

Harmonic influence in large-scale networks: Analysis, optimization, and applications to opinion dynamics and distributed estimation

Giacomo Como

Fabio Fagnani

Paolo Frasca

MINI-COURSE DESCRIPTION

Discrete harmonic functions have long proved to be a fundamental analytical tool in applications as diverse as electrical networks, random walks, and statics. Classical results include, e.g., the maximum principle, variational characterizations (Thompson's principle), and monotonicity properties with respect to the addition/removal of links or the merging of nodes [1], [2], [3], [4]. Recently, discrete harmonic functions have emerged as models for the stationary state of some ergodic opinion dynamics over social networks [5]. In this context, the stationary expected opinions of a possibly very large population of individuals can be characterized as the solution of a Laplace equation on the graph describing the network of social interactions. Boundary conditions, *i.e.*, poles of the expected opinion function, correspond to so-called *stubborn agents*, which are meant to model external and possibly conflicting sources of influence on the society, such as political parties, religious leaders, media sources, or commercial advertising. Stationary *opinion fluctuations* can also be described in terms of harmonic functions on suitably defined *graph products*.

Other current applications can be found in the analysis of several problems of distributed control and estimation. These include *leader-follower* models in robotic networks [6] and estimation from relative measurements. The latter problem, motivated by problems of localization in robotic networks and of synchronization of clock networks and camera networks [7], [8], [9], consists in reconstructing an agent-indexed state from noisy measurements of differences between the states of neighbor nodes.

This mini-course provides an introduction to such models and an overview of some recent results in the field. Using the developed theory of discrete harmonic functions, we illustrate emerging phenomena in large-scale social networks such as *homogeneous influence in highly fluid* networks and *persistent disagreement* and fluctuations [5]. We also observe how modelling stubborn agents may be relaxed to allow for shades of obstinacy [10], [11]. As an application of this framework, the problem of *optimal stubborn placement* [12] will be addressed: one stubborn has to be placed in the social network in such a way to maximally deviate the asymptotic opinion of the population. This question, primarily motivated by marketing applications, will be approached as a problem of maximization of the harmonic influence centrality of an individual on a network. An algorithm for its solution [13]

will be derived, based on the interpretation of harmonic functions as electrical potentials. We will also discuss the problem of *estimation from relative measurements*. First, we show how fundamental limitations on achievable performance can be expressed in terms of effective resistances [7]; next, we show how randomized algorithms for its solution may exploit ergodic dynamics akin to those of opinions [9].

LECTURE SCHEDULE

Giacomo Como: *Discrete harmonic functions and opinion dynamics* (50')

Fabio Fagnani: *Message-passing optimization of harmonic influence centrality* (25')

Paolo Frasca: *Estimation from relative measurements* (25')

REFERENCES

- [1] C. S. J. A. Nash-Williams, "Random walk and electric currents in networks," *Mathematical Proceedings of the Cambridge Philosophical Society*, vol. 55, no. 2, pp. 181–194, 1959.
- [2] P. Doyle and J. Snell, *Random Walks and Electric Networks*, ser. Carus Monographs. Mathematical Association of America, 1984.
- [3] L. Lovász, "Discrete analytic functions: an exposition," *Surveys in differential geometry*, vol. 9, pp. 241–273, 2004.
- [4] R. Lyons and Y. Peres, *Probability on Trees and Networks*, monograph in preparation, 2013.
- [5] D. Acemoglu, G. Como, F. Fagnani, and A. Ozdaglar, "Opinion fluctuations and disagreement in social networks," *Mathematics of Operations Research*, vol. 38, no. 1, pp. 1–27, 2013.
- [6] M. Ji, G. Ferrari-Trecate, M. Egerstedt, and A. Buffa, "Containment control in mobile networks," *IEEE Transactions on Automatic Control*, vol. 53, no. 8, pp. 1972–1975, 2008.
- [7] P. Barooah and J. P. Hespanha, "Estimation from relative measurements: Algorithms and scaling laws," *IEEE Control Systems Magazine*, vol. 27, no. 4, pp. 57–74, 2007.
- [8] D. Borra, E. Lovisari, R. Carli, F. Fagnani, and S. Zampieri, "Autonomous calibration algorithms for networks of cameras," in *American Control Conference*, Montreal, QC, 2012, pp. 5126–5131.
- [9] C. Ravazzi, P. Frasca, H. Ishii, and R. Tempo, "A distributed randomized algorithm for relative localization in sensor networks," in *European Control Conference*, Zürich, Switzerland, Jul. 2013, pp. 1776–1781.
- [10] N. E. Friedkin and E. C. Johnsen, "Social influence networks and opinion change," in *Advances in Group Processes*, E. J. Lawler and M. W. Macy, Eds. JAI Press, 1999, vol. 16, pp. 1–29.
- [11] P. Frasca, C. Ravazzi, R. Tempo, and H. Ishii, "Gossips and prejudices: Ergodic randomized dynamics in social networks," in *IFAC Workshop on Estimation and Control of Networked Systems*, Koblenz, Germany, Sep. 2013, pp. 212–219.
- [12] E. Yildiz, D. Acemoglu, A. Ozdaglar, A. Saberi, and A. Scaglione, "Discrete opinion dynamics with stubborn agents," LIDS, MIT, Tech. Rep. 2858, 2011.
- [13] L. Vassio, F. Fagnani, P. Frasca, and A. Ozdaglar, "Message passing optimization of harmonic influence centrality," To appear in the *IEEE Transactions on Control of Network Systems*, 2014.