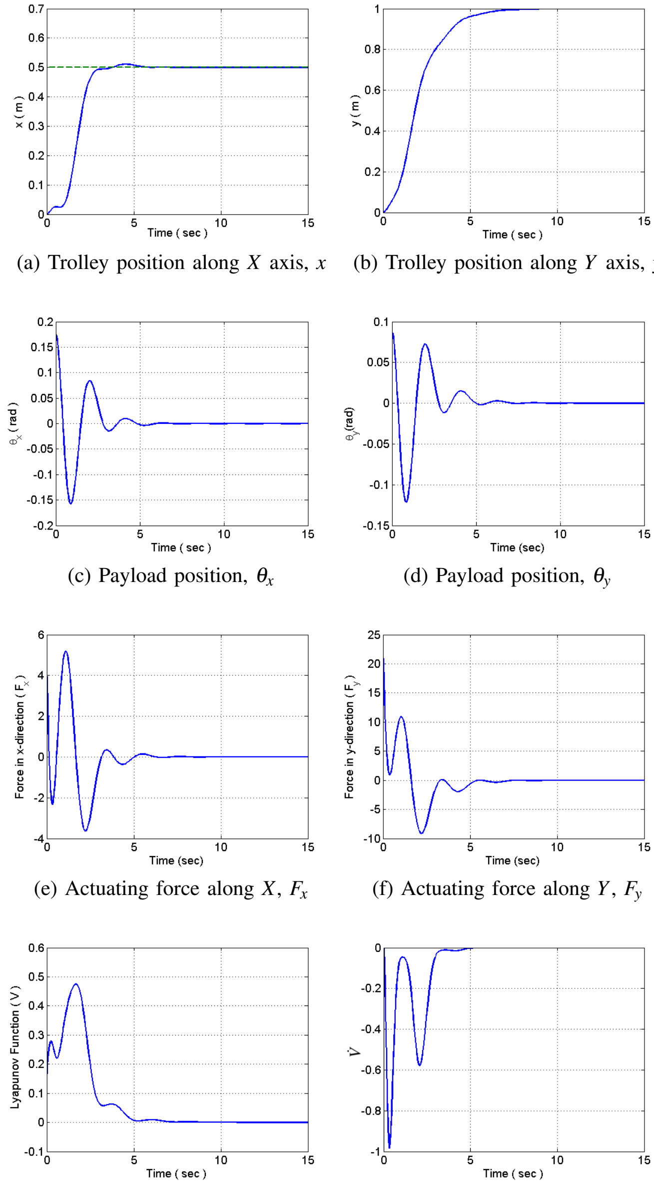
Thus $\dot{V}$ is negative semidefinite (for $k_d > 0$). The system is made passive and is stable under the action of control u_x and u_y. The total control effort F_x, F_y can be determined by substituting values of control u_x, u_y from (28) and (29), and $\ddot{\theta}_x$, $\ddot{\theta}_y$ from (21) and (22) in (13) and (14).

Remark: The nonlinear control law requires the full state feedback which includes the information of trolley/girder positions (x, y), trolley/girder velocities ($\dot{x}$, $\dot{y}$) and swing angular rate ($\dot{\theta}_x$, $\dot{\theta}_y$).

V. Simulation Results

MATLAB simulation results for the 4–DOF overhead crane system are presented in this section. The system parameters chosen for the simulation are $m_x = 6.5$ kg, $m_y = 22$ kg, $m = 1.025$ kg, $l = 0.8$ m and $g = 9.81$ m/sec^2. The desired trolley position is $x_d = 0.5$ m and $y_d = 1$ m. The Fig. 2 represents the time response plots for the closed loop system under the control law with initial condition $(x, y, \theta_x, \theta_y, \dot{x}, \dot{y}, \dot{\theta}_x, \dot{\theta}_y) = (0, 0, 10^{\text{o}}, 5^{\text{o}}, 0, 0, 0, 0)$. The gains in the control law were set to $k_p = 1$, $k_1 = 1.5$, $k_2 = 1$ and $k_{d1} = k_{d2} = 2$. The system comes to the desired position within 7 sec with pendulum angle $\theta_x = \theta_y = 0$. As mentioned in [16], constraint on the maximum force generated by the actuating motors are $F_{mx} = 20$ N and $F_{my} = 30$ N for the experimental purpose. It can be verified from Fig. 2e and 2f that the forces required are within the constraint limit.

The performance of Geometric - passivity based controller (G-PBC) is validated in comparison with gantry kinetic energy coupling (GKEC) control law [15] and energy coupling based output feedback (OFB) control scheme [16]. It can be clearly seen that both the G-PBC and GKEC controller schemes drives the trolley to desired position in approx. 8 seconds, however the G-PBC controller gives better results for elimination of payload swings. The GKEC controller exceeds the input constraints due to which the controller is partially saturated. When compared with OFB controller, the performance of G-PBC controller is comparable in areas of desired location time and payload swing suppression. The control efforts required in G-PBC controller are lower than OFB controller.



(a) Trolley position along X axis, x (b) Trolley position along Y axis, y

(c) Payload position, θ_x (d) Payload position, θ_y

(e) Actuating force along X, F_x (f) Actuating force along Y, F_y

(g) Lyapunov Function, V (h) $\dot{V}_c$

Fig. 2: Simulation results for stabilization of 4–DoF crane system

VI. Conclusions

A nonlinear control law is obtained for 4-DOF overhead crane using Geometric –passivity based control approach which achieves the precise payload positioning with quick swing motion elimination. The new approach eliminates the need of matching conditions and hence solving PDEs is not required. The constraints on input forces and other system parameters are considered for simulation purpose. In future, the method can be extended for tracking case with conditions such as initial swing and variation in payload. An important direction is extending the method to obtain almost global asymptotic result for 4-DOF crane system. The observer design using velocity estimators can be explored in future.

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