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Cubic Mobile Robot under Rolling Constraints

Tetsuro Hirano[1], Masato Ishikawa[2] and Koichi Osuka[3]

Abstract— **In this paper, the authors present a new kind of cubic mobile robot whose locomotion is dominated by rolling constraints. We discuss the controllability on the robot in a discrete state space. The behaviors of the robot are described as motions of a rolling cube with special constraints originated in their actuation structure. Considering some cases of input patterns, we concluded that the robot has the global reachability on its position.**

I. INTRODUCTION

Simplification of dynamics evolving continuous state spaces to discrete ones help us to analyze and compute the system behaviors in qualitative sense. On spatio-temporal discretization of the continuous systems, however, we should be careful not to ruin the systems characteristics which accompany the control purposes. Such discretization methods have been developed by several researchers [1][2].

In this paper, the authors consider to apply such discretization method to specific mobile robots. Previously, we proposed a new category of rolling mobile robots called *MRR*s (Mass-driver Rolling Robots) in [3]. Locomotion of *MRR*s are based on rigid rolling constraints generated by active manipulations of internal mass distribution. Shapes surrounding the robots determine the rolling constraints and characterize the locomotion patterns of them.

Among various shapes for rolling mobile robots, the sphere is a simple choice for the bodies because of its multi-directional locomotion. In this study, we regard the cubic rolling robot as the discretization of a spherical rolling robot. We have tried to produce such cubic robot as an approach to reveal the common locomotion principles in *MRR*s.

In this paper, we propose a new kind of cubic mobile robot with two internal actuators and describe the analysis on the reachability of the robot in a discretized state space.

II. CUBIC MOBILE ROBOT

As a member of *MRR*s, we have developed a cubic mobile robot whose locomotion is based on the principle of rolling between a cube and a plane. Such cubic rolling robot is also developed by D'Andrea *et al.* [4]. While their robot is equipped with many actuators and sensors for locomotion of the robot for issues on robotics , we concerned with the relationship between the locomotion principle of cubic rolling robots and that of spherical rolling robots.

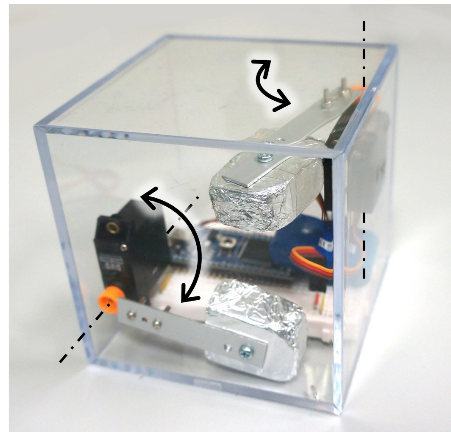

Fig. 1. Overview of prototype of the cubic rolling robot

According to the research on the spherical rolling robot "volvot" [5], rolling of the spherical robot can be generated by actuators in two degrees of freedom. This fact suggests that only two degrees of freedom actuators also can move the rolling robot with a cubic body, which is as a discretization of a spherical body.

In this study, two independent actuation units make the two-degrees-of-freedom input. Each unit consists of a servo-motor and a mass on the end of its servo arm. Based on the concepts, we produced a prototype of the cubic mobile robot (see Fig. 1).

The actuator units can make radical shifts on angular momentum by knocking on the faces from inside the robot. The robot rolls rigidly at the result of the angular momentum preservation around the edges of the cubic body. So, the higher the knocked point is from the ground plane, the more easily the rolling around the edge occurs. The robot has two actuator units and they can knock at most four point on the faces apart from the centers of the faces (see Fig. 2). Fig. 2 also indicates that each face of the cube is assigned the numbers from 1 to 6 for identification.

For simplicity, we assume that the robot can roll around the edges only when the knocked point exceeds the center of a side wall. The robot, for example, can roll to the next positions where the face number of the top side is 3, 4 or 5 from the state shown in Fig. 2(a). On the other hand, the robot can transit to only the state where the face labeled 5 becomes the top side from the state shown in Fig. 2(b).

