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Sensitivity and Related Gradient Methods in Electric Power Systems

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***Abstract*— With this contribution to the study of electric power system behavior we bring into focus the role of eigenvalue sensitivity under incremental changes of stationary power flow solutions. This analysis is motivated by the ubiquity of high spectral sensitivity in a large class of physical systems that are distributed in space, where the underlying mechanism is related to the nonnormality of the linearized dynamics. To this end we relate high sensitivity in stressed power grids to elements in the analysis of nonnormal systems, namely the ill-conditioning due to spatial transport processes. In doing so we present two recently, and independently derived novel formulas for eigenvalue deviations under changes in operating point and relate them to each other. It turns out that these eigenvalue sensitivity formulas play a fundamental role for power system behavior and we establish relations to recently proposed phase-coupled oscillator models for power systems. To conclude we discuss the use of the proposed eigenvalue sensitivity formulas for more flexible operation architectures in which PMU data may be incorporated for real-time coordination of local controls.**

I. SENSITIVITY IN NONNORMAL AND ELECTRIC POWER SYSTEMS: SIMILARITIES AND DIFFERENCES

The term nonnormality in systems refers to linear dynamics, where the system matrix $\boldsymbol{A}$ is nonselfadjoint, i.e. $\boldsymbol{A}^*\boldsymbol{A} \neq \boldsymbol{A}\boldsymbol{A}^*$, where $(\cdot)^*$ denotes the adjoint. A prototypical example is given by

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}, \quad \boldsymbol{A} = \begin{pmatrix} -\alpha & 0 \\ \gamma & -\beta \end{pmatrix}, \tag{1}$$

where nonnormality is caused by the coupling term γ. When $\boldsymbol{A}$ is regarded as system operator of a distributed system described by a PDE, (i.e. α, β, γ are differential operators, or appropriately fine discretizations of it), then coupling, and by that nonnormality, can be related to *convective transport* of mass, (or any quantity that has a density), which is *directed in space*, see e.g. [1] and [2] for a collection of examples. It is well-known that these cross terms may be a source of high sensitivity of the spectrum of $\boldsymbol{A}$ with respect to matrix disturbances or external forcings. For asymptotically stable small perturbation dynamics this sensitivity becomes manifest in the time domain by the transient amplification of system energy over several orders in magnitude, which results in a small domain of attraction.

To see this, suppose $\boldsymbol{A} \in \mathbb{R}^{2\times 2}$ with stable eigenvalues, i.e. $\lambda_{1,2} = -\alpha, -\beta \in \mathbb{R}^-$. The response of an eigenvalue λ_i with respect to an *internal perturbation*, i.e. $\boldsymbol{A} + \epsilon\Delta\boldsymbol{A}$ determines the dynamics, can be expressed for small ϵ as

$$\delta\lambda_i = \frac{\langle \boldsymbol{w}_i, \Delta\boldsymbol{A}\boldsymbol{v}_i\rangle}{\langle \boldsymbol{w}_i, \boldsymbol{v}_i\rangle}, \tag{2}$$

where $\boldsymbol{v}_i, \boldsymbol{w}_i$ are the right (direct) and left (adjoint) eigenvectors associated to λ_i of $\boldsymbol{A}$, and $||\Delta\boldsymbol{A}|| = 1$. For nonselfadjoint $\boldsymbol{A}$ the common support of the direct and adjoint eigenvectors may become very small resulting in a small divisor in (2). This leads to a large $\delta\lambda_i$, because a perturbation structure $\Delta\boldsymbol{A}$, may be chosen such that $\langle \boldsymbol{w}_i, \Delta\boldsymbol{A}\boldsymbol{v}_i\rangle \geq 1$, cf. to [1].

The effect of an *external disturbance* $\boldsymbol{d}$, s.t. $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{d}$, can be understood from considering $\boldsymbol{d}(t^-), -\infty < t^- \leq 0$, driving the (stable) system to some state $\boldsymbol{x}_0 = \boldsymbol{x}(t=0) \neq \boldsymbol{0}$. The transient behavior of $||\boldsymbol{x}(t)||$, i.e. the square root of the system energy, is bounded from above by

$$||\boldsymbol{x}(t)|| \leq ||e^{\boldsymbol{A}t}||\,||\boldsymbol{x}_0|| \leq ||\boldsymbol{T}|| \cdot ||\boldsymbol{T}^{-1}|| \max_{\lambda_i} e^{\Re(\lambda_i)t}||\boldsymbol{x}_0||, \tag{3}$$

