MTNS 2014 Mini-course on

Stability and Stabilization of Distributed Port-Hamiltonian Systems

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***Abstract*— This mini-course is devoted at illustrating the latest results on the stability analysis, and on the (boundary) stabilisation of distributed port-Hamiltonian systems. Several related topics will be discussed in five short lessons. More in detail, at the beginning, the problems of well-posedness of distributed port-Hamiltonian systems, existence of solutions for the associated systems of PDEs, and definition of a boundary control system in port-Hamiltonian form are addressed. These topics can be seen as the foundations of the remaining part of the course, that is more focused on stability criterions in case of static and dynamic boundary control, and on the energy-based control of this class of infinite dimensional systems. The emphasis is on linear systems with one-dimensional domain, but it is worth of mentioning that most of the proposed techniques can be in principal adapted to the nonlinear scenario. However, this extension is not trivial, and it is still an open problem.**

***Index Terms*— Control of Distributed Parameter Systems; Stability; Infinite Dimensional Systems Theory**

I. INTRODUCTION

It is more than two centuries that partial differential equations (PDEs) are used to model physical systems. However, one of the most recurring assumption is that no external signals are present. In this respect, only since the sixties and seventies of the last century that a mathematical theory has been developed in order to cope with (boundary) control and observation. This fact, combined with the increasing computation power provided by computers, makes it possible to study practical engineering problems modelled by PDEs, such as controlling the water level in a river, or estimating the temperature distribution in a room. Moreover, by introducing inputs and outputs, the distributed parameter system is no longer a "closed" system since it can be easily interconnected with other (sub-)systems.

From a physical point of view and with the bond-graph modelling formalism [1] in mind, the interaction between different systems can be interpreted as an exchange of energy through a set of well-defined power ports. Port-Hamiltonian systems [2], [3] have been introduced about twenty years ago as the mathematical formalisation of bond-graphs to describe lumped parameter physical systems in an unified manner, [4]. For this class of systems, the dynamic results from the power conserving interconnection of a limited set of components, each characterised by a particular "energetic behaviour," i.e. storage, dissipation, generation and conversion. The generalisation to the infinite dimensional scenario leads to the definition of distributed port-Hamiltonian systems [5] that have been introduced about one decade ago, and that have proved to represent a powerful framework for modelling, simulation and control physical systems described by PDEs. Distributed port-Hamiltonian systems share analogous geometric properties with their finite dimensional counterpart, and they are treated in the proposed MTNS 2014 mini-course with title *Geometric Structures for the modelling, analysis and discretization of infinite-dimensional port-Hamiltonian Systems* by the same organisers of this one. Moreover, also the development of stabilising control laws follows the same rationale of the lumped parameter case. Since in most of the cases the Hamiltonian is the total energy of the system, stabilising the system could be done by driving the Hamiltonian to zero. As a consequence, having such a physical quantity at our disposal simplifies the controller design considerably.

Most of the current research on the stability and stabilisation of distributed port-Hamiltonian systems deals with the development of boundary controllers. For example, in [6]–[11], this task has been accomplished by looking at, or generating, a set of Casimir functions in the closed-loop system that robustly (i.e., independently from the Hamiltonian function) relates the state of the infinite dimensional port-Hamiltonian system with the state of the controller. The controller is a finite dimensional port-Hamiltonian system which is interconnected to the boundary of the distributed parameter system. The shape of the closed-loop energy function is changed by choosing the Hamiltonian of the controller e.g. to introduce a minimum in a desired configuration. This procedure is the generalisation of the control by interconnection via Casimir generation developed for finite dimensional systems, [3], [12]. The result is an energy-balancing passivity-based controller that is not able to deal with equilibria that require an infinite amount of supplied energy in steady state, i.e. with the so-called "dissipation obstacle."

