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On the decentralized H^2 optimal control of cooperative bilateral teleoperation systems

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***Abstract*— Control of cooperative bilateral teleoperation systems over delayed communication is considered. A recently proposed quadratically invariant control architecture is adopted. It is shown that within this architecture, the H^2 optimal controller can be found analytically in spite of the decentralized nature of the problem. As a first step, we show that the problem can be split into a number of independent centralized control problems with delays. This, in turn, allows us to apply the recent loop shifting techniques for finding an efficient solution and revealing the optimal controller structure.**

I. INTRODUCTION

Teleoperation systems are used to expand operator abilities to remote and/or hazardous environments. In such systems the operator uses an interface device, called a “master” in order to manipulate a robotic systems, referred to as a “slave”. *Bilateral teleoperation* notion refers to the case in which haptic feedback is provided to operator, contributing to a telepresence [1], i.e., to the operator’s ability to feel the remote site. In this case, not only the slave but also the master device constitutes an actuated dynamical system and needs to be controlled.

In this paper, we concentrate on a control problem arising in cooperative bilateral teleoperation, i.e., in the case when multiple operators are manipulating a number of slave devices in a shared task environment. A situation like this may occur, for example, in a surgery with dual master consoles [2]. One of the major challenges associated with such problems arises from communication delays, which in many practical cases may not be neglected, and render the associated control problem decentralized.

A common approach to delayed bilateral teleoperation control is based on passivity, see [3], [4] and the references therein. While guaranteeing delay-independent stability, passivity-based methods restrict a controller structure and are somewhat less intuitive to use for performance optimization. Development of alternative optimization-based methods, however, is impeded by the decentralized nature of the problem [5]. The question underlying this work is whether it is possible to formulate a tractable optimization for the cooperative teleoperation control setup.

In [6], the problem of H^2-optimal control of teleoperation system with a single master-slave couple was studied. It was shown that this problem is quadratically invariant (QI) and, moreover, a complete analytical solution for it was derived. In [7] it was shown, however, that the problem with two master/slave pairs is, generally, not QI. Yet, a modified control architecture was proposed in [7] rendering the problem QI and allowing derivation of a non restrictive stabilizing control structure.

In this work we continue the research line from [7] and adopt the modified control architecture from the aforementioned reference. The main contribution of this work is in revealing the structure of the optimal controller and proposing a way to derive an explicit formulae for it.

The remainder of this paper is organized as follows. In Section II a brief overview of previous works is provided as a preliminary. The H^2 optimization problem is formulated in Section III. In Section IV the solution outline is presented and the optimal controller structure is presented in Section V.

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II. PRELIMINARIES

Prior to the mathematical problem formulation, a short overview of our previous work shall be presented below. Let us start with describing the control problem associated with the cooperative bilateral teleoperation setup (CBTS).

A. Cooperative teleoperation as a control problem

Block diagram for the control of CBTS with two haptic devices is presented in Fig. 1.

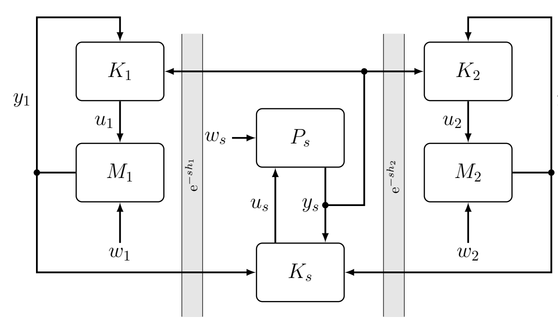

Fig. 1. Natural control architecture

The block P_s on this diagram represents the overall dynamics of a slave robotic systems. It’s inputs are the control signal u_s and the environment disturbance w_s. The output signal y_s represents all the available measurements from the robotic system. We will partition P_s with respect to its inputs as follows:

$$y_s = \begin{bmatrix} P_{sw} & P_{su} \end{bmatrix} \begin{bmatrix} w_s \\ u_s \end{bmatrix}.$$

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Note that in addition to the dynamics of the robotic system, usually, P_s incorporates also the modeled part of the environment dynamics.