where $\boldsymbol{T}$ represents the eigenbasis s.t. $\boldsymbol{A} = \boldsymbol{T}\mathrm{diag}(\lambda_i)\boldsymbol{T}^{-1}$. That is, the upper bound of any energy defined by a bilinear form $\langle \boldsymbol{x}, \boldsymbol{P}\boldsymbol{x}\rangle$, $\boldsymbol{P} \succ 0$, essentially depends on the condition number of the eigenbasis

$$\kappa(\boldsymbol{T}) = ||\boldsymbol{T}|| \cdot ||\boldsymbol{T}^{-1}|| = \frac{\sigma_{\max}(\boldsymbol{T})}{\sigma_{\min}(\boldsymbol{T})} \geq 1, \tag{4}$$

where σ means singular value. Whenever the eigenvectors contained in $\boldsymbol{T}$ are nonnormal, i.e. mutually not orthogonal, the value of κ will be strictly greater than one; it will be much larger than one and increases when the strength of cross coupling is high and increasing. Thus, a nonnormal matrix $\boldsymbol{A}$ may exhibit large transient amplification of system energy before the decaying behavior sets in due to negativity of the real parts of the eigenvalues.

An alternative bound can be derived from an input-output view, where $\boldsymbol{x}_0$ as input is mapped via the transition operator $e^{\boldsymbol{A}t}$ to the output $\boldsymbol{x}(t)$, which results in

$$\frac{||\boldsymbol{x}(t)||}{||\boldsymbol{x}_0||} \leq \sigma_{\max}(e^{\boldsymbol{A}t}). \tag{5}$$

Remark 1: In the control community, the transient behavior and energy amplification has been studied in [3] in the context of fluid flows.

To summarize, spatially distributed systems that have directed transport in space, e.g. convection terms, yield nonnormal system matrices. By that, linear models may show high sensitivity and fragility of the behavior w.r.t.

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external and internal uncertainty. This sensitivity becomes manifest in a large maximal singular value of the transition operator and/or in a large condition number of the associated eigenbasis, resp. an ill-conditioned system matrix.

In electric power systems, experience shows that the dynamic behavior "moves from being elastic to brittle", i.e. the system looses the inherent tolerance or robustness to small disturbances, when the grid becomes highly loaded [4]. However, the commonly adopted linear dynamic models do not show large amplification of perturbation energy, and the system operators that are classically studied are highly symmetric. By that the condition number of the eigenbasis is close to or equals one, and $\sigma_{\max}$ of the transition operator coincides with the maximal real eigenvalue, which is negative for stable systems. Thus, while for many examples of distributed physical systems high sensitivity is reflected in the nonnormality of the system matrix obtained from linearization, in electric power systems fragility of the dynamic behavior is not reflected in classically adopted models of small signal dynamics.

Yet, in certain problems discussed in the power systems literature similarities can be found. In [5] it is argued that interacting complex modes may cause subcritical oscillatory behavior, especially in situations when two modes are almost collinear, hence the eigenbasis is far from being normal. The analysed models, however, rather serve academic purposes than practical considerations. Another interesting feature of power systems is that processing real data sets using power flow equations leads to ill-conditioned problems [6]. In [7] it is observed that the power flow Jacobian becomes ill-conditioned for a variety of grid structures when the grid is highly loaded. To make this point more precise, we introduce by $\boldsymbol{z}^T = (\boldsymbol{\theta}^T, \boldsymbol{V}^T)$ the vector of angles and voltage magnitudes in a power grid. Using the scalar potential function

$$\begin{aligned}\Pi(\boldsymbol{z}) = &- \sum_{(i,j)\in\mathcal{E}} \Im(Y_{ij}) V_i V_j \cos(\theta_i - \theta_j) \\ &- \sum_{i=1}^{n} (P_i\theta_i + \frac{1}{2}\Im(Y_{ii})V_i^2 + Q_i \ln V_i), \end{aligned} \quad (6)$$

where $\mathcal{E}$ denotes the set of edges or lines (i, j) with admittance Y_{ij}, and i, j are network buses, the power flow equations can be stated as $\nabla_{\boldsymbol{z}}\Pi(\boldsymbol{z}) = \boldsymbol{0}$. Then, solving this equality for the active and reactive powers $\boldsymbol{P}$ and $\boldsymbol{Q}$, the Jacobian $\boldsymbol{J}_{\mathrm{PF}}$ is obtained as the operator that maps small increments in angles and voltage magnitudes to incremental powers such that

$$\begin{pmatrix}\mathrm{d}\boldsymbol{P}\\ \mathrm{d}\boldsymbol{Q}\end{pmatrix} = \boldsymbol{J}_{\mathrm{PF}} \begin{pmatrix}\mathrm{d}\boldsymbol{\theta}\\ \mathrm{d}\boldsymbol{V}\end{pmatrix}. \quad (7)$$

