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Distributed Control of Reactive Power and Storage in Microgrids

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***Abstract*— A dual decomposition approach for the distributed control of both reactive power and storage control in microgrids is presented. This approach considers physical constraints inherent to storage systems, as well as voltage regulation constraints. The objective is to minimize a cost function consisting on a linear combination of the overall active power cost over a finite time horizon, and the transmission losses over the microgrid. A distributed algorithm is introduced for which we present the corresponding convergence result. In addition, we introduce a novel approximate distributed voltage prediction algorithm. Simulations demonstrate the dual decomposition algorithm performance on a particular microgrid case.**

I. Introduction

Motivation: Solar energy is today one of the most promising renewable energies that can be used to replace the traditional fossil-fuel-based energy and the modern, but also contaminating, nuclear energy. As solar irradiance depends on highly random phenomena such as climate, pollution, and even the cleanness conditions on the photovoltaic generation systems (PV), maintaining power-grid voltages within desirable levels while satisfying user loads poses significant challenges. In order to solve these problems, a possibility is to support additional backup generation plants. However, this would incur into significant higher costs for both the utilities and users and so several alternatives are being considered. On the one hand, storage systems have been proposed to reserve the energy generated during low-demand hours or high solar generation hours for the high-demand times. On the other hand, inverters placed at the PV themselves can be leveraged to inject reactive power for both voltage regulation and optimize the network performance [1]. As PV systems and batteries can be locally owned and distributed throughout the network, distributed optimization and control algorithms that can be used to achieve the power network objectives tend to become most useful.

Literature Review: Since it was introduced in [2], the optimal power flow problem (OPF) has been widely studied in the literature. Its difficulty lies in that the feasible set is determined by inequality constraints that are highly nonlinear, yielding a nonconvex problem. Recent approaches present centralized solutions and consider voltage as a decision variable [3], [4]. In particular, the papers [5], [6], [4] are able to circumvent the nonconvexity of the OPF problem under very general conditions that guarantee a zero duality gap between primal and dual formulations of the problem. The papers [7], [8] address an OPF setting with storage integration in which battery systems are added to some nodes in a grid. In [7], voltages and battery charging/discharging rates are taken as decision variables of an OPF problem simplified by assumptions such as small-angles and infinite charging/discharging capacities. Electricity prices are assumed to vary over time, and the objective function to minimize is the cost of the energy provided by the utility. The paper [8] presents a more general setting in which the assumptions above are avoided. In this case, the solution relies on the aforementioned zero-duality-gap approach.

Reactive power control has been addressed in a significant set of works. In [9] a problem of optimal sizing for capacitors is solved using a relaxation of the power flow equations that is valid for radial networks. Optimal operation was studied in [10], where an algorithm for optimal reactive power generation was introduced. More recently, works have been oriented to control of smart inverters which allow changes in the generated reactive power. In [11], a convexification of the OPF problem is presented for grids fulfilling some assumptions on the input voltage and the impedance in the transmission lines. The objective is to minimize transmission losses in the grid by varying the injected reactive power at all generators. The decision process is designed to operate in a distributed setting. No voltage regulation constraints are taken into account. In [12], a reactive power control problem is addressed via dual decomposition approach. In this case, a lower bound for the voltage at generation nodes is used as a constraint. However, this work does not consider upper bounds on the voltage, and the only decision variables are reactive power values at generation nodes. Furthermore, storage capacity and associated constraints are not considered either.

Contributions: In the present work, we propose an algorithm to compute both the optimal reactive power generation and storage control profiles for a microgrid over a time horizon. We use a convexification approach which is valid under some assumptions on the input voltage and the transmission lines, in order to present a

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distributed algorithm based on the dual decomposition method. We consider a microgrid with PV-generation-dedicated nodes, which can also have storage capacity, while storage capacity is also considered at some of the load nodes. Our scenario considers a discrete-time horizon, which at each instant has an associated electricity cost, and also a forecasted active PV generation, and node-wise demand. The algorithm utilizes measures and predictions of voltage in the microgrid (which have implicit information on the power injected at the nodes) to choose the optimal reactive power that is injected by each PV generator, and also the optimal charging/discharging rate of each storage device at each time in the horizon. The goal is to minimize a cost function that represents a weighted sum of the transmission losses and the cost of the active power provided by the utility. The solution provided by the distributed optimization algorithm is meant to respect voltage regulation constraints. As the microgrid model is highly nonlinear, the optimization problem is nonconvex, therefore we follow the relaxation method presented in [11], [12] which linearizes the relation between power and voltage in the grid, to convexify the problem. We also use this linearization to design a novel distributed model for the voltage prediction that is required by the algorithm. This model computes future values of voltage at some nodes of the microgrid using the well known Jacobi overrelaxation. Convergence of the dual decomposition algorithm is studied using a Lyapunov stability analysis, which due to space limitations we include in [13]. Simulations are presented to illustrate the algorithm performance.

A. Preliminary notation

Define $\mathbb{R}$ as the set of real numbers, $\mathbb{R}_{>0}$ as the open set of real positive numbers, and $\mathbb{C}$ as the set of complex numbers. The complex exponential function is denoted as e^{jx}, for a scalar x. Let $\mathbf{1}_{n,l}$ be a vector in $\mathbb{R}^n$, such that the entry $l+1$ is one, for $l \in \{0, \dots, n-1\}$, while all other entries are zero. Let $\mathbf{1}_n$ be a vector in $\mathbb{R}^n$, such that all its entries have ones. Let I_n, for $n = 1, 2, \dots$ be the identity matrix with size $n \times n$. Let us denote $\|X\|_{\mathbb{C}} \in \mathbb{R}^{m\times n}_{\geq 0}$ for $X \in \mathbb{C}^{m\times n}$ as the component-wise complex norm of the variable X. The operator $\|\cdot\|$ denotes the euclidean norm of a variable. Let $\mathcal{A}$ be a finite set. $|\mathcal{A}|$ denotes the cardinality of $\mathcal{A}$. Then $I_{\mathcal{A}}$ denotes the identity matrix with size $|\mathcal{A}| \times |\mathcal{A}|$. Consider a set $\tau = \{1, \dots, T\}$. Let x be a vector in $\mathbb{C}^{|\mathcal{A}|T \times |\mathcal{A}|T}$, whose entries are $x_l(t)$ for all $l \in \mathcal{A}$, $t \in \tau$, i.e., $x = [x_1(1) \dots, x_{|\mathcal{A}|}(1), \dots, x_1(T), \dots, x_{|\mathcal{A}|}(T)]^T$. Then $x(t)$ denotes a vector $x(t) = [x_1(t), \dots, x_{|\mathcal{A}|}(t)]^T$, for each $t \in \tau$. Similarly, x_l denotes a vector $x_l = [x_l(1), \dots, x_l(T)]^T$, for each $l \in \mathcal{A}$. For any vector $x \in \mathbb{R}^n$, the notation $x \succeq 0$ denotes componentwise nonnegativity of x. Let B be a complex variable in $\mathbb{C}^{n\times n}$. The spectral radius of B is denoted by $\rho(B)$, and corresponds to the magnitude of the largest eigenvalue of B. Let x be a vector in $\mathbb{C}^n$, then $D(x)$ denotes the diagonal matrix B in $\mathbb{C}^{n\times n}$ such that $b_{ii} = x_i$, for $i \in \{1, \dots, n\}$. Let C be a complex variable in $\mathbb{C}^{m\times n}$. $\hat{C}$ denotes the conjugate transpose of C. For the matrices A, B, $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{p\times q}$, $C = A \otimes B$ represents the Kronecker product [14], where $C \in \mathbb{R}^{mp\times nq}$ is such that C is a block matrix where the block $C_{ij} = a_{ij}B$.