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[1]Tetsuro Hirano is with Department of Mechanical Engineering, Osaka University, 5650871 Suita Osaka, Japan hirano@dsc.mech.eng.osaka-u.ac.jp
[2]Masato Ishikawa is with Department of Mechanical Engineering, Osaka University, 5650871 Suita Osaka, Japan ishikawa@mech.eng.osaka-u.ac.jp
[3]Koichi Osuka is with Department of Mechanical Engineering, Osaka University, 5650871 Suita Osaka, Japan osuka@mech.eng.osaka-u.ac.jp

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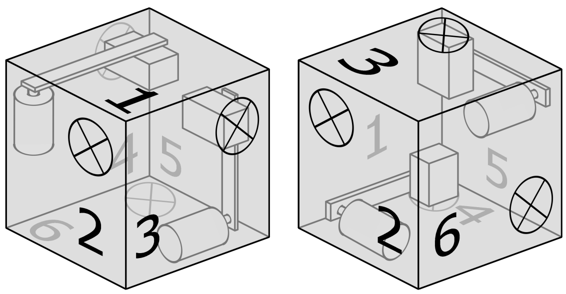

(a) Model of *CuRoRo*

(b) Alternate state with same configuration as (a)

Fig. 2. Configurations of the hammers, their knocked points and definition of the face numbers. Marks (⊗) indicate the knocked points. Face numbers are mainly used to identify which face appearing on top.

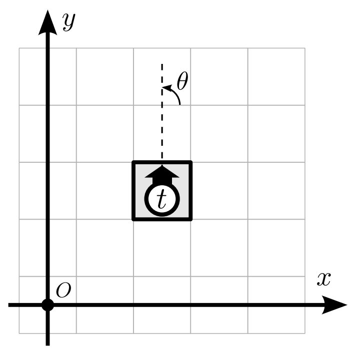

Fig. 3. Discrete coordinate system for cubic robot. θ represents the direction and t represents the face number of top side of robot.

So, the behavior of the robot can be considered as the rolling cube model with the extra constraint depending on the orientation of the robot. This extra constraint are called structural constraint since it originates in the robot structure.

III. CONTROL PROBLEM UNDER ROLLING CONSTRAINTS

In our assumptions, the robot body is an ideal cube and rolls on a smooth plane around its grounded edge without any direction of slip. There are a few differences from the ordinary rolling cube models such as described in [6].

First is that, inputs of the system are generated from inside the cube different from those which is applied by external forces. This affects the dependence of the input vector on the robot orientation. The second extra constraint comes from configuration of the actuators. The robot has only two actuators and can knock at most four walls. In other words, there are two sides which can never come to the top one depending on the initial states.

In this section, we describes a discrete modeling of the robot and a basic position control strategy under the structural constraints.

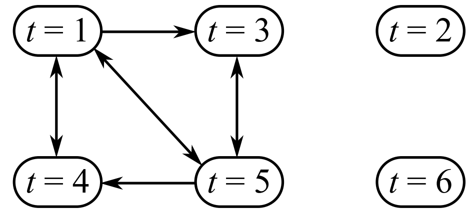

Fig. 4. State transition graph with respect to the face number on top t. Two edges are directed and two nodes are isolated because of the structural constraint

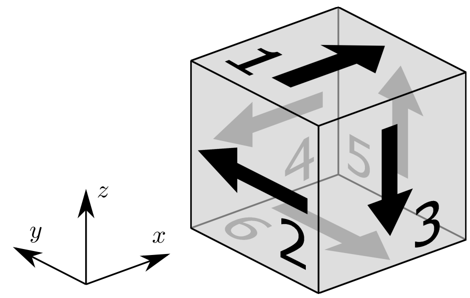

Fig. 5. Definition of cube orientation. Angle between x-axis and direction of arrow appearing on the top side determines θ. Configuration of the face numbers corresponds to those which illustrated in Fig. 2