Note that $\boldsymbol{J}_{\mathrm{PF}}$ is a Hessian of the terms in Π relating to differences across lines. Similar to nonnormal systems, κ is of the order 10^3 in stressed grids and approaches infinity at the point of voltage collages, see [7]. However, the power flow Jacobian is an element in solving a static problem, e.g. in an iterative scheme such as the Newton Raphson method; in that it is important to note that a particular node has to be taken as constant reference ("infinite mass") with respect to which all other variables are computed. The relation to dynamics represented by LTI models is not clear. Another difference to ill-conditioning of nonnormal LTI system models is the source of ill-conditioning: here it is not a large maximum singular value of the "input-output" mapping $\boldsymbol{J}_{\mathrm{PF}}$, but a small $\sigma_{\min}(\boldsymbol{J}_{\mathrm{PF}})$ cf. to [7].

II. Dynamics of Electric Power Systems and Eigenvalue Deviations under Incremental Power Flow Changes

A. *Multi-Machine Power System Model*

The simplest dynamic model is that of oscillating masses (generators) that are coupled through the electric network. Let n denote the number of buses in the power systems, and m the number of generator buses, Then the dynamics of a generator can be written as

$$m_i\ddot{\theta}_i + d_i\dot{\theta}_i = -\frac{\partial\Pi}{\partial\theta_i}, \quad i = 1,\ldots,n \quad (8a)$$

$$0 = -\frac{\partial\Pi}{\partial V_i}, \quad i = m+1,\ldots,n \quad (8b)$$

Note that for $i = m+1,\ldots,n$ (8a) becomes a balance equation for active power.

Speaking about stability of small signal dynamics, one has to look at the small perturbation dynamics that evolve about a steady state $\boldsymbol{z}^{\mathrm{ss}}$. A steady state is obtained from solving the power flow equations. The solution $\boldsymbol{z}^{\mathrm{ss}}$ then serves as setpoint or operating condition, e.g. for locally controlled dynamics. Denote by $\Delta\boldsymbol{z}$ small perturbations of the equilibrium point $\boldsymbol{z}^{\mathrm{ss}}$. The small signal dynamics of the multi-machine system satisfy the quadratic equation

$$\boldsymbol{M}\Delta\ddot{\boldsymbol{z}} + \boldsymbol{D}\Delta\dot{\boldsymbol{z}} + \boldsymbol{L}(\boldsymbol{z}^{\mathrm{ss}})\Delta\boldsymbol{z} = \boldsymbol{0}, \quad (9)$$

where $\boldsymbol{M}, \boldsymbol{D}$ are diagonal matrices, and $\boldsymbol{L} = D^2\Pi(\boldsymbol{z})|_{\boldsymbol{z}^{\mathrm{ss}}}$, where D^2 denotes the Hessian, is symmetric.

Remark 2: More detailed models for generator dynamics lead to nonlinear DAE systems of the form

$$\dot{\mathbf{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}) \quad (10a)$$

$$\boldsymbol{0} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}) \quad (10b)$$

where $\boldsymbol{y}$ are algebraic variables. The associated small perturbation dynamics evolve according to

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\Delta\boldsymbol{x}\\ \boldsymbol{0}\end{pmatrix} = \begin{bmatrix}\nabla_{\boldsymbol{x}}\boldsymbol{f}|_{\boldsymbol{z}^{\mathrm{ss}}} & \nabla_{\boldsymbol{y}}\boldsymbol{f}|_{\boldsymbol{z}^{\mathrm{ss}}}\\ \nabla_{\boldsymbol{x}}\boldsymbol{g}|_{\boldsymbol{z}^{\mathrm{ss}}} & \nabla_{\boldsymbol{y}}\boldsymbol{g}|_{\boldsymbol{z}^{\mathrm{ss}}}\end{bmatrix}\begin{pmatrix}\Delta\boldsymbol{x}\\ \Delta\boldsymbol{y}\end{pmatrix}. \quad (11)$$