The limits of the energy-Casimir method are intrinsic, and due to the fact that Casimir functions are invariants that do not depend on the particular Hamiltonian, i.e. they are completely determined by the Dirac structure of the system, [3], [9]. Main advantage, however, is that it provides a constructive way to develop the feedback law, and to choose the closed-loop Hamiltonian with desired stability properties. Moreover, this approach is able to provide a control action without explicitly dealing with the trajectories

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of open and closed-loop systems. On the other hand, this is also the main problem of the method, since the controller is developed somehow independently from the "real" evolution of the state, which is in fact a "consequence" of a specific Hamiltonian.

In *all* the first works on the boundary stabilisation of infinite dimensional port-Hamiltonian systems, several interesting and fundamental problems were not investigated. Among them, the well-posedness of the system of PDEs associated to the port-Hamiltonian systems, the selection of inputs and outputs for having a well-defined boundary control system, and even the proof of asymptotic stability of the proposed control laws are worth of mentioning. Roughly speaking, the connection between distributed port-Hamiltonian systems and classical functional analysis tools for infinite dimensional systems was not established yet. One of the first attempt in this direction has been [13], where the problem of existence of solutions for the associated system of PDEs, and of the selection of the boundary conditions in order to have a well-defined boundary control system in the sense of e.g. [14] has been solved in case of linear, distributed port-Hamiltonian systems with one-dimensional spatial domain. The latest results that combine abstract functional analytical approach with the physical approach of port-Hamiltonian system theory have been collected in [15], in which easily verifiable conditions for well-posedness and stability are given. This is the starting point of the proposed mini-course.

With the framework proposed in [13], [15] in mind, this mini-course is then focused on recent advances in the stability analysis and stabilisation via boundary control action of distributed port-Hamiltonian systems. Since the basic idea for having asymptotic stability in closed-loop is to interconnect a controller that dissipates the internal energy of the system, a first approach is to use an algebraic control action that mimics the behaviour of a sort of generalised resistance or damper. In other words, asymptotic stability is obtained via damping injection. In case of linear, distributed port-Hamiltonian systems, this problem has been studied e.g. in [16], [17].

On the other hand, the energy-Casimir method discussed before provides a constructive way for developing a dynamical control system that is able to shape the open-loop Hamiltonian to obtain desired stability properties in closed-loop, namely to introduce a (possibly) isolated minimum in a desired equilibrium configuration. Then, asymptotic stability is obtained via damping injection. Clearly, everything works only if the distributed port-Hamiltonian system with this dynamical extension, which is not strictly required to be a control system only, is well-posed, i.e. if the associated system of PDEs, ODEs and algebraic constraints admits a unique solution. Some results in this direction have been discussed in [16], [18], [19] with reference to linear positive real controllers, while the energy-Casimir method has been revised in light of these new advances in [20]–[22].

As stated before, the energy-Casimir method is not able to stabilise equilibra that require a non-zero supplied power by the controller. In other words, the resulting regulator is an energy-balancing control system. The class of controllers can be enlarged beyond the dissipation obstacle by focusing on the trajectories that correspond to a particular Hamiltonian, rather than on the geometric structure (i.e., the Dirac structure), of the system only. Then, the regulator is developed to map the open-loop trajectories into the trajectories of a target system with (at least) a different Hamiltonian and, clearly, characterized by the desired stability properties. This is the same concept adopted for finite dimensional in case of stabilization with state-modulated sources [12], or with the more general IDA-PBC control technique, [23].