B. Notions of graph theory

Let $\mathbb{G} = (\mathcal{V}, \mathcal{E}, B)$, be a directed weighted graph with a set $\mathcal{V}$ of N vertices, a set of edges $\mathcal{E}$ and a weight matrix B. Each edge in $\mathcal{E}$ can be written as (h, l), for $h, l \in \mathcal{V}$. The Laplacian matrix of $\mathbb{G}$ is given by $L(\mathbb{G}) = D(\sum_{j=1}^{N}(b)_{\cdot,j}) - B$, where $(a)_{\cdot,j}$ represents the j^{th} row of B. Consider some labeling of the set $\mathcal{E}$ with the set of indices $\{1, \dots, |\mathcal{E}|\}$. The incidence matrix A of $\mathbb{G}$ is a matrix in $\{0, \pm 1\}^{|\mathcal{E}|\times N}$ such that:

$$a_{dl} = \begin{cases} -1 & \text{if } \varepsilon_d \in \mathcal{E} \text{ is an out-edge of } l \in \mathcal{V}, \\ 1 & \text{if } \varepsilon_d \in \mathcal{E} \text{ is an in-edge of } l \in \mathcal{V}, \\ 0 & \text{otherwise.} \end{cases}$$

Consider a diagonal matrix $C \in \mathbb{C}^{|\mathcal{E}|\times|\mathcal{E}|}$ such that c_{ii} is the weight of $\varepsilon_i \in \mathcal{E}$. Then, the Laplacian can be also written as $L = A^T C A$.

***Definition** 1.1:* Consider the graph $\mathbb{G}$. A path $\mathcal{P}(l, h)$, for nodes $l, h \in \mathcal{V}$, is defined as a sequence of nodes $\{n_1, \dots, n_n\}$ such that $n_1 = l$, $n_n = h$ and (n_i, n_{i+1}) is an edge of $\mathbb{G}$, for all $i = 1, \dots, n-1$.

II. Problem definition

Consider a microgrid which is connected to the grid at a single point. The microgrid is modeled as an undirected weighted graph $\mathbb{G} = (\mathcal{V}, \mathcal{E}, Y)$, $Y \in \mathbb{C}^{|\mathcal{V}|\times|\mathcal{V}|}$, where nodes $\mathcal{V} = \{0, \dots, N-1\}$ represent buses and edges $\mathcal{E}$ represent the interconnection lines. Weights are given by the matrix Y and correspond to the line admittances of such interconnection lines. We consider a microgrid with generation and storage capacities. This microgrid has four different types of nodes: i) a single connection point to the grid, represented by node 0, which acts as a slack node, with fixed voltage and unlimited power generation, ii) a subset of nodes $\mathcal{G}$ with PV generation and storage capacity, iii) a subset of nodes $\mathcal{S}$, which only has storage capacity but no generation, and iv) a set $\mathcal{M} = \mathcal{V} \setminus (\{0\} \cup \mathcal{G} \cup \mathcal{S})$, of nodes with neither generation nor storage. Nodes in $\mathcal{V} \setminus (\mathcal{G} \cup \{0\})$ have a load that must be satisfied.

Let us consider a discrete, finite-time window τ, with T time slots, i.e., $\tau = \{1, \dots, T\}$. Each time slot t has an electricity cost $c(t)$ associated to it, which is the price per kW/h, given by a map $c : \tau \to \mathbb{R}_{>0}$, and depends on the overall demand satisfied by the utility. Although the cost function is not known to any node of the grid, the value of $c(t)$ for each $t \in \tau$ is assumed to be known from a forecasting process. Similarly, for each time slot there is a minimal amount of active power that each PV can provide during the whole time represented by the slot t, called $p_l(t) > 0$, for each $l \in \mathcal{G}$. This value is also assumed to be known from forecasting techniques. In order to make the problem slightly more general, the load at each node may also depend on the time slot $t \in \tau$. For each $l \in \mathcal{V} \setminus (\mathcal{G} \cup \{0\})$, let $p_l(t) + jq(t)$ be the load that must be satisfied at each $t \in \tau$, where $p_l(t) < 0$.

A. Mathematical model of a microgrid

Let $u_l(t) \in \mathbb{C}$ be the voltage at node $l \in \mathcal{V}$, at time $t \in \tau$. Let $\iota_l(t) \in \mathbb{C}$ be the current injected at node $l \in \mathcal{V}$. Let A be the incidence matrix associated to $\mathbb{G}$, based on some given ordering on $\mathcal{E}$. For node 0, let $u_0(t)$ be fixed and expressed as $u_0(t) = U_0 e^{j\phi}$. Let $s_l(t) = u_l(t)\hat{\iota}_l(t)$ be the complex power injected at node l at time t. Recall that $s_l(t) = P_l(t) + jQ_l(t)$, where $P_l(t)$, $Q_l(t)$ are injected active and reactive power at node $l \in \mathcal{V}$.

We introduce some compact-form notation next. Define $u(t) = [u_0, u_G^T(t), u_S^T(t), u_L^T(t)]^T$ as the voltage vector for all nodes in the set $\mathcal{V}$, at time $t \in \tau$, where $u_G(t)$ is the vector of all voltages at nodes in $\mathcal{G}$, $u_S(t)$ is the vector of voltages at nodes in $\mathcal{S}$, and $u_L(t)$ is the vector of all voltages in the set $\mathcal{M}$. Likewise, $s(t) = [s_0(t), s_G^T(t), s_S^T(t), s_L^T(t)]^T$, with $s(t) = P(t) + jQ(t)$, $P(t) = [P_0(t), P_G^T(t), P_S^T(t), P_L^T(t)]^T$, $Q(t) = [q_0(t), q_G^T(t), q_S^T(t), q_L^T(t)]^T$.