The blocks $M_{1/2}$ represent interface device and operator arm dynamics for each of the master sites. The inputs are the control signals — $u_{1/2}$ and exogenous operator inputs (muscle forces) — $w_{1/2}$. The signals $y_{1/2}$ represent the available measurements. The blocks $M_{1/2}$ will be partitioned as:

$$y_1 = \begin{bmatrix} M_{1w} & M_{1u} \end{bmatrix} \begin{bmatrix} w_1 \\ u_1 \end{bmatrix}, \; y_2 = \begin{bmatrix} M_{2w} & M_{2u} \end{bmatrix} \begin{bmatrix} w_2 \\ u_2 \end{bmatrix}.$$

The controller of the slave device is denoted as K_s. As follows from the nature of the considered application, K_s has an immediate access to the local measurements but a delayed access for the measurements from the master sites. Thus, the natural partitioning of the slave controller with respect to its inputs is:

$$u_s = \left[\; K_{ss} \quad K_{s1} \quad K_{s2} \;\right] \begin{bmatrix} y_s \\ y_1 \mathrm{e}^{-sh_1} \\ y_2 \mathrm{e}^{-sh_2} \end{bmatrix},$$

where $h_{1/2}$ are the delays in communication between the slave and the corresponding master site. Similarly, controllers of the master devices, denoted as $K_{1/2}$, have immediate access to their local measurements and a delayed access to the measurements from the slave site. A partitioning of these controllers with respect to their inputs is:

$$u_1 = \begin{bmatrix} K_{11} & K_{1s} \end{bmatrix} \begin{bmatrix} y_1 \\ y_s \mathrm{e}^{-sh_1} \end{bmatrix}, u_2 = \begin{bmatrix} K_{22} & K_{2s} \end{bmatrix} \begin{bmatrix} y_2 \\ y_s \mathrm{e}^{-sh_2} \end{bmatrix}.$$

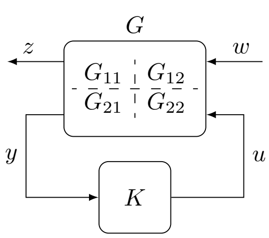

Fig. 2. General control setup

We may cast the control problem described above as a generalized setup, shown in Fig. 2, with

$$w = \begin{bmatrix} w_s \\ w_1 \\ w_2 \end{bmatrix}, u = \begin{bmatrix} u_s \\ u_1 \\ u_2 \end{bmatrix} \text{ and } y = \begin{bmatrix} y_s \\ y_1 \\ y_2 \end{bmatrix}.$$

The sub-blocks G_{11} and G_{12} define regulated output z, which typically incorporates weighted coupling errors and control efforts[1]. The other sub-blocks and the controller are restricted by the problem topology and have the following form:

$$\left[\, G_{21} \,\vdots\, G_{22} \,\right] = \begin{bmatrix} P_{sw} & 0 & 0 & P_{su} & 0 & 0 \\ 0 & M_{1s} & 0 & 0 & M_{1u} & 0 \\ 0 & 0 & M_{2s} & 0 & 0 & M_{2u} \end{bmatrix},$$

$$K = \begin{bmatrix} K_{ss} & \mathrm{e}^{-sh_1} K_{s1} & \mathrm{e}^{-sh_2} K_{s2} \\ \mathrm{e}^{-sh_1} K_{1s} & K_{11} & 0 \\ \mathrm{e}^{-sh_2} K_{2s} & 0 & K_{22} \end{bmatrix}. \quad (1)$$

Note that in the resulting generalized setup, the controller (1) is structurally constrained. We will denote this constraint with $K \in \mathcal{S}_1$, where

$$\mathcal{S}_1 : \begin{bmatrix} * & \mathrm{e}^{-sh_1} * & \mathrm{e}^{-sh_2} * \\ \mathrm{e}^{-sh_1} * & * & 0 \\ \mathrm{e}^{-sh_2} * & 0 & * \end{bmatrix} \quad (2)$$

with $*$ representing an arbitrary transfer function. This constraint impedes the use of centralized optimization techniques and constitutes one of the major challenges of the considered problem.

B. Modified control architecture

In this work we assume that the original problem dynamics — P_s, $M_{1/2}$ and, as a result, the generalized plant G are stable[2]. Thus, the set of all stabilizing controllers can be characterized using Youla parametrization [8]

$$K = \tau_n(Q, G_{22}) = Q\left(I + G_{22} Q\right)^{-1}, \quad (3)$$

where Q is any arbitrary yet stable parameter of the same dimension as K. The difficulty in adopting this parameterization for our problem is due to the fact that it covers all stabilizing controllers regardless of their structure. It is unclear then how to restrict the parameter Q in order to guarantee that $K \in \mathcal{S}_1$.