A. REDUCTION TO DISCRETE LOCOMOTION

We define a discrete coordinate system and the robot orientation. The robot position is described as the ground position of the center of it on the lattice plane $[x, y]^T \in \mathbb{Z}^2$. Fig. 3 indicates the discrete coordinate system. The robot on the horizontal plane is expressed as a bold square. t denote the face number appearing on the top side of the cube. Where let F be a set of the face numbers, t belongs to $F = \{1, 2, \cdots, 6\}$. The orientation of the robot θ is determined by the angle between the x-axis and the arrow on the face appearing on the top side (see Fig. 5). The arrows are set to be symmetry in the sense of the equivalence for one to the others. θ can be represented in discrete values since there are only four values for indicating the orientation of the robot. In our study, θ is labeled in 0 to 3 for each possible direction (see Fig. 6).

So, the state space of the robot is $\boldsymbol{x} = [x, y, \theta, t]^T \in X = \mathbb{Z}^2 \times \mathbb{Z}_4 \times \mathbb{Z}_6$ where$\mathbb{Z}_i$ indicates an integer space which consists of only i integers.

Under the coordinate settings, transitions among the robot states are described as the directed graph shown in Fig. 4 because of the structural constraint.

Considering the extra constraints, the difference equation which expresses the transition of the robot state is as follows:

$$\Delta \boldsymbol{x}_n = \sum_{t \in T_p} g_t(\theta_n, t_n) u_t, \tag{1}$$

where $\Delta \boldsymbol{x}_n$ is the transition vector from the n^{th} to the $n{+}1^{th}$ step. θ_n and t_n are θ and t at n^{th}. T_p is a set of the face numbers which have the possibility to appear on the top side (in this case $T_p = \{1, 3, 4, 5\}$). u_t is apparent control input to t and are binary values indicating whether the face

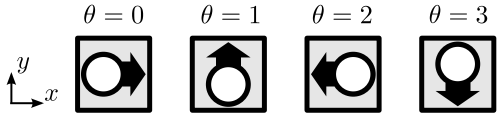

Fig. 6. Definition of direction of top side in discrete coordinate system. The values are limited to four integers identifying the robot direction.

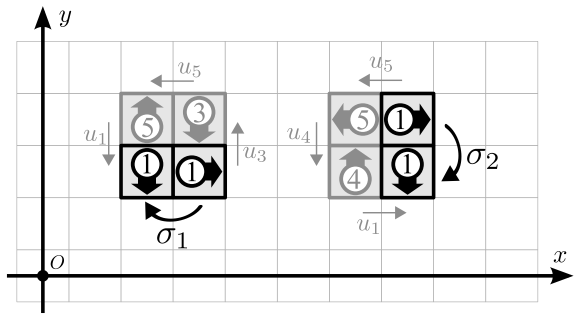

Fig. 7. Input processes of σ_1 and σ_2. Robot backs 1 unit behind and rotates clockwise from initial state in σ_1. Robot shifts 1 unit to the right and rotates clockwise from initial state in σ_2.

labeled t comes on top or not at the next step. For example, if we want to take the face labeled "1" to the top at next step, the inputs are described as $u_1 = 1$ and the others are $u_3 = u_4 = u_5 = 0$. g_t is 4×1 matrix corresponding to each u_t.

B. EXAMPLES OF CONTROL STRATEGY

We consider the state transition from the $[x, y, \theta, t]^T = [0, 0, 0, 1]^T$ as the initial state and constructed the input rules to transit the state with preserving t on $t = 1$. According to the transition graph, there are only two independent paths which return to the state $t = 1$. All input series are generated by combinations of the two paths on the graph.

