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Leader Following of Partially Heterogeneous Multi-Agent Systems via Cooperative Adaptive Control

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***Abstract*— In this paper, we deal with a leader following problem of heterogeneous uncertain multi-agent systems. We propose a cooperative adaptive control framework which is composed of two parts: a cooperative adaptive controller and a direct relative state feedback controller. Then we show that in this proposed multi-agent system, all the state variable errors and adaptive gain errors are Lyapunov stable and also all the signals used in the controller are bounded even if the leader is not asymptotically stable. From this property, we explain that the proposed controller can deal with an important application of multi-agent systems, a vehicle formation problem.**

***keywords*— multi-agent system, leader following, adaptive control, distributed control**

***AMS subject classifications*— 93A14, 93C40, 93C55, 93D21**

I. Introduction

In recent years, cooperative control of multi-agent systems (MAS) composed of many dynamical systems has been actively investigated [21], [8], [5], [19], [18], [7], [6], [22]. Its main topics are consensus, synchronization, and leader following problems and they can be formulated as synchronization of (part of) the state variables or the outputs of agents. Their applications include sensor networks, cooperative robots control, vehicle formation, and so on [4], [20], [3].

The standard setting of the conventional cooperative control problems is to assume that the dynamics of agents are identical, that is, homogeneous MAS. However, it is more realistic that the dynamics of agents are different, that is, heterogeneous MAS [22], [12], [1], and (partially) unknown in many applications. Adaptive control and robust control [11] are possible approaches to such situations and we deal with the former in this paper. This approach for the uncertain heterogeneous MAS has been investigated in several works [9], [10], [14], [15]. In particular, [9], [10] extended the standard centralized model reference adaptive control [16], [2] to a cooperative adaptive synchronization problem of MAS. Moreover, [17] proposed the corresponding cooperative adaptive controller for a leader following problem in discrete time.

In the results of [9], [10], however, the reference models are assumed to be asymptotically stable. Therefore, they are not applicable to the cases of vehicle formation in which the positions of the vehicles are usually supposed to be unbounded, although vehicle formation is one of the most important applications of MAS. In order to solve this difficulty, in this paper, we consider cooperative adaptive control for MAS, where the reference models are not necessarily stable. The dynamics of each agent is supposed to be decomposed into a partially unknown subsystem and a partially known and identical subsystem. Hence, we consider a partially heterogeneous MAS.

The proposed cooperative adaptive controller has a mechanism of parameters update law by using relative errors of state variables of neighbors and making the dynamics of the agents close to the reference model. Simultaneously, it also has a mechanism of a direct feedback of the relative errors between the state variables with the neighbors in order to stabilize the relative dynamics of the entire MAS. This method can also be applicable to the case where a part of the state variables of the reference model is unstable and unbounded.

The organization of this paper is as follows: In Section II, we provide the preliminaries for representing MAS with graphs and then introduce the formulation of MAS and a leader following problem considered in this paper. In Section III, we propose a cooperative adaptive controller and show the main result of this paper which clarifies that the proposed controller can attain the leader following. In Section IV, we illustrate our results by a numerical example of vehicle formation problems in order to show the efficiency of our proposed approach. Finally, we conclude this paper in Section V.

II. Formulation

A. Graph representation of information exchange

In this paper, by following the standard manner, we represent the structure of the information exchange between agents by a weighted directed graph. A graph is represented by $\mathcal{G} := (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} := \{1, 2, \ldots, N\}$ is the set of vertices and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of directed edges between the vertices. Each vertex $i \in \mathcal{V}$ represents the ith agent and edge $(i, j) \in \mathcal{E}$ represents information transmission from agent j to i. The neighbor $\mathcal{N}_i := \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$ of agent i represents the set of agents from which information

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is transmitted to agent i. Furthermore, define the weighted incident matrix $M = [w_{ij}] \in \mathbb{R}^{N\times N}$ where w_{ij} is a positive weight on a directed edge $(i,j) \in \mathcal{E}$ and $w_{ij} = 0$ when $(i,j) \notin \mathcal{E}$.

With a diagonal matrix $D \in \mathbb{R}^{N\times N}$ such that its ith diagonal element is equal to row sum of the ith row of M, the graph Laplacian matrix is given by $L := D - M$. Therefore, it is known that the graph Laplacian obviously has a zero eigenvalue and the corresponding eigenvector $\mathbf{1}_N = [1\ 1\ \cdots\ 1]^{\mathrm{T}} \in \mathbb{R}^N$. The real parts of the other eigenvalues are positive and the zero eigenvalue is simple if and only if the corresponding graph has a directed spanning tree [19].

In this paper, we define "a leader" agent of index $l\ (=1)$ as an ideal model which the other agents follow. Those agents are called "followers" with unknown dynamics and the set of their indices is defined by $\mathcal{V}_{\mathrm{f}} := \{2, 3, \dots, N\}$.