It was shown in [6] that in the case of teleoperation with a single master-slave pair, this difficulty can be overcome since the associated problem is quadratically invariant [9] (QI). The QI notion refers to decentralized problems in which structural constraint on the generalized controller $K \in \mathcal{S}$ satisfies $\forall K \in \mathcal{S} : K G_{22} K \in \mathcal{S}$. It can be shown that in this case the structural constraint is invariant under Youla parameterization, i.e., $K \in \mathcal{S} \Leftrightarrow Q \in \mathcal{S}$. Note, though, that the structural constraint in our problem, $\mathcal{S}_1$, does not admit the QI property. Thus, there is no direct extension for the results from [6] to the cooperative teleoperation case.

Recently it was shown in [7] that a reasonable extension in communication can still render the cooperative teleoperation control problem QI. The idea is to allow direct or indirect communication between the master sites. In this work we adopt a modified control architecture from [7], depicted in Fig. 3, where additional communication between the master sites is allowed with a delay h_3. Note that this does not necessarily require opening a new communication channel, as all the required information can be transferred via the slave site. In this case $h_3 = h_1 + h_2$, see [7] for more details.

[1]The regulated output z should reflect a requirement on the transparency of the resulting teleoperation system. Its choice is an interesting question for a separate study and is not in the scope of the current paper.

[2]This assumption is made for simplicity. All results presented in this work can be extended for the unstable case.

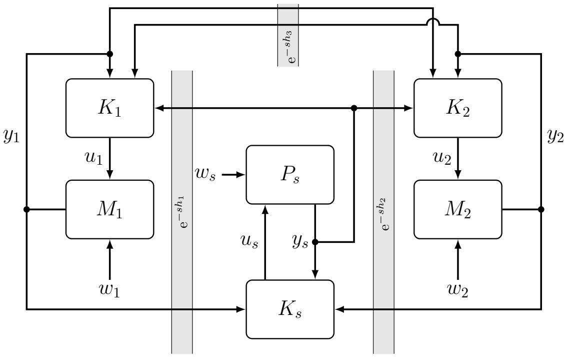

Fig. 3. Modified control architecture

Within this modified architecture, each of the master controllers has three inputs and should be partitioned as

$$u_1 = \begin{bmatrix} K_{11} & K_{1s} & K_{12} \end{bmatrix} \begin{bmatrix} y_1 \\ y_s e^{-sh_1} \\ y_2 e^{-sh_3} \end{bmatrix},$$

$$u_2 = \begin{bmatrix} K_{22} & K_{2s} & K_{21} \end{bmatrix} \begin{bmatrix} y_2 \\ y_s e^{-sh_2} \\ y_1 e^{-sh_3} \end{bmatrix}.$$

The resulting generalized controller for Fig. 2 has a form

$$K = \begin{bmatrix} K_{ss} & e^{-sh_1} K_{s1} & e^{-sh_2} K_{s2} \\ e^{-sh_1} K_{1s} & K_{11} & e^{-sh_3} K_{12} \\ e^{-sh_2} K_{2s} & e^{-sh_3} K_{21} & K_{22} \end{bmatrix}, \quad (4)$$

i.e., the original structural constraint on the controller (2) is transformed into

$$S: \begin{bmatrix} * & e^{-sh_1} * & e^{-sh_2} * \\ e^{-sh_1} * & * & e^{-sh_3} * \\ e^{-sh_2} * & e^{-sh_3} * & * \end{bmatrix}. \quad (5)$$

It is shown in [7] that under a set of mild assumptions this modified structural constraint is QI. Moreover, it is shown that applying Youla parameterization reveals an insightful structure of controllers. This structure guarantees delay-independent stability but, unlike passivity-based structures, is not restrictive and covers all possible linear stabilizing controllers. In this work we aim at making the next step and finding the H^2-optimal controller within the proposed control architecture.

III. Problem formulation

Our goal in this work is to find a stabilizing H^2 optimal controller for CBTS within the control architecture depicted in Fig. 3. Following the arguments from Section II we cast the problem as a general control setup in Fig. 2 with

$$\left[\begin{array}{c|c} G_{11} & G_{12} \\ \hline G_{21} & G_{22} \end{array}\right] = \left[\begin{array}{ccc|ccc} G_a & G_b & G_c & G_d & G_e & G_f \\ \hline P_{sw} & 0 & 0 & P_{su} & 0 & 0 \\ 0 & M_{1s} & 0 & 0 & M_{1u} & 0 \\ 0 & 0 & M_{2s} & 0 & 0 & M_{1u} \end{array}\right], \quad (6)$$

where $G_{a,\dots,f}$ are the columns of G_{11} and G_{12}, that define the regulated output z. Note that the block-diagonal structure of G_{22} results from the fact that the original dynamics of the slave and the two master devices are decoupled. The block-diagonal structure of G_{21}, in turn, results from the fact that each of the sites has an independent disturbance.