The convention for the active power sign is that if $P_l(t) < 0$, power must be injected into node l in order to maintain the power balance, while $P_l(t) > 0$ means that power must be drained out of node l to maintain the power balance, for all $l \in \mathcal{V}$, $t \in \tau$. As an example, for a node $l \in \mathcal{M}$, it must hold that $P_l(t) < 0$ for all $t \in \tau$. Likewise, for a node l with only generation, and no storage, $P_l(t) > 0$ for all $t \in \tau$. Then, using the Kirchoff's current and voltage laws, the relation between voltage and current is given by:

$$\begin{aligned} A^T \mathcal{I}(t) + \iota(t) &= 0 \\ Au(t) + Z\mathcal{I}(t) &= 0, \end{aligned} \tag{1}$$

where Z is the diagonal matrix in $\mathbb{C}^{|\mathcal{E}| \times |\mathcal{E}|}$ whose elements are the line impedances in the microgrid, and $\mathcal{I}(t)$ is the vector in $\mathbb{R}^{|\mathcal{E}|}$ whose components are the currents on the transmission lines.

B. Mathematical model of a battery

We model a battery with the following dynamics:

$$x_l(t) = x_l(t-1) + \frac{1}{\beta_l} v_l(t), \qquad \forall l \in \mathcal{G} \cup \mathcal{S}, \quad \forall t \in \tau, \tag{2}$$

where $x_l(t) \in [0,1]$ is the battery state, β_l corresponds to the maximum capacity of the battery, and $v_l(t)$ is the charged (resp. discharged) amount of energy to (resp. from) the battery during the time slot $t \in \tau$. Clearly, the battery has physical constraints, formulated as $v_l \in [V_{\text{dis}}^l, V_{\text{ch}}^l]$, where $V_{\text{dis}}^l < 0$, $|V_{\text{dis}}^l|$ is the maximum discharge rate allowed for the battery, and $V_{\text{ch}}^l > 0$ is the maximum charging rate. Assume that the initial state for the battery at node $l \in \mathcal{G} \cup \mathcal{S}$ is $x_l(0)$. Then, for each $t \in \tau$, the battery charge is given by:

$$x_l(t) = x_l(0) + \frac{1}{\beta_l} \sum_{\ell=1}^{T} v_l(\ell), \qquad \forall l \in \mathcal{G} \cup \mathcal{S} \quad \forall t \in \tau.$$

In compact form, denote by $v_G(t)$ the vector of battery charge/discharge rates at time t, for all nodes in the set $\mathcal{G}$, and similarly, $v_S(t)$ denotes the same variables for all nodes in the set $\mathcal{S}$. Moreover, let us define $v(t) = [v_G^T(t), v_S^T(t)]^T$.

Notice that $v_l(t) > 0$ implies that the battery at node $l \in \mathcal{G} \cup \mathcal{S}$ is charging at time $t \in \tau$, hence this is seen by the microgrid as consumed power at the node. Similarly, $v_l(t) < 0$ means that power is being injected by the battery at the node. Therefore, the net power injected at all nodes in $\mathcal{G}$ is given by $P_G(t) = p_G(t) - v_G(t)$, and the net power injected at all nodes in $\mathcal{S}$ is $P_S(t) = p_S(t) - v_S(t)$, for all $t \in \tau$.

Based on the microgrid and battery models that we have discussed, it can be established that actuation over the microgrid will take place through the decision variables $q_l(t)$, for all $l \in \mathcal{G}$, $t \in \tau$ and $v_l(t)$ for all $l \in \mathcal{G} \cup \mathcal{S}$, $t \in \tau$, while the input voltage u_0, the energy cost $c(t)$, the loads $p_l(t)$ for all $l \in \mathcal{V} \setminus (\mathcal{G} \cup \{0\})$ and the forecasted active power generation $p_l(t)$ for $l \in \mathcal{G}$ are system parameters.

For the remaining of this paper, we will consider the following assumptions on the system parameters. These assumptions have already been used in [11], [12], and are rather accurate for actual microgrids.

***Assumption** 2.1 (Large input voltage):* The value of U_0 is very large as compared to he currents provided by the inverters and batteries, or supplied to the loads.

***Assumption** 2.2 (Reactance/resistance ratio):* The microgrid has transmission lines with the same reactance/resistance ratio. Therefore, z_h can be written as $z_h = |z_h| e^{j\theta}$, for each edge $h \in \mathcal{E}$.

The following definition introduces a notion of communication-between-agents topology, for both generators and storage systems. This notion is related to the microgrid topology and the location of the generation/storage nodes in the microgrid.

***Definition** 2.1:* Let l be a node in $\mathcal{G}$. The set of generation neighbors $\mathcal{N}_G(l)$ is given by $\mathcal{N}_G(l) = \{h \in \mathcal{V} \mid \mathcal{P}(h,k) \cap (\mathcal{G} \cup \{0\}) = \{h,l\}, \forall \mathcal{P}(h,k)\}$. Similarly, for a node $l \in \mathcal{G} \cup \mathcal{S}$, the set of storage neighbors is given by $\mathcal{N}_S(l) = \{h \in \mathcal{V} \mid \mathcal{P}(h,k) \cap (\mathcal{G} \cup \{0\} \cup \mathcal{S}) = \{h,l\}, \forall \mathcal{P}(h,k)\}$.

C. Optimization problem

Our objective is to compute in a distributed way the optimal reactive power generation $q_G(t)$ and storage control profiles $v(t)$ for each t in the time horizon τ. This must be done in such a way that the voltage at each node in $\mathcal{G}$ is maintained within desirable range. The optimality is defined with respect to the cost function defined by:

$$J(u) = \sum_{t\in\tau}(J_{\mathrm{loss}}(u(t)) + \delta J_{\mathrm{power}}(u(t))),$$

where $J_{\mathrm{loss}}(u(t))$ represents the distribution loss at time slot t, and $J_{\mathrm{power}}(u(t))$ is the overall cost of the active power provided by the utility at time slot t. The parameter $\delta > 0$ is used to scale the value of $J_{\mathrm{power}}(u(t))$ with respect to $J_{\mathrm{loss}}(u(t))$. Following the same framework as in [12], the power distribution loss can be expressed as $J_{\mathrm{loss}}(u(t)) = \hat{u}^T(t)Lu(t)$, where $L = A^T Z_{\mathrm{mag}}^{-1} A$, $Z_{\mathrm{mag}} = \|Z\|_{\mathbb{C}}$, and the power cost is given by $J_{\mathrm{power}}(u(t)) = -c(t)\,\mathrm{Re}(s_0(t)) = -c(t)\,\mathrm{Re}(\hat{i}_0(t)u_0(t))$. Since $i(t)$ can be approximated by assumption 2.2 on the reactance/resistant ratio as $i(t) = e^{-j\theta}Lu(t)$, it can be written as $J_{\mathrm{power}}(u(t)) = -\mathrm{Re}\{e^{j\theta}\hat{u}^T(t)L\mathbf{1}_{N,0}\mathbf{1}_{N,0}^T u(t)\}c(t)$.