B. *Small Signal Stability and Eigenvalue Sensitivity*

Asymptotic stability of the operating point $\boldsymbol{z}^{\mathrm{ss}}$ can be devised from negative real parts of the eigenvalues of the quadratic eigenvalue problem

$$\boldsymbol{Q}(\lambda_i)\boldsymbol{v}_i = \boldsymbol{0}, \quad \boldsymbol{Q}(\cdot) = \lambda_i^2\boldsymbol{M} + \lambda_i\boldsymbol{D} + \boldsymbol{L}. \quad (12)$$

In the following we omit indexing of eigenpairs.

Remark 3: For certain DAE models the representation is determined by a system matrix $\boldsymbol{A}(\boldsymbol{z}^{\mathrm{ss}}) = \boldsymbol{J}_{\mathrm{PF}}|_{\boldsymbol{z}^{\mathrm{ss}}}$.

We are interested in changes of an eigenvalue λ, when the operating point changes. That is, we want to estimate the effect of a change $\delta\boldsymbol{z}^{\mathrm{ss}} = \boldsymbol{z}_+^{\mathrm{ss}} - \boldsymbol{z}^{\mathrm{ss}}$ on an eigenvalue resulting in $\delta\lambda = \lambda_+ - \lambda$. In that a change in operating point can be seen as a means of controlling $\delta\lambda$ in terms of exploiting the nonlinearity in the power flow equations by which incremental power flow (setpoint) changes act.

C. Sensitivity from Externally Induced Incremental Changes in Operating Point

Suppose an external power disturbance $\boldsymbol{d}$ affects the system as a stationary forcing. The steady state power flows internally rebalance such that the forced version of the power flow equations

$$\nabla_{\boldsymbol{z}}\Pi(\boldsymbol{z}_+^{\mathrm{ss}}) + \boldsymbol{d} = \boldsymbol{0} \tag{13}$$

holds. The variation in $\delta\lambda$ can be formally stated as the Gateaux differential

$$\delta\lambda = \lim_{\epsilon\to 0^+} \frac{\lambda_+(\boldsymbol{z}^{\mathrm{ss}} + \epsilon\delta\boldsymbol{z}^{\mathrm{ss}}(\boldsymbol{d})) - \lambda(\boldsymbol{z}^{\mathrm{ss}})}{\epsilon} := \langle \boldsymbol{S_d}, \boldsymbol{d}\rangle \tag{14}$$

where $\boldsymbol{S_d}$ denotes the most sensitive direction of the eigenvalue w.r.t. the action of the external disturbance $\boldsymbol{d}$ such that $||\boldsymbol{d}|| = 1$. In addition, λ_+ is required to satisfy the eigenvalue problem parameterized by the post disturbance operating point $\boldsymbol{z}_+^{\mathrm{ss}}$. The desired gradient / sensitivity direction $\boldsymbol{S_d}$ along which $\boldsymbol{d}$ acts on $\delta\lambda$ can be derived from the Lagrangian

$$\mathcal{L}(\lambda^+, \boldsymbol{\mu}, \boldsymbol{z}_+^{\mathrm{ss}}(\boldsymbol{d})) = ||\delta\lambda|| - \langle \boldsymbol{\mu}, \boldsymbol{Q}(\boldsymbol{z}_+^{\mathrm{ss}}(\boldsymbol{d}))\boldsymbol{v}\rangle, \tag{15}$$

where the vector $\boldsymbol{\mu}$ denotes the vector of Lagrange multipliers, together with stationarity of the Lagrangian as necessary optimality condition, i.e.

$$\delta\mathcal{L} = \langle \boldsymbol{S_d}, \boldsymbol{d}\rangle \overset{!}{=} 0. \tag{16}$$

This approach has been presented for the general system class (10) in [8], and here we apply the method on the power system model (8) using the associated quadratic eigenvalue problem as constraint.