A series of ordered inputs is a finite sequence of integers chosen from 1, 3, 4 and 5. Here, we propose two basic input series:

$$\sigma_1 = \{3, 5, 1\} \tag{2}$$

$$\sigma_2 = \{5, 4, 1\} \tag{3}$$

where $\sigma = \{t_1, t_2, \cdots, t_j\}\,(t_j \in T_p)$ indicates that inputs are chosen in sequence $u_{t_1} \to u_{t_2} \to \cdots \to u_{t_j}$. Note that the order of each input series follows the directions of the edges in the state transition graph. After each $\sigma_1$and σ_2 acts on the robot, the state transition goes to

$$\Delta_{\sigma_1}\boldsymbol{x} = [-\cos(\frac{\pi}{2}\theta_n), -\sin(\frac{\pi}{2}\theta_n), -1, 0]^T \tag{4}$$

$$\Delta_{\sigma_2}\boldsymbol{x} = [\sin(\frac{\pi}{2}\theta_n), -\cos(\frac{\pi}{2}\theta_n), -1, 0]^T. \tag{5}$$

Fig. 7 illustrates the state transition processes of each input series.

We can make some further input series combining σ_1 and σ_2. It takes four times of both input series to return the state oriented the same direction. So, if we take σ_1 or σ_2 at four times, the robot can transit the states with preserving θ and

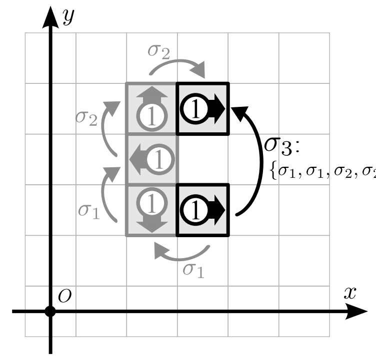

Fig. 8. Input process of σ_3. Robot shifts 2 units to the left from initial state with preserving orientation

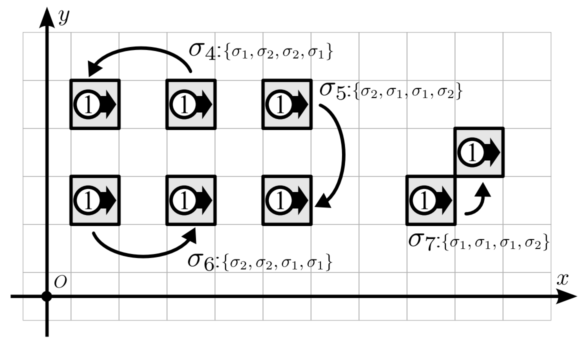

Fig. 9. State transitions of each σ_4 to σ_7. σ_4, σ_5 and σ_6 make the robot shift 2 units behind, to the right and forward from initial state with preserving its orientation. σ_7 make the robot shift 1 unit to the left and 1 unit forward.

t. In other words, such input patterns generate translational movement. We describe σ_3 as an example of them.

$$\sigma_3 = \{\sigma_1, \sigma_1, \sigma_2, \sigma_2\} \tag{6}$$

This input series makes translation for the left side of the robot (see also Fig. 8). The state transition generated by σ_3 is as follows:

$$\Delta_{\sigma_3}\boldsymbol{x} = \left[-2\sin\left(\frac{\pi}{2}\theta_n\right), 2\cos\left(\frac{\pi}{2}\theta_n\right), 0, 0\right]^T. \tag{7}$$

The other input series σ_4, σ_5 and σ_6 also make the translation following the other directions. Their patterns are as follows:

$$\sigma_4 = \{\sigma_1, \sigma_2, \sigma_2, \sigma_1\} \tag{8}$$

$$\sigma_5 = \{\sigma_2, \sigma_1, \sigma_1, \sigma_2\} \tag{9}$$

$$\sigma_6 = \{\sigma_2, \sigma_2, \sigma_1, \sigma_1\}\,. \tag{10}$$

Each of them make the 2 units of rectilinear translation for the directions following x and y-axis as follows:

$$\Delta_{\sigma_4}\boldsymbol{x} = -\left[2\cos\left(\frac{\pi}{2}\theta_n\right), 2\sin\left(\frac{\pi}{2}\theta_n\right), 0, 0\right]^T \tag{11}$$

$$\Delta_{\sigma_5}\boldsymbol{x} = \left[2\sin\left(\frac{\pi}{2}\theta_n\right), -2\cos\left(\frac{\pi}{2}\theta_n\right), 0, 0\right]^T \tag{12}$$

$$\Delta_{\sigma_6}\boldsymbol{x} = \left[2\cos\left(\frac{\pi}{2}\theta_n\right), 2\sin\left(\frac{\pi}{2}\theta_n\right), 0, 0\right]^T. \tag{13}$$

They enable the robot to translate for each direction of the lattice plane space.