The following results give us a convenient way to address the problem, in order to write $u(t)$ as a linear function of the decision variables.

***Lemma** 2.1 (Matrix X existence [11]):* There exists a unique symmetric, positive semidefinite matrix $X \in \mathbb{R}^{N\times N}$, which can be written as:

$$X = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & W & F & H \\ 0 & F^T & R & Z \\ 0 & H^T & Z^T & S \end{bmatrix},$$

such that:

$$XL = I_N - \mathbf{1}_N(\mathbf{1}_{N,0})^T,$$
$$X\mathbf{1}_{N,0} = 0,$$

where $W \in \mathbb{R}^{|\mathcal{G}|\times|\mathcal{G}|}$, $R \in \mathbb{R}^{|\mathcal{S}|\times|\mathcal{S}|}$, $S \in \mathbb{R}^{|\mathcal{M}|\times|\mathcal{M}|}$, $F \in \mathbb{R}^{|\mathcal{G}|\times|\mathcal{S}|}$, $H \in \mathbb{R}^{|\mathcal{G}|\times|\mathcal{M}|}$, $Z \in \mathbb{R}^{|\mathcal{S}|\times|\mathcal{M}|}$.

The physical meaning of the matrix X is widely discussed in [11], [12]. The product $(\mathbf{1}_{N,h} - \mathbf{1}_{N,l})^T X(\mathbf{1}_{N,h} - \mathbf{1}_{N,l})$ corresponds to the effective impedance from node $h \in \mathcal{V}$ to node $l \in \mathcal{V}$.

The following result provides a linearization of the relation between voltages and powers on the microgrid.

***Lemma** 2.2 (Voltage approximation [12]):* Consider the set of equations in (1), along with $s_l(t) = u_l(t)\hat{\imath}(t)$. Then voltages satisfy:

$$\begin{bmatrix} u_0(t) \\ u_G(t) \\ u_S(t) \\ u_L(t) \end{bmatrix} = \frac{e^{j(\theta+\phi)}}{U_0} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & W & F & H \\ 0 & F^T & R & Z \\ 0 & H^T & Z^T & S \end{bmatrix} \begin{bmatrix} 0 \\ \hat{s}_G(t) \\ \hat{s}_S(t) \\ \hat{s}_L(t) \end{bmatrix} + e^{j\phi}U_0\mathbf{1}_n + o\left(\frac{1}{U_0}\right), \tag{3}$$

where $o\left(\frac{1}{U_0}\right)$ means that $\lim_{U_0\to\infty}\frac{o(f(U_0))}{f(U_0)} = 0$.

Notice that by assumption 2.1 on the magnitude of the input voltage, the relaxation given by Lemma 2.2 is accurate, as the term $o\left(\frac{1}{U_0}\right)$ vanishes for large values of U_0.

Having established some relaxations on the nonlinear equations of the microgrid physics, we describe our problem as follows:

$$\min_{q_G,v,u} J(u)$$

s.t.

$$\forall t\in\tau \begin{cases} U_{\min} \le \|u_l(t)\|_{\mathbb{C}} \le U_{\max}, & \forall l \in \mathcal{G}, \\ V_{\mathrm{dis}}^l \le v_l(t) \le V_{\mathrm{ch}}^l, & \forall l \in \mathcal{G}\cup\mathcal{S}, \\ 0 \le x_l(0) + \frac{1}{\beta_l}\sum_{s=1}^T v_l(s) \le 1, & \forall l \in \mathcal{G}\cup\mathcal{S}, \\ \text{Equation (3) holds}, \end{cases} \tag{4}$$

where $\|\cdot\|_{\mathbb{C}}$ represents the component-wise magnitude of a complex vector. Note that the lower bound constraint on the voltage magnitude introduces a non-convex constraint to the problem. Therefore, we follow the convexification idea in [12], and define $w_G(t) = \frac{\|u_G(t)\|_{\mathbb{C}}\odot\|u_G(t)\|_{\mathbb{C}}}{U_0^2}$, where $\odot$ represents the Hadamard product. After some manipulation, from (3), it can be obtained that:

$$\begin{aligned} w_G(t) =& \mathbf{1}_{|\mathcal{G}|} + \frac{2}{U_0^2}\Bigg(\cos\theta\Big(\begin{bmatrix} W & F \end{bmatrix}\begin{bmatrix} p_G(t) \\ p_S(t) \end{bmatrix} \\ &+ Hp_L(t)\Big) + \sin\theta\Big(\begin{bmatrix} W & F \end{bmatrix}\begin{bmatrix} q_G(t) \\ q_S(t) \end{bmatrix} \\ &+ Hq_L(t)\Big) - \cos\theta\Big(\begin{bmatrix} W & F \end{bmatrix}v(t)\Big)\Bigg) + o\left(\frac{1}{U_0^2}\right). \end{aligned} \tag{5}$$

Thus, we can write the constraints on the voltage magnitude as:

$$W_{\min} = \frac{U_{\min}^2}{U_0^2} \le w_G(t) \le \frac{U_{\max}^2}{U_0^2} = W_{\max},$$

for all $t \in \tau$. Clearly, as $U_{\min}$, $U_{\max}$ have in practice a similar order-of-magnitude than that of U_0, it holds that $\frac{U_{\min}^2}{U_0^2}$, $\frac{U_{\max}^2}{U_0^2}$ are close to one. For large values of U_0, (5) is affine in the decision variables $q_G(t)$, $v(t)$, hence the constraint above is convex.

III. Distributed algorithm

In order to solve this problem in a distributed way, we propose an extension of the dual decomposition approach presented in [12]. The extension optimizes not only on the reactive power injection, but also on the battery charge/discharge, considering physical constraints on the battery control. The fact that we also consider a different cost function, modifies the algorithm dynamics. The dual decomposition algorithm consists of performing a gradient ascent on the Lagrangian with respect to the dual variables of the problem, while an unconstrained optimization with respect to the primal variables, parameterized by the estimated dual variables, is executed.

A. Communication network

Our solution approach for this problem is based on two communication networks, hereinafter denoted *generation network* and *storage network*. The *generation network* is determined by the graph built according to the definition of neighbors $\mathcal{N}_G$ and the *storage network* follows from the topology of the graph built by the definition of neighbors $\mathcal{N}_S$.

The following result follows directly from Lemmas 3 and 4 in [12].