We consider the case where each follower is given a controller composed of an adaptive controller and an LTI feedback controller. Those controllers are with the use of the relative errors of state variables between the agent and its neighbors. For representing the neighbors, we define directed graphs $\mathcal{G}^{\mathrm{A}} = (\mathcal{V}, \mathcal{E}^{\mathrm{A}})$ and $\mathcal{G}^{\mathrm{R}} = (\mathcal{V}, \mathcal{E}^{\mathrm{R}})$ which represent information structures used by the adaptive controller and the LTI feedback controller, respectively. In this paper, we suppose that $\mathcal{E}^{\mathrm{A}}$ and $\mathcal{E}^{\mathrm{R}}$ may be different in general and this allows a variety of controller designs. We also represent notations $\mathcal{N}_i^{\mathrm{A}}$, w_{ij}^{A}, L^{A} or $\mathcal{N}_i^{\mathrm{R}}$, w_{ij}^{R}, L^{R} in $\mathcal{G}^{\mathrm{A}} = (\mathcal{V}, \mathcal{E}^{\mathrm{A}})$ or $\mathcal{G}^{\mathrm{R}} = (\mathcal{V}, \mathcal{E}^{\mathrm{R}})$, respectively.

We also assume that no edge to the leader $l = 1$ exist in the graphs $\mathcal{G}^{\mathrm{A}}$ and $\mathcal{G}^{\mathrm{R}}$, which implies that the first row of L is 0. Related to this, let $\bar{L} \in \mathbb{R}^{(N-1)\times(N-1)}$ be the matrix such that the first row and the first column are removed from L. Note that

$$\{\lambda_i(L)\} = \{\lambda_i(\bar{L})\} \cup \{0\}, \tag{1}$$

where $\lambda_i(\bullet)$ represents the ith eigenvalue of $\bullet$.

Moreover, we assume that the graph $\mathcal{G}^{\mathrm{A}}$ satisfies the following conditions:

Condition 1:

1. When agent i and j are both followers and $(i,j) \in \mathcal{E}^{\mathrm{A}}$, then $(j,i) \in \mathcal{E}^{\mathrm{A}}$ and $w_{ij}^{\mathrm{A}} = w_{ji}^{\mathrm{A}}$.
2. The graph $\mathcal{G}^{\mathrm{A}}$ has a directed spanning tree from leader l as the root.

The first condition in Condition 1 implies that a graph composed of $\mathcal{G}^{\mathrm{A}}$ and the edges to the leader becomes a connected undirected graph. Then, the zero eigenvalue of L^{A} is simple. Now, we get the following lemma from (1):

Lemma 1: The matrix $\bar{L}^{\mathrm{A}}$ is positive definite.

B. Dynamics of agents

We consider the discrete-time MAS and the dynamics of the leader and the followers are given by

$$\begin{aligned} x_l(k+1) &= A_l x_l(k) + Br(k), \quad k \in \mathbb{N}, \\ x_l(0) &= x_{l0}, \end{aligned} \tag{2}$$

$$\begin{aligned} x_i(k+1) &= A_i x_i(k) + Bu_i(k), \quad k \in \mathbb{N}, \\ x_i(0) &= x_{i0}, \quad i \in \mathcal{V}_{\mathrm{f}}, \end{aligned} \tag{3}$$

where $x_l(k)$ and $x_i(k) \in \mathbb{R}^n$ represent the state vectors of the leader and the follower i, respectively, $u_i(k) \in \mathbb{R}^m$ is the control input to follower i with $n \geq m$, $r(k) \in \mathbb{R}^m$ is a finite reference and supposed to be known to all the agents, $A_l \in \mathbb{R}^{n\times n}$ is a known matrix, $B \in \mathbb{R}^{n\times m}$ is a known matrix with column-full-rank, and (A_l, B) is stabilizable.

Moreover, assume the following:

Assumption 1: An integer $n_1 \leq n$ is known and the state variable of each agent is decomposed to $x_i(k) = [z_i^{\mathrm{T}}(k)\ v_i^{\mathrm{T}}(k)]^{\mathrm{T}}$, where $z_i(k) \in \mathbb{R}^{n-n_1}$ and $v_i(k) \in \mathbb{R}^{n_1}$. Then,

1. (Partially heterogeneous MAS) For $i \in \mathcal{V}$, decompose $A_i \in \mathbb{R}^{n\times n}$ to
$$A_i = \begin{bmatrix} A_{1,i} & A_{2,i} \end{bmatrix} \tag{4}$$
where $A_{1,i} \in \mathbb{R}^{n\times(n-n_1)}$ is known and equal to $A_{1,l}$ and $A_{2,i} \in \mathbb{R}^{n\times n_1}$ is unknown but there exists $K_i \in \mathbb{R}^{m\times n_1}$ satisfying the following matching condition given by
$$A_{2,l} = A_{2,i} + BK_i \tag{5}$$
2. The partial state variable $v_l(k)$ of the leader is bounded, that is, there exists $\delta > 0$ satisfying
$$\|v_l(k)\|_2 < \delta, \quad k \in \mathbb{N}. \tag{6}$$

Remark 1: The matching condition (5) implies that there exists a "true" K_i which makes the follower's dynamics to that of the leader by a feedback $K_i v_i(k)$. The basic idea of our proposed cooperative adaptive controller given later is to adjust the dynamics of each agent to that of the leader by the adaptive law.