We will consider the problem under the following set of assumptions on the communication delays:

$\boldsymbol{\mathcal{A}_1}$: $h_1 \le h_2 + h_3$

$\boldsymbol{\mathcal{A}_2}$: $h_2 \le h_3 + h_1$

$\boldsymbol{\mathcal{A}_3}$: $h_3 \le h_1 + h_2$

These assumptions are needed in order to guarantee that the problem is QI, [7]. Note that they are satisfied in most of the practical cases. In particular, they hold for the case when the communication between the two master devices is via the slave site and, as a result, $h_3 = h_1 + h_2$. For the sake of simplicity, we will assume also that

$\boldsymbol{\mathcal{A}_4}$: $h_3 \ge h_2 \ge h_1$

Note, however, that this is a technical assumption and the solution presented below can be reformulated for any other relation between the delay lengths as along as $\boldsymbol{\mathcal{A}_{1\text{-}3}}$ hold.

OK: Given the transfer matrices $G_{11}, G_{12}, G_{21}, G_{22} \in H^\infty$ as in (6) and the structural constraint $\mathcal{S}$ as in (5), find a stabilizing $K \in \mathcal{S}$ that minimizes the H^2 norm of the closed loop transfer function

$$T_{zw} = G_{11} + G_{12} K (I - G_{22} K)^{-1} G_{21}. \quad (7)$$

IV. Solution outline

Substituting Youla parameterization (3) into (7), yields the following parameterization of closed loop the transfer matrix:

$$T_{zw} = G_{11} + G_{12} Q G_{21}. \quad (8)$$

Since our problem is QI, the structural constraint $K \in \mathcal{S}$ can be transferred to the Youla parameter. This allows us to reformulate the original problem **OK** in terms of the Youla parameter:

OQ: Given the transfer matrices $G_{11}, G_{12}, G_{21} \in H^\infty$ as in (6) and the structural constraint $\mathcal{S}$ as in (5), find $Q \in \mathcal{S} \cap H^\infty$ that minimizes the H^2 norm of the closed loop transfer function T_{zw} given in (8).

This problem falls into the category of model matching optimizations with structural constraints. Problems of this type may constitute a substantial theoretical challenge, [10],[11], and for the best of our knowledge, there is no ready to use solution available for the general case. It turns out, however, that **OQ** can be solved analytically due to some structural properties inherent in the considered problem. To this end, let us rewrite (8) in a more explicit form:

$$T_{zw} = \left[\begin{array}{c|c|c} G_a & G_b & G_c \end{array}\right] + \begin{bmatrix} G_d & G_e & G_f \end{bmatrix} \begin{bmatrix} Q_{ss} & e^{-sh_1} Q_{s1} & e^{-sh_2} Q_{s2} \\ e^{-sh_1} Q_{1s} & Q_{11} & e^{-sh_3} Q_{12} \\ e^{-sh_2} Q_{2s} & e^{-sh_3} Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} P_{sw} & 0 & 0 \\ 0 & M_{1s} & 0 \\ 0 & 0 & M_{2s} \end{bmatrix}.$$

One may notice now that the remedy lies in the properties of G_{21}. Its block-diagonal structure allows to solve the H^2

optimization column-wise. Let us consider the three columns of T_{zw} separately:

$$T_1 = G_a + \left[\begin{array}{ccc} G_d & G_e & G_f \end{array}\right] \left[\begin{array}{c} Q_{ss} \\ \mathrm{e}^{-sh_1} Q_{1s} \\ \mathrm{e}^{-sh_2} Q_{2s} \end{array}\right] P_{sw}$$

$$T_2 = G_b + \left[\begin{array}{ccc} G_d & G_e & G_f \end{array}\right] \left[\begin{array}{c} \mathrm{e}^{-sh_1} Q_{s1} \\ Q_{11} \\ \mathrm{e}^{-sh_3} Q_{21} \end{array}\right] M_{1s}$$

$$T_3 = G_c + \left[\begin{array}{ccc} G_d & G_e & G_f \end{array}\right] \left[\begin{array}{c} \mathrm{e}^{-sh_2} Q_{s2} \\ \mathrm{e}^{-sh_3} Q_{12} \\ Q_{22} \end{array}\right] M_{2s}$$