***Lemma** 3.1 (Matrices G_G, G_S):* There are unique symmetric matrices G_G, G_S, such that:

$$\begin{bmatrix} 0 & 0 \\ 0 & B \end{bmatrix} G_S = I_{|\mathcal{G}\cup\mathcal{S}|+1} - \mathbf{1}_{|\mathcal{G}\cup\mathcal{S}|+1}(\mathbf{1}_{|\mathcal{G}\cup\mathcal{S}|+1,0})^T,$$
$$G_S\mathbf{1}_{|\mathcal{G}\cup\mathcal{S}|+1} = 0$$
$$\begin{bmatrix} 0 & 0 \\ 0 & W \end{bmatrix} G_G = I_{|\mathcal{G}|+1} - \mathbf{1}_{|\mathcal{G}|+1}(\mathbf{1}_{|\mathcal{G}|+1,0})^T,$$
$$G_G\mathbf{1}_{|\mathcal{G}|+1} = 0,$$

where B is defined as $B = \begin{bmatrix} W & F \\ F^T & R \end{bmatrix}$. Moreover, the matrix G_G has a sparsity induced by the definition of neighbor $\mathcal{N}_G$, while the matrix G_S has a sparsity induced by the definition of neighbor $\mathcal{N}_S$, this is, $(G_G)_{ij} \neq 0$ if and only if $j \in \mathcal{N}_G(i)$, and $(G_S)_{ij} \neq 0$ if and only if $j \in \mathcal{N}_S(i)$. The matrices B and W are blocks of the matrix X described in Lemma 2.1.

From the result above, it is immediately noted that the matrices W and B, are invertible. We can also see that their inverse matrices correspond to blocks in the matrices G_G and G_S respectively, which means that they also have a sparsity induced by the definition of neighbors $\mathcal{N}_G$ and $\mathcal{N}_S$.

B. Algorithm derivation

The Lagrangian for the optimization problem becomes:

$$\begin{aligned}\mathcal{L}(q_G,v,\psi) = {} & J(q_G,v) + \sum_{t=1}^{T}\underline{\lambda}^T(t)(W_{\min} - w_G(t)) \\ & + \sum_{t=1}^{T}\overline{\lambda}^T(t)(w_G(t) - W_{\max}) \\ & + \sum_{t=1}^{T}\underline{\eta}^T(t)(V_{\text{dis}} - v(t)) + \sum_{t=1}^{T}\overline{\eta}^T(t)(v(t) - V_{\text{ch}}) \\ & - \sum_{t=1}^{T}\underline{\mu}^T(t)\left(x(0) + (D(\beta^{-1}))\sum_{h=1}^{t}v(h)\right) \\ & + \sum_{t=1}^{T}\overline{\mu}^T(t)\left(x(0) + (D(\beta^{-1}))\sum_{h=1}^{t}v(h) - 1\right),\end{aligned} \tag{6}$$

where $\underline{\lambda}(t), \overline{\lambda}(t), \underline{\eta}(t), \overline{\eta}(t), \underline{\mu}(t), \overline{\mu}(t) \succeq 0$ for all $t \in \tau$, are Lagrange multipliers, $\psi = [\underline{\lambda}^T, \overline{\lambda}^T, \underline{\eta}^T, \overline{\eta}^T, \underline{\mu}^T, \overline{\mu}^T]^T$ and $(D(\beta^{-1})) \in \mathbb{R}^{|\mathcal{G}\cup\mathcal{S}|\times|\mathcal{G}\cup\mathcal{S}|}$ is a diagonal matrix such that $(D(\beta^{-1}))_{ii} = \frac{1}{\beta_i}$, for all $i \in \{1,\ldots,|\mathcal{G}\cup\mathcal{S}|\}$.

It is easy to see that since the Lagrangian is a quadratic function of (q_G, v), a minimizer for $\mathcal{L}(q_G, v, \psi)$, given a fixed ψ, can be found directly by solving $\nabla_{q_G,v}\mathcal{L}(q_G, v, \psi) = 0$. After some algebraic manipulations that follow Lemma 2 in [12]:

$$q_G^k(t) = -W^{-1}[\ F \quad H\ ]\begin{bmatrix} q_S(t) \\ q_L(t) \end{bmatrix} - \sin\theta[\overline{\lambda}^k(t) - \underline{\lambda}^k(t)] \tag{7}$$

$$\begin{aligned} v^k(t) = {} & \begin{bmatrix} p_G(t) \\ p_S(t) \end{bmatrix} + B^{-1}\begin{bmatrix} H \\ Z \end{bmatrix} p_L(t) \\ & - \frac{U_0^2}{2}B^{-1}\left(\sum_{h=t}^{T} D(\beta^{-1})(\overline{\mu}^k(h) - \underline{\mu}^k(h)) + \overline{\eta}^k(t)\right. \\ & \left. - \underline{\eta}^k(t) + \delta c(t)\mathbf{1}_{|\mathcal{G}\cup\mathcal{S}|}\right) + \cos\theta I_{\mathcal{G}}^e[\overline{\lambda}^k(t) - \underline{\lambda}^k(t)],\end{aligned} \tag{8}$$

where $I_{\mathcal{G}}^e = [I_{\mathcal{G}} \quad 0_{|\mathcal{G}|\times|\mathcal{S}|}]^T$, for all $t \in \tau$. Since the Lagrangian is linear in the dual variables, the derivative with respect to each of them is merely the expression representing the constraint associated to that dual variable. Thus, the gradient ascent for the dual variables is given by:

$$\begin{aligned} \overline{\lambda}^{k+1}(t) &= \left[\overline{\lambda}^k(t) + \gamma(w_G^k(t) - W_{\max})\right]_+, \\ \underline{\lambda}^{k+1}(t) &= \left[\underline{\lambda}^k(t) + \gamma(W_{\min} - w_G^k(t))\right]_+, \\ \overline{\eta}^{k+1}(t) &= \left[\overline{\eta}^k(t) + \gamma(v^k(t) - V_{\text{ch}})\right]_+, \\ \underline{\eta}^{k+1}(t) &= \left[\underline{\eta}^k(t) + \gamma(V_{\text{dis}} - v^k(t))\right]_+, \\ \overline{\mu}^{k+1}(t) &= \left[\overline{\mu}^k(t) + \gamma\left(x(0) + \sum_{h=1}^{t}(D(\beta^{-1}))v^k(h) - \mathbf{1}_{|\mathcal{G}\cup\mathcal{S}|}\right)\right]_+, \\ \underline{\mu}^{k+1}(t) &= \left[\underline{\mu}^k(t) - \gamma\left(x(0) + D(\beta^{-1})\sum_{h=1}^{t}v^k(h)\right)\right]_+, \end{aligned} \tag{9}$$

for all $t \in \tau$, with a common parameter γ.