Remark 2: On the second condition (6) in Assumption 1, when the asymptotically stable eigenspace of A_l includes the space of v_l, such δ always exists for an arbitrary bounded r.

The special case $n_1 = n$ in Assumption 1 implies that all the elements of A_i are unknown and all the elements of the state vector of the leader should be bounded. On the other hand, when $n_1 < n$, it is possible to utilize the partially known information $A_{1,i}$ and apply an adaptive control in order to compensate the unknown $A_{2,i}$.

Example 1: A typical example of MAS satisfying Assumption 1 is vehicle formation control. Let h_i, $q_i(k)$, $p_i(k)$ and $x_i(k) = [q_i(k) - h_i, p_i(k)]^{\mathrm{T}}$ represent the ideal relative location of agent i to a reference point in the formation of the agents, the actual location, the velocity and the state vector, respectively. Then, define the dynamics (2) and (3) with

$$A_i = \begin{bmatrix} 1 & 1 \\ 0 & \alpha_i \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{7}$$

where $0 < \alpha_i < 1$; attenuation of the velocities. The velocity $p_l(k)$ of the leader is bounded for a bounded $r(k)$ and this case satisfies Assumption 1 with $n_1 = 1$. Note that the first column of A_i, that is, $\begin{bmatrix} 1 & 0 \end{bmatrix}^{\mathrm{T}}$, represents the obvious relationship between the position and the velocity in the state vectors and it is reasonable to assume that this part is known to all the agents.

C. Leader following problem

In this subsection, we introduce the problem of this paper and the related facts:

Problem 1: (Leader following problem) For the MAS (2) and (3), find controllers $u_i(k)$ satisfying:

$$\lim_{k\to\infty} e_{il}(k) := \lim_{k\to\infty} (x_i(k) - x_l(k)) = 0, \ i \in \mathcal{V}_{\mathrm{f}}. \tag{8}$$

The condition (8) implies the convergence of all the followers' states to that of the leader. With a vector $e_l(k) := \begin{bmatrix} e_{2l}^{\mathrm{T}}(k), \ldots, e_{Nl}^{\mathrm{T}}(k) \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{(N-1)n}$, (8) is represented as $\lim_{k\to\infty} e_l(k) = 0$.

Hereafter in this subsection, we introduce the related well-known results on the state synchronization problem of homogeneous MAS.

For agent i, define feedback control input u_i^{R} by

$$u_i^{\mathrm{R}}(k) = F \sum_{j \in \mathcal{N}_i^{\mathrm{R}}} w_{ij}^{\mathrm{R}} (x_i(k) - x_j(k)), \tag{9}$$

where $F \in \mathbb{R}^{m\times n}$ is a common gain for all the agents. Graph $\mathcal{G}^{\mathrm{R}}$, its weights, and F are chosen satisfying the next condition:

Condition 2: Let $\lambda_1(L^{\mathrm{R}}) = 0$, where L^{R} is the Laplacian of $\mathcal{G}^{\mathrm{R}}$. Then it follows that

$$\begin{aligned} &|\lambda_i(A_l + \lambda_j(L^{\mathrm{R}})BF)| < 1, \\ &i = 1, 2, \ldots, n, \ j = 2, 3, \ldots, N. \end{aligned} \tag{10}$$

Remark 3: In the case where A_l is unstable and (A_l, B) is stabilizable, the existence of F satisfying Condition 2 depends on the locations of the eigenvalues of L^{R}. In particular, note that a condition that the zero eigenvalue of L^{R} is simple is necessary for the existence of such F, but it is not sufficient. In this paper, further discussion on the existence and the design of F is omitted. See [13] for the details.

Under Condition 2, we can derive the following lemma:

Lemma 2: For the MAS (2) and (3), suppose $A_i = A_l$, $i \in \mathcal{V}$. Then, (8) is attained by the control input $u_i(k) = u_i^{\mathrm{R}}(k) + r(k)$.