These three columns can be represented in a convenient unified fashion. To this end, we shall define

$$\Lambda_i := \begin{bmatrix} I & 0 & 0 \\ 0 & I\mathrm{e}^{-sh_j} & 0 \\ 0 & 0 & I\mathrm{e}^{-sh_k} \end{bmatrix}, \qquad (j,k) = \begin{cases} (1,2) & \text{if } i = 1 \\ (1,3) & \text{if } i = 2 \\ (2,3) & \text{if } i = 3 \end{cases}$$

and perform some row/column manipulations in order to get

$$T_1 = G_a + \left[\begin{array}{ccc} G_d & G_e & G_f \end{array}\right] \Lambda_1 \left[\begin{array}{c} Q_{ss} \\ Q_{s1} \\ Q_{2s} \end{array}\right] P_{sw}, \qquad (9)$$

$$T_2 = G_b + \left[\begin{array}{ccc} G_e & G_d & G_f \end{array}\right] \Lambda_2 \left[\begin{array}{c} Q_{11} \\ Q_{s1} \\ Q_{21} \end{array}\right] M_{1s}, \qquad (10)$$

$$T_3 = G_c + \left[\begin{array}{ccc} G_f & G_d & G_e \end{array}\right] \Lambda_3 \left[\begin{array}{c} Q_{22} \\ Q_{s2} \\ Q_{12} \end{array}\right] M_{2s}. \qquad (11)$$

Our original problem is now reduced to the following three optimizations:

$\mathbf{OQ_1}$- Given G_{11}, G_{12}, G_{21} find $Q_{ss}, Q_{1s}, Q_{2s} \in H^\infty$ that minimize $||T_1||_2$ for T_1 as in (9);
$\mathbf{OQ_2}$- Given G_{11}, G_{12}, G_{21} find $Q_{11}, Q_{s1}, Q_{21} \in H^\infty$ that minimize $||T_2||_2$ for T_2 as in (10);
$\mathbf{OQ_3}$- Given G_{11}, G_{12}, G_{21} find $Q_{22}, Q_{s2}, Q_{12} \in H^\infty$ that minimize $||T_3||_2$ for T_3 as in (11);

Note that for each of the columns, the delay is extracted outside of the design parameters. Thus, by splitting we overcome its decentralized nature and to reduce it to three independent tractable centralized optimization problems with multiple delays. Those delayed optimizations can now be solved using Pade approximation, discretization and state augmentation, or other available techniques for delayed control.

V. Optimal controller structure

One way to solve $\mathbf{OQ}_i$ is by using the recent loop shifting technique from [12]. This approach was adopted in [6] for the case of a single master-slave couple. In our case, the problems are more complex since they contain two delayed channels. However, the results from [12] can still be applied to $\mathbf{OQ}_i$ iteratively, revealing the structure of the optimal controller, depicted in Fig. 4. All Π_* blocks on this block diagram represent FIR systems while $\tilde{Q}_*$ stand for finite dimensional systems. Using the solution procedure from [12], explicit state-space formulae for each controller block can be derived, which is the subject of the ongoing work.

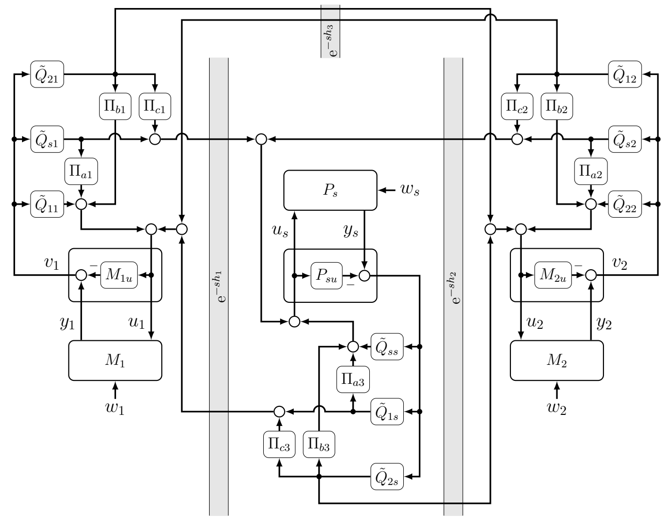

Fig. 4. Optimal control structure

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