***Lemma** 3.2 (Distributed algorithm [12]):* The expressions in (7), (8) can be recast as:

$$\begin{aligned} q_G^k(t) = {} & \operatorname{Im}\left(e^{-j\theta}[\ 0 \quad D(\overline{u}_G^{k-1}(t))\ ]G_G\begin{bmatrix} u_0^{k-1}(t) \\ u_G^{k-1}(t) \end{bmatrix}\right) \\ & + q_G^{k-1}(t) + o\left(\frac{1}{U_0^2}\right) - \sin\theta[\overline{\lambda}^k(t) - \underline{\lambda}^k(t)], \end{aligned} \tag{10}$$

$$\begin{aligned} v^k(t) = {} & \operatorname{Re}\left(e^{-j\theta}\begin{bmatrix} 0 & D(\overline{u}_G^{k-1}(t)) & 0 \\ 0 & 0 & D(\overline{u}_S^{k-1}(t)) \end{bmatrix}\right. \\ & \left. G_S\begin{bmatrix} u_0^{k-1}(t) \\ u_G^{k-1}(t) \\ u_S^{k-1}(t) \end{bmatrix}\right) + v^{k-1}(t) + o\left(\frac{1}{U_0^2}\right) \\ & + \cos\theta I_{\mathcal{G}}^e[\overline{\lambda}^k(t) - \underline{\lambda}^k(t)] - \frac{U_0^2}{2}B^{-1}\left((\overline{\eta}^k(t) - \underline{\eta}^k(t))\right. \\ & \left. + \sum_{h=t}^{T}(D(\beta^{-1}))(\overline{\mu}^k(h) - \underline{\mu}^k(h)) + \delta c(t)\mathbf{1}_{|\mathcal{G}\cup\mathcal{S}|}\right), \end{aligned} \tag{11}$$

for all $t \in \tau$.

The result above provides a dynamics that can be executed by each of the nodes where some type of decision can be made, which in turn is distributed according to the sparsity of G_G and G_S. Algorithm 1 represents the iteration executed by each node that has actuation capacity.

Algorithm 1: Execution for node $l \in \mathcal{G} \cup \mathcal{S}$

Input: $\overline{\eta}_l^0(t)$, $\underline{\eta}_l^0(t)$, $\overline{\mu}_l^0(t)$, $\underline{\mu}_l^0(t)$, $q_l^0(t)$, $v_l^0(t)$ (also $\overline{\lambda}_l^0(t)$, $\underline{\lambda}_l^0(t)$, if $l \in \mathcal{G}$), for all $t \in \tau$

for $k > 0$ **do**

  for $t = \{1, \ldots, T\}$ **do**

    if $l \in \mathcal{G}$ **then**

      $\overline{\lambda}_l^k(t) = \left[\overline{\lambda}_l^{k-1}(t) + \gamma(w_l^{k-1}(t) - W_{\max})\right]_+$

      $\underline{\lambda}_l^k(t) = \left[\underline{\lambda}_l^{k-1}(t) + \gamma(W_{\min} - w_l^{k-1}(t))\right]_+$

    end

    $\overline{\eta}_l^k(t) = \left[\overline{\eta}_l^{k-1}(t) + \gamma(v_l^{k-1}(t) - V_{\mathrm{ch}}^l)\right]_+$

    $\underline{\eta}_l^k(t) = \left[\underline{\eta}_l^{k-1}(t) + \gamma(V_{\mathrm{dis}}^l - v_l^{k-1}(t))\right]_+$

    $\overline{\mu}_l^k(t) = \left[\overline{\mu}_l^{k-1}(t) + \gamma\left(x_l(0) + \frac{\alpha_l}{\beta_l}\sum_{h=1}^t v_l^{k-1}(h) - 1\right)\right]_+$

    $\underline{\mu}_l^k(t) = \left[\underline{\mu}_l^{k-1}(t) + \gamma\left(-x_l(0) - \frac{\alpha_l}{\beta_l}\sum_{h=1}^t v_l^{k-1}(h)\right)\right]_+$

  end

  Gather $\overline{\eta}_h^k(t)$, $\underline{\eta}_h^k(t)$, $\overline{\mu}_h^k(t)$, $\underline{\mu}_h^k(t)$, for all $t \in \tau$, for all $h \in \mathcal{N}_S(l) \setminus \{0\}$

  Gather $u_h^{k-1}(t)$ for all $t \in \tau$, for all $h \in \mathcal{N}_S(l)$ (for all $h \in \mathcal{N}_S(l) \cup \mathcal{N}_G(l)$ if $l \in \mathcal{G}$)

  for $t = \{1, \ldots, T\}$ **do**

    $v_l^k(t) = v_l^{k-1}(t) + \sum_{h \in \mathcal{N}_S(l)} (G_S)_{lh} \Big(\|u_l^{k-1}(t)\|_{\mathbb{C}} \|u_h^{k-1}(t)\|_{\mathbb{C}} \cos(\angle u_h^{k-1}(t) - \angle u_l^{k-1}(t) - \theta)\Big) + \frac{U_0^2}{2} \sum_{h \in \mathcal{N}_S(l)\setminus\{0\}} (G_S)_{lh} \Big(-(\overline{\eta}_h^k(t) - \underline{\eta}_h^k(t)) - \frac{1}{\beta_h}\sum_{b=t}^T (\overline{\mu}_h^k(b) - \underline{\mu}_h^k(b)) - \delta c(t)\Big)$

    if $l \in \mathcal{G}$ **then**

      $v_l^k(t) = v_l^k(t) + \cos\theta(\overline{\lambda}_l^k(t) - \underline{\lambda}_l^k(t))$

      $q_l^k(t) = q_l^{k-1} - \sin\theta(\overline{\lambda}_l^k(t) - \underline{\lambda}_l^k(t)) + \sum_{h \in \mathcal{N}_C(l)} (G_G)_{lh} \Big(\|u_l^{k-1}(t)\|_{\mathbb{C}} \|u_h^{k-1}(t)\|_{\mathbb{C}} \sin(\angle u_h^{k-1}(t) - \angle u_l^{k-1}(t) - \theta)\Big)$

    end

  end

end

The following result, which is proven in [13] states that the distributed procedure represented in Algorithm 1 converges asymptotically to the optimal solution of the problem defined in (4), provided the parameter γ is small enough.

***Theorem** 3.1 (Algorithm convergence):* Let assumptions 2.1, 2.2 hold. Assume that the optimization problem in (4) is feasible. Then, for $\gamma < \frac{2}{\rho(M)}$ the execution of Algorithm 1 by each node $l \in \mathcal{G} \cup \mathcal{S}$ leads to $q_G^k(t) \to q_G^\star(t)$, $v^k(t) \to v^\star(t)$, for all $t \in \tau$, where $q_G^\star$, $v^\star$ is the unique optimizer of the problem in (4).

IV. Voltage prediction

Notice that the dual decomposition algorithm requires information about the system for all times $t \in \tau$. In order to compute $q_G(t), v(t)$, we need to know values of $u(t)$, which cannot be directly provided by the physical response of the microgrid. Thus, it is necessary to use a prediction model for values of $u(t)$, for $t \in \{1, \ldots, T\}$. Recall that a microgrid is modeled by the nonlinear memoryless system of equations formulated in (1). Finding the solution for voltages $u(t)$ given $P(t)$, $Q(t)$ and u_0 is not only a computationally expensive procedure, but it is also not distributed. We formulate a novel solution for this problem, which is based on lemma (2.2).