Note that Lemma 2 holds even if an edge to agent 1 exists in $\mathcal{G}^{\mathrm{R}}$, that is, no leader exists. The proof for such a case is essentially the same to that in [5] and hence we omit it. In the case where a leader exists, define

$$\bar{A} := I_{N-1} \otimes A_l + \bar{L}^{\mathrm{R}} \otimes BF, \tag{11}$$

then we get

$$\begin{aligned} &\{\lambda_i(\bar{A}), \ i = 1, 2, \ldots, n(N-1)\} \\ &\quad = \big\{\lambda_i(A_l + \lambda_j(L^{\mathrm{R}})BF), \\ &\qquad i = 1, 2, \ldots, n, \ j = 2, 3, \ldots, N\big\}, \end{aligned} \tag{12}$$

because

$$(U \otimes I_n)\bar{A}(U^{-1} \otimes I_n) = I_{N-1} \otimes A_l + J \otimes BF, \tag{13}$$

where U is a transformation matrix for $\bar{L}^{\mathrm{R}}$ to the Jordan canonical form $J = U\bar{L}^{\mathrm{R}}U^{-1}$. Then Lemma 2 follows from Lemma 1.

From (12), it is clear that Condition 2 corresponds to the fact that $\bar{A}$ is asymptotically stable.

III. Leader following via a cooperative adaptive control strategy

A. Distributed adaptive controller

We define the control input $u_i(k)$ for follower i by

$$u_i(k) = u_i^{\mathrm{A}}(k) + u_i^{\mathrm{R}}(k) + r(k), \tag{14}$$

where $r(k)$ is the reference in (2). Note that u_i^{R} is the control input given in (9) and let it satisfy Condition 2. In the case where K_i in (5) is known, the leader following is achieved by setting $u_i^{\mathrm{A}} = K_i v_i(k)$ from Lemma 2. However, since K_i is in fact unknown, we consider to replace it by its estimate $\hat{K}_i(k) \in \mathbb{R}^{m\times n_1}$ at k and get the following adaptive controller

$$u_i^{\mathrm{A}}(k) = \hat{K}_i(k) v_i(k), \tag{15}$$

with the adaptive law of $\hat{K}_i(k)$ given by

$$\begin{aligned} &\hat{K}_i(k+1) \\ &\quad = \hat{K}_i(k) - Q_i \sum_{j \in \mathcal{N}_i^{\mathrm{A}}} \left(w_{ij}^{\mathrm{A}} \rho_{ij}(v,k) \tilde{u}_{ij}^{\mathrm{A}}(k) \right) v_i(k)^{\mathrm{T}}, \end{aligned} \tag{16}$$

where $Q_i \in \mathbb{R}^{m\times m}$ is a positive-definite symmetric matrix satisfying $W_i^{\mathrm{A}} \lambda_{\max}(Q_i) < 2$ for $W_i^{\mathrm{A}} := \sum_{j\in\mathcal{N}_i^{\mathrm{A}}} w_{ij}^{\mathrm{A}}$, $\lambda_{\max}(\bullet)$ represents the largest eigenvalue of a positive semi-definite matrix $\bullet$,

$$\rho_{ij}(v,k) := \frac{1}{c + \|v_i(k)\|_2^2 + \|v_j(k)\|_2^2}, \tag{17}$$

$$\begin{aligned} \tilde{u}_{ij}^{\mathrm{A}}(k) := &B^{\dagger}(e_{ij}(k+1) - A_l e_{ij}(k)) \\ &- (u_i^{\mathrm{R}}(k) - u_j^{\mathrm{R}}(k)), \end{aligned} \tag{18}$$

and c is an arbitrary fixed positive number. Note that each controller can be operated only with local information and in this sense, the whole control system is distributed. The block diagram of the closed-loop system is shown in Fig. 1.

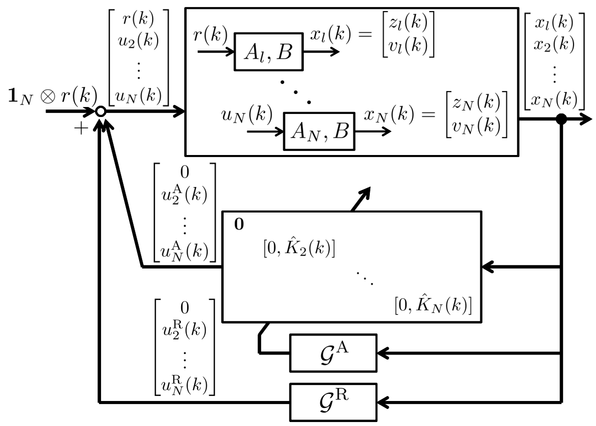

Fig. 1: Schematic structure of proposed distributed adaptive controller

B. Main result

We show the main result of this paper in this subsection. Define the feedback gain error and the matrix given by

$$\tilde{K}_i(k) := \hat{K}_i(k) - K_i \quad (19)$$

$$\tilde{K}(k) := \left[\tilde{K}_2(k), \dots, \tilde{K}_N(k)\right] \in \mathbb{R}^{m\times(N-1)n_1} \quad (20)$$

Then we get the following theorem:

Theorem 1: Consider the closed-loop system consisting of MAS (2), (3). Then the controller (9), (14)–(16) achieves the leader following (8), keeping control inputs $u_i(k)$ and components $v_i(k)$ of states bounded. Moreover, the equilibrium points $\tilde{K}(k) = 0$ and $(e_l(k), \tilde{K}(k)) = (0, 0)$ are Lyapunov stable and $e_l(k), \tilde{K}(k)$ are bounded. Furthermore, for all $i \in \mathcal{V}_\mathrm{f}$,

$$\lim_{k\to\infty} \tilde{K}_i(k)v_i(k) = 0 \quad (21)$$

$$\lim_{k\to\infty} (\hat{K}_i(k+1) - \hat{K}_i(k)) = 0 \quad (22)$$

are satisfied.