The result can be expressed by means of the power flow Jacobian, the scalar potential Π and the external steady power disturbance $\boldsymbol{d}$, as

$$\boxed{\delta\lambda = \frac{\epsilon}{Z}\langle (\boldsymbol{J}_{\mathrm{PF}}^{-1})^* \boldsymbol{S_z}, \boldsymbol{d}\rangle} \tag{17}$$

$$\text{with} \quad \boldsymbol{S_z} := \nabla_{\boldsymbol{z}}[D^2\Pi(\boldsymbol{z})\boldsymbol{v}]_{\boldsymbol{z}^{\mathrm{ss}}}]^*\boldsymbol{v} \tag{18}$$

denoting the most sensitive direction of $\delta\lambda$ w.r.t. an internal redispatch induced by $\delta\boldsymbol{z}^{\mathrm{ss}}$; the quantity $Z := \langle \boldsymbol{v}, [2\lambda\boldsymbol{M} + \boldsymbol{D}]\boldsymbol{v}\rangle$ defines a necessary scaling parameter s.t. $\boldsymbol{\mu} = Z^{-1}\boldsymbol{v}$.

D. Sensitivity from Internal Incremental Redispatch

Eigenvalue deviations from internal redispatch can be estimated using the inner product formula

$$\boxed{\delta\lambda = \frac{1}{Z}\langle \boldsymbol{v}, \mathrm{d}\boldsymbol{L}\boldsymbol{v}\rangle}. \tag{19}$$

In [9] a formula has been derived for $\mathrm{d}\boldsymbol{L}$. With that, equation (19) can be expressed in terms of steady state angle and voltage differences across lines and at buses, so that with $e = (i,j) \in \mathcal{E}$,

$$\delta\lambda = -\frac{1}{Z}\left[\sum_{e\in\mathcal{E}}[c_1(\boldsymbol{v},e)P_e - c_2(\boldsymbol{v},e)Q_e]\delta(\theta_i^{\mathrm{ss}} - \theta_j^{\mathrm{ss}}) + \right. \tag{20}$$

$$\left.\sum_{i=m+1}^{n}\left[\sum_{j\in\mathcal{N}_i}[c_3(\boldsymbol{v},e)Q_e + c_4(\boldsymbol{v},e)P_e] + c_5 Q_i^{\mathrm{ss}}\right]\delta\ln V_i^{\mathrm{ss}}\right] \tag{21}$$

where $c_{1,\dots,5}$ are constants that depend on the entries of the eigenvector $\boldsymbol{v}$, $\mathcal{N}_i$ denotes the neighborhood of i, and

$$P_e = \Im(Y_{ij})V_iV_j\sin(\theta_i^{\mathrm{ss}} - \theta_j^{\mathrm{ss}}), \tag{22a}$$

$$Q_e = -\Im(Y_{ij})V_iV_j\cos(\theta_i^{\mathrm{ss}} - \theta_j^{\mathrm{ss}}). \tag{22b}$$

That is, eigenvalue deviations depend linearly on steady state power flows weighted by some eigenvector dependent constants. Note that $\delta(\theta_i^{\mathrm{ss}} - \theta_j^{\mathrm{ss}})$ can be regarded as variation of a new line coordinate θ_e^{ss} which highlights the importance of relative changes within the network.

III. Discussion

A. Power Flow Jacobian and Sensitive Spectrum

From formula (19) no specific argument can be derived about highly sensitive behavior. However, using (7) an external, small and steady power disturbance $\epsilon\boldsymbol{d}$ induces changes $\delta\boldsymbol{z}^{\mathrm{ss}}$ via mapping with $\boldsymbol{J}_{\mathrm{PF}}$. By that we can relate formulas (19) and (17) as

$$\boxed{\delta\lambda = \frac{1}{Z}\langle \boldsymbol{v}, \mathrm{d}\boldsymbol{L}\boldsymbol{v}\rangle = \frac{1}{Z}\langle \boldsymbol{S_z}, \delta\boldsymbol{z}^{\mathrm{ss}}\rangle = \frac{\epsilon}{Z}\langle \boldsymbol{S_z}, \boldsymbol{J}_{\mathrm{PF}}^{-1}\boldsymbol{d}\rangle}. \tag{23}$$

Knowing that $\kappa(\boldsymbol{J}_{\mathrm{PF}}) \approx 10^3$ in stressed situations, and using (4), we see that a weighting with $\boldsymbol{J}_{\mathrm{PF}}^{-1}$ can result in large amplification of the energy provided by the input $\boldsymbol{d}$, because $||\boldsymbol{J}_{\mathrm{PF}}^{-1}|| = \frac{1}{\sigma_{\min}(\boldsymbol{J}_{\mathrm{PF}})}$ tends to very large numbers along certain external disturbance patterns, because $\sigma_{\min} \to 0$ with increasing loading factor.