A. Distributed approximation

Let us assume that the loads in nodes $l \in \mathcal{M}$ are constant for all $t \in \tau$, i.e., $s_L(t) = s_L$, for all $t \in \tau$.

Let us consider some $u_0 = [U_0 e^{j\phi}, u_{0,G}^T, u_{0,S}^T, u_{0,L}^T]^T \in \mathbb{R}^N$ and $s_0 = [S_0, s_{0,G}^T, s_{0,S}^T, s_{0,L}^T]^T \in \mathbb{R}^N$, with $s_{0,S} = p_{0,S} + jq_{0,S}$ and $s_{0,L} = s_L$, such that the pair u_0, s_0 solves the power flow equations that model the microgrid. Assume that each node $l \in \mathcal{G} \cup \mathcal{S}$ knows $(u_0)_l$ and $(s_0)_l$.

Using the expression in (3), we have that:

$$\Delta u_t = e^{j\phi}\left(\frac{e^{j\theta}}{U_0} B \Delta \hat{s}_t\right),$$

where B is as defined in Lemma 3.1, $\Delta u_t = \begin{bmatrix} u_G(t) - u_{G,0} \\ u_S(t) - u_{S,0} \end{bmatrix}$, and $\Delta \hat{s}_t = \begin{bmatrix} \hat{s}_G(t) - \hat{s}_{G,0} \\ \hat{s}_S(t) - \hat{s}_{S,0} \end{bmatrix}$, for all $t \in \{1, \ldots, T\}$. Then, we obtain:

$$\mathrm{Re}(\Delta u_t) = B\left(\frac{\cos(\theta+\phi)}{U_0}\mathrm{Re}(\Delta\hat{s}_t) - \frac{\sin(\theta+\phi)}{U_0}\mathrm{Im}(\Delta\hat{s}_t)\right),$$

$$\mathrm{Im}(\Delta u_t) = B\left(\frac{\sin(\theta+\phi)}{U_0}\mathrm{Re}(\Delta\hat{s}_t) + \frac{\cos(\theta+\phi)}{U_0}\mathrm{Im}(\Delta\hat{s}_t)\right).$$

Notice that $\Delta\hat{s}_t$ is known for all $t \in \tau$. Since B is invertible, it is possible to solve Δu_t, using the Jacobi overrelaxation algorithm, as follows:

$$u_{e,t}^{\ell+1} = (1-h)u_{e,t}^\ell - hD(B^e)^{-1}\left((B^e - D(B^e))u_{e,t}^\ell - s_{e,t}^\ell\right), \tag{12}$$

where $B^e = \begin{bmatrix} B^{-1} & 0 \\ 0 & B^{-1} \end{bmatrix}$ and:

$$u_{e,t} = \begin{bmatrix} \mathrm{Re}(\Delta u_t) \\ \mathrm{Im}(\Delta u_t) \end{bmatrix},$$

$$s_{e,t} = \begin{bmatrix} \frac{\cos(\theta+\phi)}{U_0}\mathrm{Re}(\Delta\hat{s}_t) - \frac{\sin(\theta+\phi)}{U_0}\mathrm{Im}(\Delta\hat{s}_t) \\ \frac{\sin(\theta+\phi)}{U_0}\mathrm{Re}(\Delta\hat{s}_t) + \frac{\cos(\theta+\phi)}{U_0}\mathrm{Im}(\Delta\hat{s}_t) \end{bmatrix}.$$

By construction of B, it also holds that the diagonal elements of B^e are strictly positive. Then the matrix $D(B^e)^{-1}$ is well defined. Given that B^{-1} is symmetric and positive definite, B^e is also symmetric and positive definite. Then, it holds that if $h < 2/|\mathcal{G} \cup \mathcal{S}|$, the Jacobi overrelaxation converges from any initial condition and also presents a linear rate of convergence. Since B^{-1} is distributed in the sense of storage neighbors $\mathcal{N}_S$, the computation can be made using only local and neighboring information.

In Algorithm 2 we describe the execution that each node $l \in \mathcal{G} \cup \mathcal{S}$ must perform in order to carry out the voltage prediction.

Algorithm 2: Execution for node $l \in \mathcal{G} \cup \mathcal{S}$

Input: $v_l^k(t)$, $q_l^k(t)$ (only $v_l^k(t)$ if $l \in \mathcal{S}$), for all $t \in \tau$.

for $t = \{1, \ldots, T\}$ **do**
- $\delta^0_{t,Re}(l) = 0$
- $\delta^0_{t,Im}(l) = 0$
- **if** $l \in \mathcal{G}$ **then**
  - $s_{l,t} = p_l(t) - v_l^k(t) + jq_l^k(t)$
- **else**
  - $s_{l,t} = p_l(t) - v_l^k(t) + jq_l(t)$
- **end**
- $b_{t,Re} = \frac{\cos(\theta+\phi)}{U_0}\,\mathrm{Re}(s_{l,t} - s_{l,0}) - \frac{\sin(\theta+\phi)}{U_0}\,\mathrm{Im}(s_{l,t} - s_{l,0})$
- $b_{t,Im} = \frac{\sin(\theta+\phi)}{U_0}\,\mathrm{Re}(s_{l,t} - s_{l,0}) + \frac{\cos(\theta+\phi)}{U_0}\,\mathrm{Im}(s_{l,t} - s_{l,0})$

end

for $\ell \in \{1, \ldots, \ell_{\max} - 1\}$ **do**
- **for** $t = \{1, \ldots, T\}$ **do**
  - $\delta^{\ell+1}_{t,Re}(l) = (1-h)\delta^{\ell}_{t,Re}(l) - \frac{h}{(G_S)_{ll}}\left(\sum_{r \in \mathcal{N}_S(l)\setminus\{0,l\}} (G_S)_{lr}\delta^{\ell}_{t,Re}(r) - b_{t,Re}\right)$
  - $\delta^{\ell+1}_{t,Im}(l) = (1-h)\delta^{\ell}_{t,Im}(l) - \frac{h}{(G_S)_{ll}}\left(\sum_{r \in \mathcal{N}_S(l)\setminus\{0,l\}} (G_S)_{lr}\delta^{\ell}_{t,Im}(r) - b_{t,Im}\right)$
- **end**

end

for $t = \{1, \ldots, T\}$ **do**
- $u_l^k(t) \approx \delta^{\ell_{\max}}_{t,Re}(l) + j\delta^{\ell_{\max}}_{t,Im}(l) + u_{l,0}$

end

Notice that a fast execution of this algorithm (until some error tolerance is reached) at the end of each iteration of the dual decomposition algorithm can give an approximation of $u_G(t)$, $u_S(t)$, $t \in \tau$, for the next iteration.