C. Proof of Theorem 1

In this subsection, we give the outline of the proof of Theorem 1.

Because $\bar{A}$ is asymptotically stable by (10) and (12), we can choose positive number ε and positive-definite matrices P and $R \in \mathbb{R}^{(N-1)n\times(N-1)n}$ such that

$$(1+\varepsilon)\bar{A}^\mathrm{T} P\bar{A} - P + R = 0 \quad (23)$$

is satisfied.

Next, we choose a Lyapunov function candidate

$$V(e_l, \tilde{K}) := V_1(e_l) + aV_2(\tilde{K}), \quad (24)$$

where

$$V_1(e_l) := \ln(1 + e_l^\mathrm{T} P e_l), \quad (25)$$

$$V_2(\tilde{K}) := \sum_{i=2}^{N} V_{2,i}(\tilde{K}_i), \quad (26)$$

$$V_{2,i}(\tilde{K}_i) := \mathrm{tr}\left[\tilde{K}_i^\mathrm{T} Q_i^{-1}\tilde{K}_i\right], \quad (27)$$

and a is an appropriate positive number determined later.

The outline of the proof is as follows. At first, we will show that $V_2(\tilde{K})$ is weakly decreasing (Lemma 3). Next, it will be shown that $V(e_l, \tilde{K})$ is weakly decreasing for sufficiently large $a > 0$, while $V_1(e_l)$ is not necessarily weakly decreasing (Lemma 4). Finally, we will prove Theorem 1 with combination of these facts.

For follower $i \in \mathcal{V}_\mathrm{f}$, the input error caused by the feedback gain errors is defined by

$$\tilde{u}_i^\mathrm{A}(k) := \tilde{K}_i(k)v_i(k). \quad (28)$$

Then, from (3), (5) and (14), we get

$$x_i(k+1) = A_l x_i(k) + B(\tilde{u}_i^\mathrm{A}(k) + u_i^\mathrm{R}(k) + r(k)) \quad (29)$$

for $i \in \mathcal{V}_\mathrm{f}$.

By settings $\tilde{u}_l^\mathrm{A}(k) \equiv 0$ and $u_l^\mathrm{R}(k) \equiv 0$, it is known that (29) is also satisfied for $i = l$ from (2). Therefore, for any $i, j \in \mathcal{V}$,

$$\begin{aligned} e_{ij}(k+1) = A_l e_{ij}(k) &+ B(\tilde{u}_i^\mathrm{A}(k) - \tilde{u}_j^\mathrm{A}(k)) \\ &+ B(u_i^\mathrm{R}(k) - u_j^\mathrm{R}(k)) \end{aligned} \quad (30)$$

is satisfied. Then, with (18), we can get

$$\tilde{u}_{ij}^\mathrm{A} = \tilde{u}_i^\mathrm{A} - \tilde{u}_j^\mathrm{A}. \quad (31)$$

Next we define the set of edges from the leader in $\mathcal{G}^\mathrm{A}$ as

$$\mathcal{E}_l^\mathrm{A} := \{(i,l) \in \mathcal{E}^\mathrm{A}\}, \quad (32)$$

and the set of the one side edges between followers as

$$\mathcal{E}_\mathrm{f}^\mathrm{A} := \{(i,j) \in \mathcal{E}^\mathrm{A} | i, j \in \mathcal{V}_\mathrm{f},\ i < j\}. \quad (33)$$

Furthermore, by choosing a positive number κ which satisfies

$$W_i^\mathrm{A}\lambda_{\max}(Q_i) \le 2 - \kappa, \quad (34)$$

we can show the next lemma:

Lemma 3: The function $V_2(\tilde{K}(k))$ is non-increasing as time k increases and satisfies

$$\begin{aligned} \Delta V_2(k) &:= V_2(\tilde{K}(k+1)) - V_2(\tilde{K}(k)) \\ &\le - \sum_{(i,j)\in(\mathcal{E}_l^\mathrm{A}\cup\mathcal{E}_\mathrm{f}^\mathrm{A})} \kappa w_{ij}^\mathrm{A}\rho_{ij}(v)\|\tilde{u}_{ij}^\mathrm{A}\|_2^2 \ \le\ 0. \end{aligned} \quad (35)$$

Lemma 3 implies that V_2 is a Lyapunov function concerning $\tilde{K}(k)$.

