# "System identification in the behavioral setting: A low-rank approximation approach"


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**Format** 1 block of 100 minutes, split into 3 talks $(50 + 25 + 25)$


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## Abstract


Established data modeling approaches are often derived in a stochastic setting. An alternative deterministic approximation approach, known in the systems and control literature as the behavioral approach, has been developed since the 80's by Jan C. Willems and co-workers. The behavioral approach differentiates between the abstract notion of a model and the concrete notion of a model representation. This distinction proves to be important for developing a coherent theory and effective algorithms for system identification, analysis, and control. The mini-course presents a behavioral approach to system identification.

The highlight of the mini-course is the low-rank approximation problem, which is a practical tool for modeling in the behavioral setting. A matrix constructed from the data being rank deficient implies that there is an exact low complexity linear model for the data. Moreover, the rank of the data matrix corresponds to the complexity of the model. In the generic case when an exact low-complexity model does not exist, the aim is to find a model that fits the data approximately. The corresponding computational problem is low-rank approximation. In the case of linear time-invariant dynamical models, the data matrix is, in addition, Hankel structured and the approximation should have the same structure.

Once the approximate system identification problem is formulated as a low-rank approximation problem, it is solved by generic methods. Except for a few special cases, however, low-rank approximation problems are nonconvex and a global solution is expensive to compute. In the mini-course, we present methods based on local optimization, which lead to fast and effective algorithms. The cost function evaluation has the system theoretic interpretation of Kalman smoothing.

In addition to the theory and algorithms for exact and approximate system identification, the mini-course presents examples from system theory (model reduction and distance to uncontrollability), computer algebra (approximate common divisor computation), and machine learning (recommender systems). Software implementation of the developed methods makes the theory applicable in practice.

## Overview of the talks

1. *Low-rank approximation approach to system identification* (50 minutes, given by I. Markovsky)

   Approximate system identification is a trade-off between model complexity and model accuracy. One particular scalarization of this biobjective problem is the low-rank approximation problem. Fundamentally different ways of measuring the model accuracy are the misfit and latency. Misfit leads to errors-in-variables problems and latency leads to the prediction error problems. Independent of the particular problem formulation, there are three conceptually different approaches for solving the problem: convex relaxations, local optimization, and global optimization. In all approaches, first, a representation of the model is chosen which leads to a corresponding parameter optimization problem. Details about two different methods for local optimization, based on kernel and image representation, are presented in Talk 2 and Talk 3, respectively.

2. *Kernel representation approach to structured low-rank approximation* (25 minutes, given by K. Usevich)

   The second talk of the mini-course presents an approach for structured low-rank approximation based on the variable projection method for the kernel representation of the rank constraint. The bilinear structure of the problem is effectively used to eliminate a part of the optimization variables. The computation of the cost function has a system theoretic interpretation of Kalman smoothing. Exploiting the matrix structure, the evaluation of the cost function and its derivatives has linear complexity in the number of data points. The remaining problem is a nonlinear least squares optimization problem. Due to invariance of the cost function with respect to a change of the kernel basis, the problem is an optimization problem on a Grassmann manifold.

3. *Factorization approach for structured low-rank approximation* (25 minutes, given by M. Ishteva)

   In the third talk of the mini-course, the low-rank constraint is imposed by representing the approximation matrix as a product of two factors with reduced dimension. The matrix structure is enforced by introducing a penalty term in the cost function. The proposed local optimization algorithm can, in addition, deal with missing and fixed elements. The proposed algorithm has an advantage over the kernel approach, in the case of small targeted rank. We compare the image and kernel approaches on numerical examples of system identification, approximate greatest common divisor problem, and symmetric tensor decomposition.

## References

The mini-course is based on the following references:


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- M. Ishteva, K. Usevich, and I. Markovsky. Regularized structured low-rank approximation. Technical report, Vrije Univ. Brussel, 2013. Submitted on 02/08/2013 to *SIAM J. Matrix Anal. Appl.*