V. Simulation results and discussion

We implement the dual decomposition algorithm in a model of the IEEE 37 standard, with the same location of PV generators as in [12]. We also add storage capacity to all generation nodes, and also to 4 load nodes. Connection to the grid is made only through node 0. The complete list of the system parameters we use can be found at `http://fausto.dynamic.ucsd.edu/andres`. Simulations have been run by coupling the dual decomposition algorithm with the nonlinear power flow equations that represent the microgrid. The parameter γ is chosen to be $0.8/\|M\|_\infty$, which clearly satisfies the bound presented in Theorem 3.1. We also solve the problem in (4) in a centralized way, in order to obtain a benchmark. Recall that this is a relaxation of the OPF problem. Figure 1 shows convergence for the reactive power and storage control variables on node 4. Figures 2 through 4 show the dual variables evolution at each iteration. We find through this simulation that the algorithm reaches the same $v^\star$ as the centralized solution of (4). The value of $q^\star$ differs slightly, which we consider is due to the fact that we are designing on a linear relaxation of the constraints in the problem. Nonetheless, the error in $q^\star$ is near 1.5×10^4, while $q^\star$ is larger than 10^5.

A drawback of this algorithm is that the convergence time is large, since the parameter γ must be very small in order to guarantee convergence. This leads multipliers to drift very slowly before reaching their steady state, as it can be seen on Figures 2 through 4. For the particular simulation scenario we show, convergence takes almost 46000 iterations. This becomes particularly problematic if we consider application scenarios in which the duration of a time slot in the prediction horizon is short.

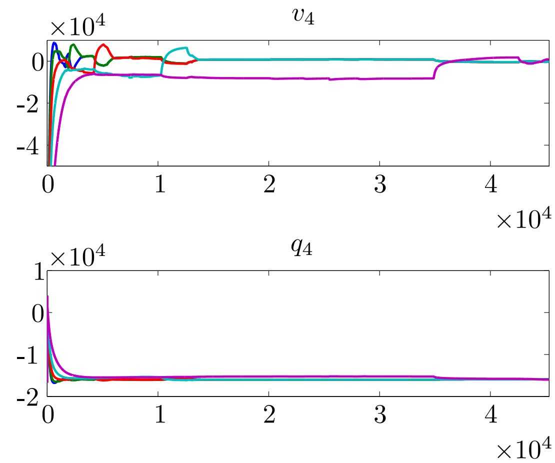

Fig. 1. a) Error in $v_4(t)$ for all $t \in \tau$, b) Error in $q_4(t)$ for all $t \in \tau$.

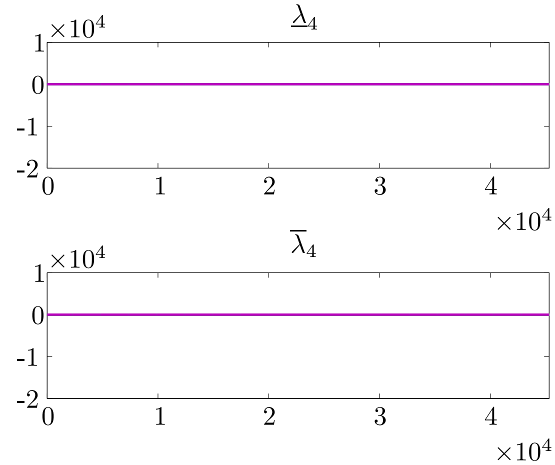

Fig. 2. a) $\underline{\lambda}_4(t)$ for all $t \in \tau$, b) $\overline{\lambda}_4(t)$ for all $t \in \tau$.

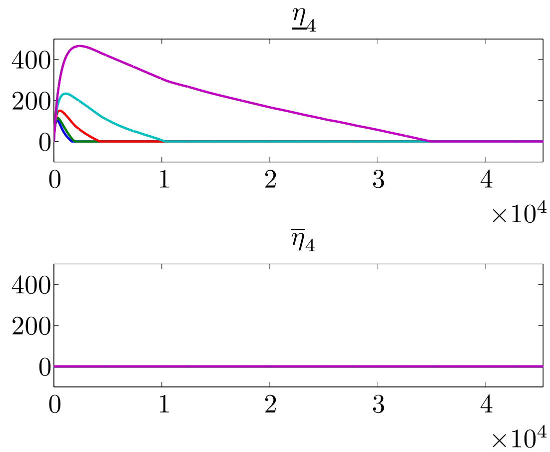

Fig. 3. a) $\underline{\eta}_4(t)$ for all $t \in \tau$, b) $\overline{\eta}_4(t)$ for all $t \in \tau$.

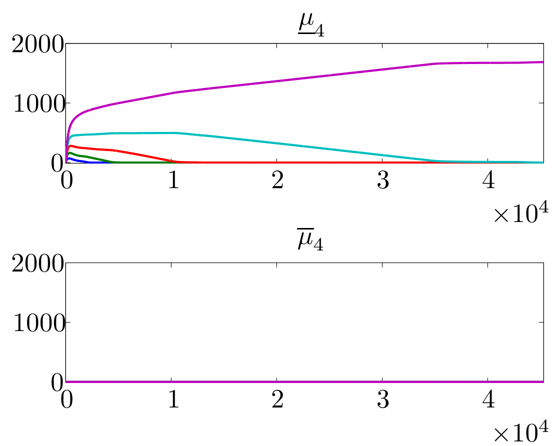

Fig. 4. a) $\underline{\mu}_4(t)$ for all $t \in \tau$, b) $\overline{\mu}_4(t)$ for all $t \in \tau$.

VI. Conclusions and future work

We present a distributed algorithm for the computation of predictive control sequences of reactive power and storage in microgrids. The algorithm uses forecasted parameters such as the electricity cost on time and also the solar power generation, in order to compute the reactive power that must be injected by PV generators and the charging/discharging rate that storage devices must follow in order to minimize electricity cost and also transmission losses. The design is based on the convexification approach of the OPF problem presented in [11], [12]. This approach relaxes the power flow equations onto a linear relation between power and voltage in a microgrid, under some assumptions on impedances in the transmission lines and the magnitude of the input voltage. Constraints on voltage regulation, and operational constraints on the storage systems are considered Then, a dual-decomposition-based dynamics is presented. We also present a convergence result whose proof can be found in [13], concluding that the dynamics globally converge to the unique optimizer of the problem. Finally, we discuss two ways of retrieving the voltage values that are required in order to obtain future values of the decision variables, i.e., reactive power generation and storage control. Simulations illustrate the algorithm performance.

As a future direction we intend to design dynamics with fast convergence, which could be practically implementable for short-time prediction horizons and relatively small discretization steps, in basic computation devices such as PLCs and RTUs. Accelerated gradient methods can be considered to this end.

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