We next define

$$\bar{B} := I_{N-1} \otimes B, \quad (36)$$

$$\tilde{u}^\mathrm{A}(k) := \left[\tilde{u}_2^\mathrm{AT}\ \dots\ \tilde{u}_N^\mathrm{AT}\right]^\mathrm{T} \in \mathbb{R}^{(N-1)m}, \quad (37)$$

and get the following lemma:

Lemma 4: In the closed-loop system, the dynamics of tracking error $e_l(k)$ to the leader state is given by

$$e_l(k+1) = \bar{A}e_l(k) + \bar{B}\tilde{u}^{\mathrm{A}}(k). \tag{38}$$

Furthermore,

$$\begin{aligned}\Delta V_1(k) &:= V_1(e_l(k+1)) - V_1(e_l(k)) \\ &\leq \frac{-e_l^{\mathrm{T}}Re_l + (1+1/\varepsilon)\nu\|\tilde{u}^{\mathrm{A}}\|_2^2}{1+e_l^{\mathrm{T}}Pe_l}\end{aligned} \tag{39}$$

is satisfied, where $\nu := \lambda_{\max}(\bar{B}^{\mathrm{T}}P\bar{B})$.

Remark 4: With the denominator of (39), we can show the existence of positive constant a in (24) which gives the statement in Lemma 5.

Lemma 5: When a in (24) satisfies $a > a_{\mathrm{LB}}$ where

$$a_{\mathrm{LB}} := \left(1+\frac{1}{\varepsilon}\right)\frac{\nu}{\kappa}\frac{\max\{c+4\delta^2, 2/\mu\}}{\lambda_{\min}(\bar{L}^{\mathrm{A}})}, \tag{40}$$

$\mu := \lambda_{\min}(P)$ and $\lambda_{\min}(\bullet)$ represents the smallest eigenvalue of a positive definite matrix $\bullet$, $V(e_l, \tilde{K})$ is monotonically increasing with respect to e_l and $\tilde{K}$ and

$$\begin{aligned}\Delta V(k) &:= V(e_l(k+1), \tilde{K}(k+1)) - V(e_l(k), \tilde{K}(k)) \\ &\leq \frac{-e_l^{\mathrm{T}}Re_l}{1+e_l^{\mathrm{T}}Pe_l} \;\leq\; 0.\end{aligned} \tag{41}$$

Now, we can give the proof of Theorem 1 by using these lemmas. At first, $V(e_l, \tilde{K})$ defined by (24) satisfies $V(0,0) = 0$. Since P and Q_i are positive definite and $a > 0$, $V(e_l, \tilde{K}) > 0$ for $(e_l, \tilde{K}) \neq (0,0)$ and $V(e_l, \tilde{K})$ is radially unbounded. Furthermore, since it is shown that $V(e_l, \tilde{K})$ is a Lyapunov function in Lemma 5, $(e_l(k), \tilde{K}(k)) = (0,0)$ is Lyapunov stable and $e_l(k)$, $\tilde{K}(k)$ are bounded. By using Lemma 3, it is also shown that $\tilde{K}(k) = 0$ is Lyapunov stable. Observing that $e_l(k)$ and $v_l(k)$ are bounded and that $v_i(k) - v_l(k)$ is a component of $e_l(k)$, $v_i(k)$ is also bounded for all $i \in \mathcal{V}$, which means that the control input $u_i(k)$ given by (14) is bounded.

Next, Lemmas 3 and 5 show that V and V_2 are monotonically non-increasing and bounded below, and then converge. Therefore ΔV and ΔV_2 converge to 0. The right-hand sides of the second lowest lines of (35) and (41) then converge to 0. From (41), it is shown that e_l converges to 0 and it implies the leader following (8). Noting that v_i is bounded, it is known that $\tilde{u}_{ij}^{\mathrm{A}}$ in (35) converges to 0 for any (i,j) in (35). This fact together with the update law (16) enables us to show (22). Finally, from the facts that $\mathcal{G}^{\mathrm{A}}$ has spanning tree and that $\tilde{u}_l^{\mathrm{A}}(k) \equiv 0$, we can conclude that $\tilde{u}_i^{\mathrm{A}}(k)$ converges to 0 for $i \in \mathcal{V}$, which is equivalent to (21).

D. Interpretation of Theorem 1

Theorem 1 guarantees that the leader following (8) is achieved even though component $z_i(k)$ of the agent state is unbounded. Even in such a case, the control inputs and signals which are measured and used in computation of them are all bounded. Moreover, only the relative values of $z_i(k)$ between agents are needed for the control systems and they are kept to be bounded.

The formulae (21) and (22) represent asymptotic property of the adaptive controller. Eq. (21) implies that u_i^{A} converges to $K_i v_i(k)$ and that the dynamics of all the agents become almost homogeneous after sufficiently long time. Eq. (22) also implies that $\hat{K}_i(k)$ converges to a constant.