With this in mind, it is clear that in case of boundary control of distributed port-Hamiltonian systems, the first issue is to understand the effects of the inputs on the state evolution. Without leaving the general framework proposed in [13], [15], and by taking advantage from the geometric structure associated to a port-Hamiltonian system, i.e. its Dirac structure, that is present also in the distributed parameter case, it is possible to easily "map" the inputs into the system dynamics. It is well-known, in fact, that in finite dimensions the Dirac structure provides an elegant way to parametrize the system dynamics once inputs, resistive structure, and Hamiltonian have been fixed, [24]. Furthermore, as proposed in [25] for implicit port-Hamiltonian systems (i.e., port-Hamiltonian systems described by DAEs), the control synthesis can take advantage from this parametrization: energy-balancing control, and control by state-modulated source are applied to this class of systems, and the solution explicitly determined on the basis of geometric and energetic properties. As far as the distributed port-Hamiltonian systems are concerned, the starting point is the definition of Dirac structures on Hilbert spaces proposed in [26], and in particular their kernel and image representations, [27]. These topics will be discussed in the other MTNS 2014 mini-course *Geometric Structures for the modelling, analysis and discretization of infinite-dimensional port-Hamiltonian Systems.* Instead, in this mini-course, it is shown how these representation are successfully used to determine how to map the open-loop dynamics into a desired one, which is associated to a distributed port-Hamiltonian systems with the same geometric structure (Dirac structure) of the original one, but with at least a different Hamiltonian. The resulting control action can be interpreted as a state-modulated (boundary) source. Some initial related results can be found e.g. in [28], [29]. Note that with this approach the geometry of a port-Hamiltonian system is going to play a central role also in the control development.

II. Speakers

The following speakers have confirmed they willingness to actively contribute in this mini-course. They are some of the most active researcher in the field, as it can be immediately checked by looking for their names in the next Reference section of this proposal. Their names are reported here, together with a very short biography.

- **Björn Augner** is research assistant at the Faculty of Mathematics and Natural Sciences, University of Wuppertal, Germany, and he is part of the working group of functional analysis. He is presenting a joint work developed with **Birgit Jacob**, full professor at the same university. She is also the co-author of *Linear Port-Hamiltonian Systems on Infinite Dimensional Spaces*, Birkhäuser, 2012, with H. Zwart, [15].

- **Yann Le Gorrec** is professor at Département Automatique et Systèmes Micro-Mécatroniques of FEMTO-ST (Franche-Comté Electronique Mécanique Thermique et Optique Sciences et Technologies) Insititute, France. He worked on modelling of physico-chemical processes, robust control, modelling and control of distributed parameter systems in port-Hamiltonian form. As far as the latter topic is concerned, actually, his interests are in the stability and stabilisation of linear and nonlinear distributed port-Hamiltonian systems, with applications to smart material based actuators, to distributed micro systems and more generally to micro actuators.
- **Alessandro Macchelli** is assistant professor at the Department of Electrical, Electronic and Information Engineering of the University of Bologna, Italy. He has been working since his PhD studies on the control of distributed port-Hamiltonian systems, with more emphasis on the extension of energy-based techniques developed for finite dimensional systems, to the distributed parameter scenario.
- **Hector Ramirez** is assistant professor at the University of Franche-Comté in Besançon and researcher at (AS2M) FEMTO-ST, France. During his PhD studies he worked on modelling and control of irreversible systems in port-Hamiltonian form. Actually, his research interests are in the fields of nonlinear control, passivity based control, port-Hamiltonian systems and distributed parameter systems.
- **Hans Zwart** is full professor at the Department of Applied Mathematics, University of Twente, The Netherlands. His research interest lies in the area of distributed parameter systems. He is the co-author *An Introduction to Linear Infinite-Dimensional System Theory,* Springer Verlag, 1995, with R.F. Curtain, [14], and of *Linear Port-Hamiltonian Systems on Infinite Dimensional Spaces*, Birkhäuser, 2012, with B. Jacob, [15]. Current research topics include controller design for UV-disinfection, controllability and observability of abstract linear systems, and system theoretic properties for systems with a Hamiltonian dynamics.

III. Schedule

The mini-course consists of the following five talks of 25 minutes each:

- **Talk 1:** "An introduction to boundary-controlled port-Hamiltonian systems and their stabilisation," by Yann Le Gorrec;
- **Talk 2:** "Basic results on stability of port-Hamiltonian systems," by Hans Zwart;
- **Talk 3:** "Exponential stabilisation via static and dynamic boundary control," by Björn Augner;
- **Talk 4:** "Energy-based control of distributed port-Hamiltonian systems," by Alessandro Macchelli;
- **Talk 5:** "Modeling and stabilisation of a class of nanotweezers using boundary-controlled port-Hamiltonian systems," by Hector Ramirez.

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