In addition, there are diagonal translation generated by, for example,

$$\sigma_7 = \{\sigma_1, \sigma_1, \sigma_1, \sigma_2\}. \tag{14}$$

The state transition generated by σ_7 is as follows:

$$\Delta_{\sigma_7}\boldsymbol{x} = \sqrt{2}\left[\cos\left(\pi\theta_n + \frac{\pi}{4}\right), \sin\left(\pi\theta_n + \frac{\pi}{4}\right), 0, 0\right]^T. \tag{15}$$

The state transition process of each σ_4 to σ_7 is illustrated in Fig. 9

The robot can reach any positions in the lattice plane using these translational input series. Let us consider a case that a desired position is given as (x_d, y_d). An example of the control algorithm for the robot with a initial state $\boldsymbol{x}_0 = [0, 0, 0, 1]^T$ is as follows:

1) Calculate $k_x = |\lfloor x_d/2 \rfloor|$ and $k_y = |\lfloor y_d/2 \rfloor|$.
2) Adjust the x position using σ_6 (if $x_d \geq 0$) or σ_4 (if $x_d < 0$) for k_x times.
3) Adjust the y position using σ_3 (if $y_d \geq 0$) or σ_5 (if $y_d < 0$) for k_y times.
4) Approach using σ_7 if x_d or y_d are odd numbers.
5) Approach using σ_1 (if x_d is even) or σ_2 (if y_d is even).

where $\lfloor x \rfloor = \max\{k \in \mathbb{Z} | k \leq x\}$

So, we can conclude that the robot has global reachability in its positions if its direction or face at the top side are not referred. Note that this control strategy, however, is feasible from any initial state except for the cases $t = 2$ or 6 because the robot can not roll to any directions from these initial states under our assumption.

IV. CONCLUSION

In this paper, the authors presented a new type of cubic mobile robot whose locomotion is dominated by rolling constraints. In order to consider a position control strategy, a discrete coordinate system was described. At the result of the discrete analysis, a position control strategy was obtained.

Theoretical approaches considering the extra constraints must be required for further development. These approaches help us to control alternate cubic rolling robots with other structures and to analyze the optimal pathways depending on each constraint. Also introducing triangles or mixed polygons to the state space, we will extend the control theories on the cubic rolling robot to those on polyhedral rolling robots.

REFERENCES

[1] A. Marigo and A. Bicchi, “Steering driftless nonholonomic systems by control quanta,” in Proc. of IEEE Int. Conf. Decision Control, 1998.
[2] Y. Tazaki and J. Imura, “Finite abstractions of discrete-time linear systems and its application to optimal control,”in Proc. of 17th IFAC World Congress, 2008.
[3] T. Hirano, M. Ishikawa and K. Osuka, “Control and Development of Cylindrical Mobile Robot, J. of Robotics and Mechatronics,” vol. 25, no. 2, Apr. 2013, pp. 392–397.
[4] M. Gajamohan, M. Merz, I. Thommen and R. D’Andrea, “The Cubli: A Cube that can Jump Up and Balance,” in Proceeding IEEE/RSJ International Conference of Intelligent Robots and Systems, pp. 3722–3727, 2012.
[5] M. Ishikawa, , R. Kitayoshi and T. Sugie, “Volvot: a Spherical Mobile Robot with Eccentric Twin Rotors,” in Proceedings of IEEE International Conference on Robotics and Biomimetics, pp.1462–1467, 2011.
[6] A. Bicchi, Y. Chitour and A. Marigo, “Reachability and Steering of Rolling Polyhedra: A Case Study in Discrete Nonholonomy,” IEEE Trans. on Automatic Control, vol. 49, no. 5, May 2004, pp. 710–726.