The proof of the theorem is based on the Lyapunov function (24) concerning the state variable errors and the feedback gain errors $(e_l(k), \tilde{K}_i(k))$. Therefore, from those errors at the initial time, we can evaluate upper-bounds of the state variable errors quantitatively. To be more precise, when we can find a smaller a in (41), it is expected that the state variable errors are smaller compared to that of the feedback gain errors.

From this point of view, we will give some discussion about the lower bound (40) given in Lemma 5. Firstly, when we set the smaller weights $\{w_{ij}^{\mathrm{A}}\}$ of $\mathcal{G}^{\mathrm{A}}$, we can find the larger κ in (34), however $\lambda_{\min}(\bar{L}^{\mathrm{A}})$ becomes smaller simultaneously. This implies that there is a trade-off in the step size of the parameter update law (16). Next, let α be the largest absolute value of the eigenvalue of the stable matrix $\bar{A}$, a constraint on ε appeared in (23) is given as $(1+\varepsilon)\alpha^2 < 1$ from (23). Therefore, when α is close to 1, ε should be close to 0 and then $1/\varepsilon$ is large. This makes the possible a larger and it is not preferable for the magnitude of the state variable errors from the above discussion. Therefore, F and L^{R} should be chosen so that α is sufficiently small. This suggests that even though A_l is asymptotically stable, the direct feedback based on the relative state variables is effective.

Finally, note that Theorem 1 includes the result of [17] as a special case with a setting that $n_1 = n$, A_l is stable, $F = 0$ (no relative feedback), and $w_{ij}^{\mathrm{A}} = 1 \forall (i,j) \in \mathcal{E}^{\mathrm{A}}$.

IV. Numerical Example

In this section, we present a numerical example of vehicle formation controls for showing the efficiency of our proposed cooperating adaptive control systems.

As explained in Example 1 in Subsection II-B, the control objective is to keep relative positions between vehicle agents which continue moving in a direction on a line. The number of agents is $N = 4$. The position of agent i is $q_i(k)$, the velocity $p_i(k)$, the target relative position among the agents is h_i and we set it as 0, -5, -10, -15 for $i = l$, 2, 3, 4. The dynamics of agent i from the input to $p_i(k)$ is given by

$$p_i(k+1) = -a_{1,i}p_i(k) - a_{2,i}p_i(k-1) + u_i(k) \tag{42}$$

where $u_i = r$ for $i = l$. The parameters $a_{1,i}$ and $a_{2,i}$ are unknown except for $i = l$. Also assume that (42) is stable for $i = l$. Let $x_i(k) = [q_i(k) - h_i, p_i(k-1), p_i(k)]^{\mathrm{T}}$ be the state vector of the agents. This formation problem can be reduced to the leader following problem (8) by setting of A and B in (2) and (3) as

$$A_i = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & -a_{2,i} & -a_{1,i} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \tag{43}$$

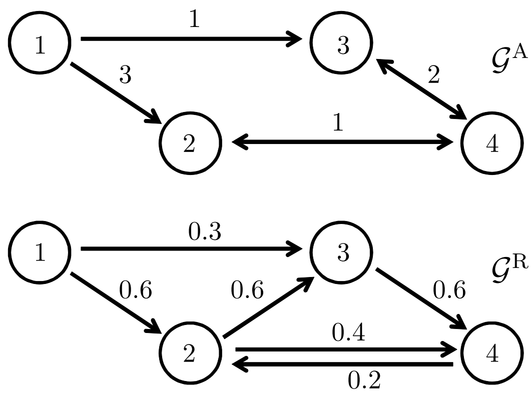

Fig. 2: Graphs $\mathcal{G}^{\mathrm{A}}$ and $\mathcal{G}^{\mathrm{R}}$ used in Section IV