From this viewpoint incremental power flow changes may indeed lead to high sensitivity and “brittle” behavior in highly loaded situations, with the mechanism being transport of power in space and the associated ill-conditioning of the related Jacobian.

B. Kuramoto Dynamics and Spectral Sensitivity

Novel insights into power system behavior have been derived from relations between Kuramoto-type coupled oscillator systems and (8), see [10]. Consider the potential function $U(\boldsymbol{\theta}) = \sum_{(i,j)\in\mathcal{E}} \Im(Y_{ij})V_iV_j(1-\cos(\theta_i-\theta_j))$, and the associated gradient flow

$$\dot{\theta}_i = -\sum_{j\in\mathcal{N}_i} \Im(Y_{ij})V_iV_j\sin(\theta_i-\theta_j) = -\nabla_{\theta_i}U(\boldsymbol{\theta}). \quad (24)$$

The dynamics (24) represent an (unforced) Kuramoto-type system with phase coupling according to the network structure. This system type has been applied in studying (transient) stability problems in electric power systems, see [10]. For the special case of having constant voltages, the terms in (21) vanish. In addition, considering only those terms that relate to active power one obtains for eigenvalue changes the relation

$$\delta\lambda = -\frac{1}{Z}\sum_{e\in\mathcal{E}} c_1(v(\boldsymbol{\theta},e))P_e\delta\theta_e. \quad (25)$$

For small variations as $\delta\theta_e \to 0^+$, edge-wise one has

$$P_e = -\frac{Z}{c_1}\frac{\partial\lambda}{\partial\theta_e}, \quad (26)$$

so that

$$-\sum_{e\in\mathcal{E}} P_e = \sum_{e\in\mathcal{E}} \frac{Z}{c_1}\frac{\partial\lambda}{\partial\theta_e}, \quad (27)$$

Using $P_e = \Im(Y_{ij})V_iV_j\sin(\theta_i-\theta_j)$ the phase-coupled dynamics can be related to eigenvalue sensitivity such that

$$\boxed{\dot{\theta}_i = -\mathrm{grad}_{\theta_i}U = Z\mathrm{div}^{c_1}_{\mathcal{N}_i}\lambda}. \quad (28)$$

By $\mathrm{div}^{c_1}_{\mathcal{N}_i}$ we denote the divergence operator $\nabla\cdot$ w.r.t. line coordinates, weighted by $c_1^{-1}(\cdot,e)$, across the neighborhood of bus i. Thus, the gradient flow of the phase-reduced first order system has a divergence form. The dual (formal adjoint) to the grad operator is the negative divergence operator, and similarly, the description in line coordinates (across) is dual to one in nodal coordinates; within this context, $\lambda\in\mathcal{C}$ appears to be dual to the additive potential $U(\boldsymbol{\theta})$ and its (local) sensitivities drive the dynamics.

While in recent Kuramoto-type models only active powers and constant voltage magnitudes are considered, the equivalence (28) and the formulas (20) with (21) hint towards the possibility to incorporate variations of these terms by using spectral sensitivity methods in Kuramoto-type approaches for the study of power system behavior.

C. Sensitivity Based Architecture

In the traditional operation architecture of power systems there is a methodological gap between static optimization based planning schemes and methods for the (local) control of dynamics, cf. to Fig. 1. It is commonly believed that coordination of local controllers at a wider geographical scale may provide additional system flexibility that is required for accommodating increasing stress and variability. Increased flexibility would be obtained through the controlled transport of power (external disturbances) from regions with high sensitivity to regions of less fragility. The sensitivity based combination of static optimization tools via power flow Jacobians, from where sensitive regions or power flow patterns may be computed, together with coordinated local control action may be one approach towards a sensitivity based novel operation architecture. In [11] the presented sensitivity formula (17) has been applied to the much discussed area of quantifying technical flexibility in power systems.

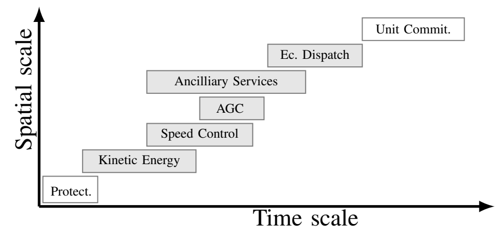

Fig. 1. Traditional operation architecture

The formulas (20), (21) could be used for calculations with PMU data, thus enabling new realt-time control schemes.

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