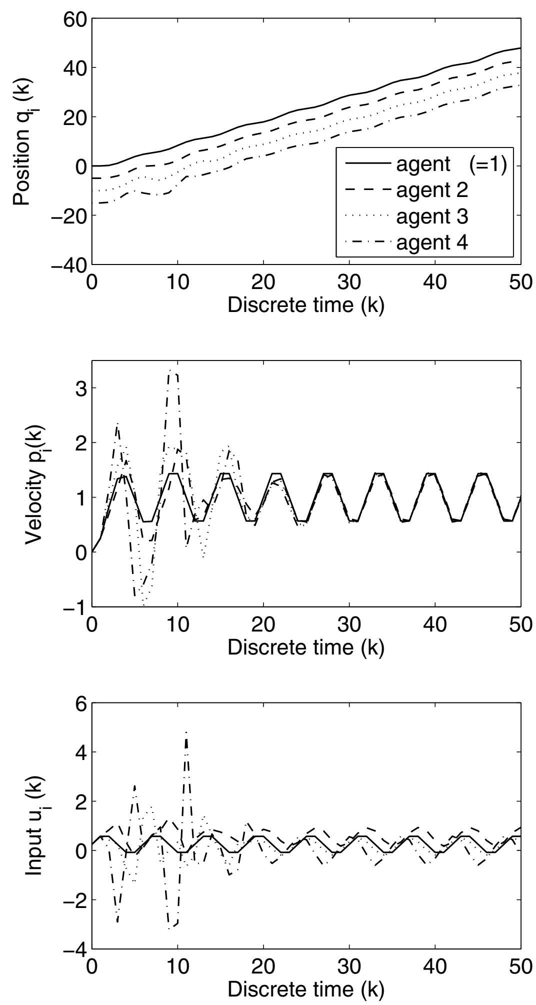

Fig. 3: Time responses of state variables and input signals

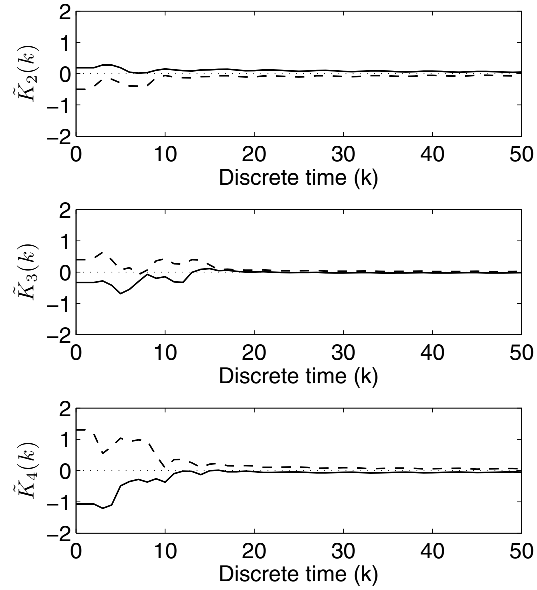

Fig. 4: Time responses of gain errors $\tilde{K}_i$ (solid line: $\tilde{K}_{i1}$, dashed line: $\tilde{K}_{i2}$)

The velocity of the leader is bounded so that this MAS satisfies Assumption 1 with $n_1 = 2$.

We set A_i such that their eigenvalues become $(0.5, 0.5)$, $(0.2, 0.3)$, $(0.7 + 0.3j, 0.7 - 0.3j)$, $(1.1, 1.2)$ except for the common eigenvalue 1 for $i = l$, 2, 3, 4, respectively. Also set the reference signal to $r(k) = 0.25 + 0.375 \sin(2\pi k/6)$ and this makes the velocity of the leader in the steady state to be $1 + 0.5 \sin(2\pi(k - 2)/6)$. The information structures $\mathcal{G}^{\mathrm{A}}$ and $\mathcal{G}^{\mathrm{R}}$ are set as in Fig. 2 and the corresponding Laplacians are

$$L^{\mathrm{A}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -3 & 4 & 0 & -1 \\ -1 & 0 & 3 & -2 \\ 0 & -1 & -2 & 3 \end{bmatrix},$$
$$L^{\mathrm{R}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.6 & 0.8 & 0 & -0.2 \\ -0.3 & -0.6 & 0.9 & 0 \\ 0 & -0.4 & -0.6 & 1.0 \end{bmatrix}.$$

Also set $F = \begin{bmatrix} -0.3830 & 0.2133 & -1.1926 \end{bmatrix}$ and the maximum of the absolute values of the eigenvalues in Condition 2 is 0.5692 so that Condition 2 is satisfied. The parameters in the adaptive law (16) are set as $c = 1$ and $Q_i = 1.8/W_i^{\mathrm{A}}$. Moreover $x_i(0) = 0$ for all i and $\hat{K}_i(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}$ for $i \in \mathcal{V}_{\mathrm{f}}$.

Figs. 3 and 4 show the time responses of the positions, velocities, control inputs of the agents, and the errors of adaptive controller gains $\tilde{K}_i$, respectively. From Fig. 3, it is clear that the agents converge to their relative positions in the formation at about $k = 20$ and their velocities follow that of the leader. On the other hand, control inputs do not converge to a same response because the gain parameter K_i is different for each agent i. Although the positions of the agents in Fig. 3 move unbounded, their relative positions and control inputs are kept bounded. From Fig. 4, we see that $\tilde{K}_i(k)$ converges to 0 at about $k = 20$ in this example.

V. Conclusion

In this paper, we proposed a cooperative adaptive control method for a leader following problem of heterogeneous uncertain MAS. The cooperative controller is composed of two parts: the previously proposed cooperative adaptive controller, which makes the agents' dynamics close to that of the leader and a relative state feedback controller. With this structure, we solved the fundamental issue in our previous results [17] and can deal with the case where the leader is not necessarily asymptotically stable. From this extension, the cooperative controller can be used in an important application: the vehicle formation problem in which the agents move unbounded. We showed that all the signals used in the controller are bounded and the state variable errors and adaptive gain errors are Lyapunov stable.

Finally, issues on

- the case that matrix B also includes uncertain parameters
- the case of output control
- data transmission delay and packet loss

are left and we will discuss them in future works.

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